

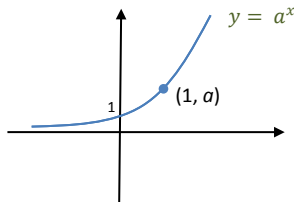
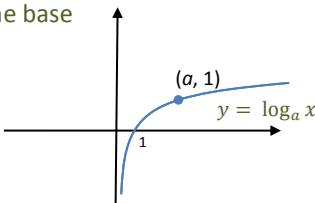
# ZETA MATHS

## Higher Mathematics Revision Checklist

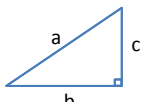
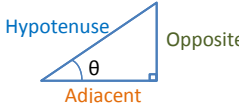
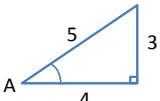
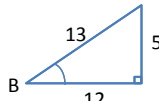
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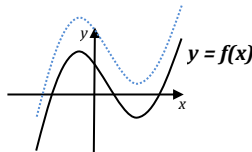
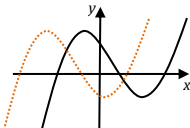
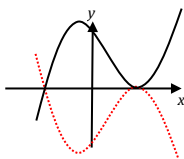
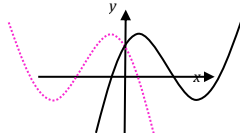
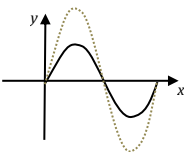
# Expressions & Functions

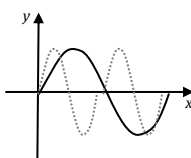
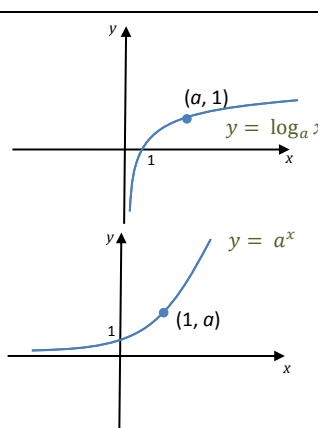
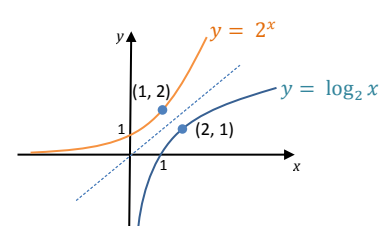
Topic	Skills	Notes			
<b>Logs and Exponentials</b>					
<b>Prior Skills</b>					
Equation of a Line	Know and use $y = mx + c$ to determine the equation of a line				
<b>New Skills</b>					
Exponential Functions	<p>An exponential function is written in the form <math>y = a^x</math> where <math>a</math> is the base, and <math>x</math> is the index or exponent</p> 				
The Logarithmic Function	<p>The logarithmic function is the inverse of the exponential function. It is written as <math>y = \log_a x</math> where <math>a</math> is the base</p> <p><b>NB:</b> On your calculator the <i>log</i> button is <math>\log_{10} x</math></p> 				
Convert Between Logarithmic and Exponential Form	<p>If <math>y = a^x</math> then <math>x = \log_a y</math></p> <p>e.g. <math>3 = \log_a 8</math>  <math>a^3 = 8</math>  <math>a = 2</math></p>				
The Exponential Function	The exponential function is written as $y = e^x$ where $e$ is the base, which is approximately 2.718				
Natural Logarithms	The natural log function is the inverse of the exponential function $y = e^x$ , it is written as $y = \ln x$ , which means $y = \log_e x$				
Laws of Logs	<ul style="list-style-type: none"> <li><math>\log_a xy = \log_a x + \log_a y</math>  e.g. <math>\log_4 8 + \log_4 2 = \log_4 (8 \times 2) = \log_4 16 = 2</math> (since <math>4^2 = 16</math>)</li> <li><math>\log_a \frac{x}{y} = \log_a x - \log_a y</math>  e.g. <math>\log_4 8 - \log_4 2 = \log_4 \frac{8}{2} = \log_4 4 = 1</math> (since <math>4^1 = 4</math>)</li> <li><math>\log_a x^n = n \log_a x</math>  e.g. <math>\frac{1}{3} \log_9 27 = \log_9 27^{\frac{1}{3}} = \log_9 3 = \frac{1}{2}</math> (since <math>9^{\frac{1}{2}} = 3</math>)</li> <li><math>\log_a a = 1</math>  e.g. <math>\log_5 5 = 1</math> (since <math>5^1 = 5</math>)</li> </ul>				
Use the Laws of Logs to Solve Log Equations	<p><b>Ex 1.</b> Solve: <math>\log_5 (x + 1) + \log_5 (x - 3) = 1</math></p> <p><b>Soln.</b> <math>\log_5 (x + 1)(x - 3) = 1</math> (using first law)  <math>(x + 1)(x - 3) = 5</math> (since <math>5^1 = 5</math>)  <math>x^2 - 2x - 3 = 5</math> (solve for <math>x</math>)  <math>x^2 - 2x - 8 = 0</math>  <math>(x + 2)(x - 4) = 0 \therefore x = -2, x = 4</math></p>				

