National Qualifications 2025

X847/77/11

Mathematics
Paper 1 (Non-calculator)

MONDAY, 12 MAY

9:00 AM – 10:00 AM

Total marks — 35

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

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FORMULAE LIST

Standard derivatives	
$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$

(Geometric series) $S_n = \frac{a\left(1-r^n\right)}{1-r},\ r \neq 1$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}, \quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \text{ where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \ldots$$

FORMULAE LIST (continued)

De Moivre's theorem

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

[Turn over

MARKS

Total marks — 35

Attempt ALL questions

1. Use the binomial theorem to expand $\left(\frac{1}{x}-3x\right)^4$.
Simplify your answer. **4**

2. Given $f(x)=\frac{2x^3+x}{3+2x}$, find $f'(x)$. Simplify your answer. **3**

3. Two complex numbers are defined as $z=11+10i$ and $w=3-2i$.
Find $\frac{z}{w}$ in the form $a+bi$, where $a,\ b\in\mathbb{R}$. **2**

4. Matrices A and B are defined by $A=\begin{pmatrix}-3 & 2\\ 0 & 1\end{pmatrix}$ and $B=\begin{pmatrix}2 & 2\\ 5 & \lambda\end{pmatrix}$ where $\lambda\in\mathbb{R}$.

(a) Find $3A+2B$. **1**

(b) (i) Find $A'B$, where A' is the transpose of A. **2**

(ii) Find an expression for the determinant of $A'B$. **1**

(iii) Determine the value of λ such that $A'B$ is singular. **2**

5. Find $\int \frac{1}{1+4x^2}\,dx$. **2**

MARKS

6. On a suitable domain a curve is given by the equation $y = f(x)$, where

$f(x) = \dfrac{x^2 + x + 5}{x - 2}$.

(a) Express $f(x)$ in the form $Ax + B + \dfrac{C}{x-2}$, where A, B and C are constants. **2**

(b) State the equations of the vertical and non-vertical asymptotes of the curve. **2**

7. Solve the differential equation

$$\frac{dy}{dx} = \frac{y}{2x-1},\ x, y > 1,$$

given that $y = 12$ when $x = 5$. Express y in terms of x. **5**

8. Three planes are defined by

$\pi_1 :\quad x - y + 4z = -6$

$\pi_2 :\ 2x + 3y + z = 15$

$\pi_3 :\ 3x + 2y - 2z = 16$

(a) Use Gaussian elimination to find T, the point of intersection of the three planes. **4**

The line L_1 is defined by $\dfrac{x+4}{3} = \dfrac{y-7}{2} = \dfrac{z-4}{1}$.

(b) Find P, the point of intersection of the line L_1 and the plane π_3. **3**

The line L_2 passes through points T and P.

(c) Find, in parametric form, the equations of the line L_2. **2**

[END OF QUESTION PAPER]

AH National Qualifications 2025

X847/77/12

Mathematics Paper 2

MONDAY, 12 MAY

10:30 AM – 1:00 PM

Total marks — 80

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

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X847712

FORMULAE LIST

Standard derivatives	
$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
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$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$

(Geometric series) $S_n = \frac{a\left(1-r^n\right)}{1-r},\ r \neq 1$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2},\quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6},\quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \text{ where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \ldots$$

FORMULAE LIST (continued)

De Moivre's theorem

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

[Turn over

MARKS

Total marks — 80

Attempt ALL questions

1. A function is defined by $f(x) = \cos^{-1} 4x$.

Find $f'(x)$. **2**

2. A curve is defined by the equation $2y^2 + 4xe^{2y} = 3x$.

Find an expression for $\frac{dy}{dx}$ in terms of x and y. **3**

3. Express $\frac{2x^2 - 18x + 4}{(x-1)(x-3)(x+5)}$ in partial fractions. **3**

4. (a) Use the Euclidean algorithm to find d, the greatest common divisor of 1118 and 416. **1**

(b) Hence find integers a and b such that $1118a + 416b = d$. **2**

5. A curve is defined by $y = x^{\cot x}$.

Use logarithmic differentiation to find $\frac{dy}{dx}$.