Topic	Skills	Notes			
	<p><b>Ex 2.</b> Find <math>x</math> if <math>4 \log_x 6 - 2 \log_x 4 = 1</math></p> <p><b>Soln.</b> <math>\log_x 6^4 - \log_x 4^2 = 1</math></p> $\log_x \frac{6^4}{4^2} = 1$ $\frac{6^4}{4^2} = x$ $x = \frac{2^4 \times 3^4}{2^4} \quad (\text{since } 6^4 = 2^4 \times 3^4 \text{ and } 4^2 = 16 = 2^4)$ $x = 3^4$ $x = 81$				
Use Laws of Logs to Solve Exponential Growth or Decay Problems	<ul style="list-style-type: none"> <li>For finding an initial value; substitute given values in to equation to determine the initial value</li> <li>For finding a half-life, make the equation equal to one half</li> </ul> <p><b>e.g.</b> In the equation, where <math>A</math> represents micrograms of a radioactive substance remaining over time <math>t</math>. Find:</p> <p>(a) the initial value if there are 500 microgram after 100 years</p> <p>(b) the half-life of the substance</p> <p>(a) <math>A_t = A_0 e^{-0.004t}</math></p> $500 = A_0 e^{-0.004 \times 100}$ $500 = 0.67 A_0$ $A_0 = 746 \text{ micograms}$ <p>(b) <math>373 = 746 e^{-0.004t}</math></p> $\frac{1}{2} = e^{-0.004t}$ $\ln \frac{1}{2} = \ln e^{-0.004t}$ $-0.004t = \ln \frac{1}{2}$ $t = 173 \text{ years}$				
Formulae for Experimental Data	<p>In experimental data questions, two types of exponential functions are considered, <math>y = kx^n</math> and <math>y = ab^x</math></p> $y = kx^n$ <p>Taking logs of boths sides, this equation may be expressed as <math>\log y = n \log x + \log k</math>. To find the unknown values <math>n</math> and <math>k</math>:</p> <ul style="list-style-type: none"> <li>If the data given is <math>x</math> and <math>y</math> data, then take logs of two sets of the data for <math>x</math> and <math>y</math> and form a new table with <math>\log x</math> and <math>\log y</math></li> <li>Substitute new values into <math>\log y = n \log x + \log k</math> and solve simultaneously to find values for <math>n</math> and <math>\log k</math></li> <li>Find <math>k</math> by solving <math>\log k</math></li> <li>Write <math>y = kx^n</math> with values of <math>k</math> and <math>n</math></li> </ul> $y = ab^x$ <p>Taking logs of boths sides, this equation may be expressed as <math>\log y = x \log b + \log a</math>. To find the unknown values <math>a</math> and <math>b</math>:</p> <ul style="list-style-type: none"> <li>If the data given is <math>x</math> and <math>y</math> data, then take logs of the data for <math>y</math></li> <li>Substitute values into <math>\log y = x \log b + \log a</math> and solve simultaneously to find values for <math>\log a</math> and <math>\log b</math></li> <li>Find <math>a</math> and <math>b</math> by solving <math>\log a</math> and <math>\log b</math></li> <li>Write <math>y = ab^x</math> with values of <math>a</math> and <math>b</math></li> </ul>				
Sketch the Graph of the Inverse Function of a Log or Exponential Function	See <i>Graphs of Functions</i>				

Topic	Skills	Notes			
<b>Addition Formulae</b>					
<b>Prior skills</b>					
Pythagoras Theorem	$a^2 = b^2 + c^2$ 				
SOHCAHTOA	$\sin \theta = \frac{Opp}{Hyp}$ , $\cos \theta = \frac{Adj}{Hyp}$ , $\tan \theta = \frac{Opp}{Adj}$ 				
Solve Trig Equations	Use the CAST diagram or graphical method to solve equations (see <i>Relationships</i> in National 5 checklist)				
Exact Values	See <i>Trigonometry</i>				
Convert from Degrees to Radians and Vice-Versa	See <i>Trigonometry</i>				
<b>New Skills</b>					
Use Exact Values to Calculate Related Obtuse Angles	<p><b>Ex 1.</b> Find the exact value of <math>\cos 225^\circ</math></p> <p><b>Soln.</b> The related acute angle is <math>45^\circ</math> since <math>180^\circ + 45^\circ = 225^\circ</math>  From the graph or CAST diagram <math>\cos 225^\circ</math> is negative.  <math>\therefore \cos 225 = -\cos 45 = -\frac{1}{\sqrt{2}}</math></p> <p><b>Ex 2.</b> Find the exact value of <math>\sin \frac{2\pi}{3}</math></p> <p><b>Soln.</b> The related acute angle is <math>\frac{\pi}{3}</math> since <math>\pi - \frac{\pi}{3} = \frac{2\pi}{3}</math>  From the graph or CAST diagram <math>\sin \frac{2\pi}{3}</math> is positive.  <math>\therefore \sin \frac{2\pi}{3} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}</math></p>				
Use Addition Formulae to Expand Expressions	$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ <b>NB:</b> For sin functions the signs are the same, for cos functions the signs are different				
Use Addition Formulae to Evaluate Exact Values of Expressions	<p><b>Ex 1.</b> Find the exact value of <math>\cos 75^\circ</math></p> <p><b>Soln.</b> <math>\cos 75^\circ = \cos(45 + 30)^\circ</math> (use addition formulae expansion)  <math>= \cos 45 \cos 30 - \sin 45 \sin 30</math> (use exact values)  <math>= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}</math>  <math>= \frac{\sqrt{3} - 1}{2\sqrt{2}}</math></p> <p><b>Ex 2.</b> Given <math>\sin A = \frac{3}{5}</math> and <math>\cos B = \frac{12}{13}</math>, show that <math>\sin(A + B) = \frac{56}{65}</math></p> <p><b>Soln.</b> Use SOHCAHTOA to sketch triangles from the info given and use Pythagoras to find unknowns</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p>Expand using Addition Formulae  <math>\sin(A + B) = \sin A \cos B + \cos A \sin B</math>  <math>= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13}</math>  <math>= \frac{56}{65}</math></p>				