Write your answer in terms of x. **5**

6. (a) Find and simplify the Maclaurin expansion, up to and including the term in x^4, for $\cos 3x$. **2**

(b) Hence find and simplify the Maclaurin expansion, up to and including the term in x^4, for $\cos^2 3x$. **2**

MARKS

7. A curve is defined on a suitable domain by the equations $x = t^2$ and $y = \tan t$.

Find in terms of t:

(a) $\dfrac{dy}{dx}$ **2**

(b) $\dfrac{d^2y}{dx^2}$. **3**

8. The matrix A has the following property:

$$A^2 = 6A - I, \text{ where } I \text{ is the identity matrix.}$$

(a) Express A^3 in the form $pA + qI$, where $p, q \in \mathbb{R}$. **2**

Matrix A is non-singular.

(b) Find a similar expression for A^{-1} in terms of A and I. **2**

9. Relative to a fixed origin, the velocity, v metres per second, of an object at time t seconds is given by $v = 2t + e^{5t}$.

(a) Find an expression for the displacement of the object, s metres, in terms of t, given that when $t = 0$, $s = 0$. **2**

(b) Show that the acceleration of the object is always positive. **2**

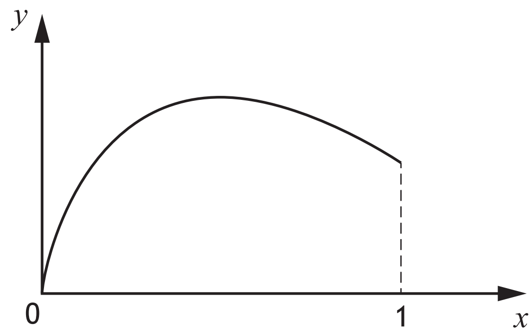
10. Find and fully factorise an expression for $\sum_{r=1}^{n}\left(r^3 - 3r\right)$. **2**

[Turn over

MARKS

11. (a) Using the substitution $u = 2x^2$, or otherwise, find $\int xe^{-2x^2}\,dx$. **2**

The diagram shows part of the curve with equation $y = \dfrac{4\sqrt{x}}{e^{x^2}}$.



A solid is generated by rotating the curve through 2π radians about the x-axis from $x = 0$ to $x = 1$.

(b) Calculate the exact value of the volume generated. **3**

12. Solve the differential equation

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 15x^2 - 31x + 40$$

given that $y = 4$ and $\dfrac{dy}{dx} = 13$ when $x = 0$. **9**

MARKS

13. An infinite geometric sequence of positive numbers has second term 100 and fourth term 16.

(a) Determine:

(i) the common ratio **2**

(ii) the first term of this sequence. **1**

(b) Explain why the associated geometric series has a sum to infinity. **1**

(c) Determine this sum to infinity. **1**

A new geometric sequence is formed by multiplying each term in the sequence above by the real number k, where $k \neq 0$.

(d) State the effect that this will have on:

(i) the common ratio **1**

(ii) the sum to infinity of the associated series. **1**

14. Find the general solution of the differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \sec^2 3x.$$

Give your answer in the form $y = f(x)$. **5**

15. Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{1}{(2r+1)(2r-1)} = \frac{n}{2n+1}.$$ **5**

[Turn over

MARKS

16. Use integration by parts to find

$$\int e^{2x} \sin 5x \, dx.$$ **5**

17. The volume, V cm^3, of water in a tank is given by

$$V = \frac{1}{5}h^3, \text{ where } h \text{ cm is the depth of water in the tank.}$$

Water is being piped into the tank at a rate of 6 cm^3/second.

Water is leaking from the bottom of the tank at a rate of $\frac{1}{10}\sqrt{h}$ cm^3/second.

Calculate the rate of change of the depth of water when $h = 400$. **4**

18. Let $z = x + iy$ be a complex number, where $x, y \in \mathbb{R}$.

(a) (i) Express $\overline{z} + iz$ in Cartesian form, where $\overline{z}$ is the complex conjugate of z. **2**

(ii) Given $x > y$, find the argument of $\overline{z} + iz$. **1**

When $x < y$, $\overline{z} + iz = r\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$ where r is the modulus of $\overline{z} + iz$.

(b) Use de Moivre's theorem to find, in polar form, both square roots of $\overline{z} + iz$. **2**

[END OF QUESTION PAPER]