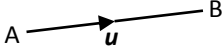
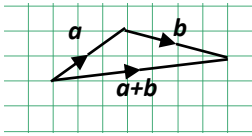
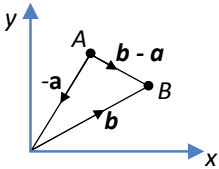
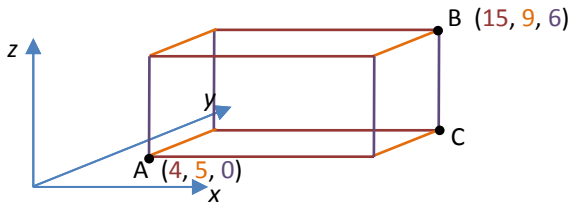
Topic	Skills	Notes			
Solve Trig Equations using Trig Identities	<ul style="list-style-type: none"> <li>Determine which part of the equation has a related identity</li> <li>Replace trigonometric term with related identity and solve</li> </ul> <p>e.g. Solve <math>\sin 2x + \sin x = 0</math>, for <math>0 \leq x \leq 180^\circ</math></p> <p><b>Soln.</b> As <math>\sin 2x = 2\sin x \cos x</math>, then</p> $2 \sin x \cos x + \sin x = 0, \text{ solve by factorising}$ <p>Factorise <math>\sin x (2\cos x + 1) = 0</math></p> $\sin x = 0 \qquad 2\cos x + 1 = 0$ <p>(using graph) <math>\cos x = -\frac{1}{2}</math> (using CAST)</p> $x = 0^\circ, 180^\circ, 360^\circ \qquad x_A = 60^\circ$ $\qquad \qquad \qquad x = 120^\circ, 240^\circ$ $\therefore x = 0^\circ, 120^\circ, 180^\circ$				
<b>Wave Function</b>					
<b>Prior Skills</b>					
Solve Trig Equations	Use the CAST diagram or graphical method to solve equations (see <i>Relationships</i> in National 5 checklist)				
Exact Values	See <i>Trigonometry</i>				
Use Addition Formulae to Expand Expressions	See <i>Addition Formulae</i>				
<b>New Skills</b>					
Write an Expression Form $k \sin(x \pm \alpha)$ or $k \cos(x \pm \alpha)$	<p><math>a \cos x + b \sin x</math> can be written in one of the following forms:</p> $k \sin(x + \alpha)$ $k \sin(x - \alpha)$ $k \cos(x + \alpha)$ $k \cos(x - \alpha)$ <p>Where <math>k = \sqrt{a^2 + b^2}</math> and <math>\tan \alpha</math> is derived from <math>a</math> and <math>b</math></p> <p><b>Ex 1.</b> Express <math>\sqrt{3} \sin x + \cos x</math> in the form <math>k \sin(x + \alpha)^\circ</math> where <math>k &gt; 0</math> and <math>0 \leq x \leq 360^\circ</math></p> <p><b>Soln.</b> <math>k \sin(x + \alpha) = k(\sin x \cos \alpha + \cos x \sin \alpha)</math> (Expand)</p> $= k \cos \alpha \sin x + k \sin \alpha \cos x$ $= \sqrt{3} \sin x + \cos x$ $\therefore k \cos \alpha = \sqrt{3} \text{ and } k \sin \alpha = 1$ <p>To find <math>k</math>: <math>k = \sqrt{3 + 1} = 2</math></p> <p>To find <math>\alpha</math>: <math>\tan \alpha = \frac{k \sin \alpha}{k \cos \alpha}</math></p> $\tan \alpha = \frac{1}{\sqrt{3}} \quad (\text{use exact values})$ $\alpha = 30^\circ$ <p>(NB: <math>k \sin \alpha</math> and <math>k \cos \alpha</math> are both positive, therefore the angle is in quadrant 1, i.e. less than <math>90^\circ</math>)</p> $\therefore \sqrt{3} \sin x + \cos x = 2 \sin(x + 30)^\circ$				

Topic	Skills	Notes			
	<p><b>Ex 2.</b> Express <math>8 \cos x - 6 \sin x</math> in the form <math>k \cos(x + \alpha)^\circ</math> where <math>k &gt; 0</math> and <math>0 \leq x \leq 360^\circ</math></p> <p><b>Soln.</b> <math>k \cos(x + \alpha) = k(\cos x \cos \alpha - \sin x \sin \alpha)</math> (Expand)</p> <p><math>= k \cos \alpha \cos x - k \sin \alpha \sin x</math></p> <p><math>= 8 \cos x - 6 \sin x</math></p> <p><math>\therefore k \cos \alpha = 8</math> and <math>k \sin \alpha = 6</math></p> <p>To find <math>k</math>: <math>k = \sqrt{8^2 + 6^2} = 10</math></p> <p>To find <math>\alpha</math>: <math>\tan \alpha = \frac{k \sin \alpha}{k \cos \alpha}</math></p> <p><math>\tan \alpha = \frac{6}{8}</math> (use calculator)</p> <p><math>\alpha = 36.9^\circ</math></p> <p><math>\therefore 8 \cos x - 6 \sin x = 10 \cos(x + 36.9)^\circ</math></p>				
Graphs of Functions					
Prior Skills					
Sketch Quadratics	Sketch quadratics of the form $y = kx^2$ and $y = a(x + b)^2 + c$				
Sketch Trig Graphs	Sketch graphs of the form $y = a \sin bx + c$ and $y = a \cos bx + c$				
New Skills					
Sketch Related Graphs	<p>Ensure all given coordinates are translated and marked on the new graph and axes and graphs are labelled</p> <p><b><math>y = f(x) + a</math></b> Graph moves up or down by <math>a</math> Up for <math>f(x) + a</math> Down for <math>f(x) - a</math></p> <p><b><math>y = f(x + a)</math></b> Graph moves left or right Left when <math>f(x + a)</math> Right for <math>f(x - a)</math></p> <p><b><math>y = -f(x)</math></b> Graph reflects in x-axis</p> <p><b><math>y = f(-x)</math></b> Graph reflects in y-axis</p> <p><b><math>y = kf(x)</math></b> Graph is stretched vertically for <math>k &gt; 1</math> Graph is squashed vertically for <math>0 &lt; k &lt; 1</math></p>	    			

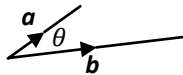
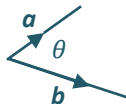
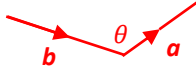
Topic	Skills	Notes			
	$y = f(kx)$ Graph is squashed horizontally for $k > 1$ Graph is stretched horizontally for $0 < k < 1$ 				
Sketch Log and Exponential Graphs	Log graphs of the form $y = \log_a x$ always cut the x-axis at the point (1, 0) and will pass through (a, 1)  Exponential graphs of the form $y = a^x$ always cut the y-axis at the point (0, 1) and will pass through (1, a)  All of the related graph transformations above apply to log and exponential graphs 				
Sketch the Graph of the Inverse Function of a Log or Exponential Function	The graph of an inverse function is reflected along the line $y = x$ . The logarithmic graph is the inverse of the exponential graph and vice-versa  <b>e.g.</b> For the graph of the function $y = 2^x$ the inverse function is $y = \log_2 x$ 				
Sketch a Trig Graph of the Form $y = k \sin(x \pm \alpha)$ or $y = k \cos(x \pm \alpha)$	See <i>Trigonometry – Sketch a Trig Graph from its Equation</i>				
<b>Sets and Functions</b>					
<b>Prior Skills</b>					
Identify the Turning Point of a Quadratic	From completed square form $y = a(x + b)^2 + c$ , turning point is $(-b, c)$				
<b>New Skills</b>					
Find Composite Functions	Composite functions consist of one function within another. <b>e.g.</b> If $f(x) = 3x - 2$ and $g(x) = x^2 - 4$ , find (a) $f(g(x))$ (b) $g(f(x))$ <b>Soln.</b> (a) $f(g(x)) = 3(x^2 - 4) - 2 = 3x^2 - 12 - 2 = 3x^2 - 14$ (b) $g(f(x)) = (3x - 2)^2 - 4 = 9x^2 - 12x + 4 - 4 = 9x^2 - 12x$				

Topic	Skills	Notes			
Evaluate Using Composite Functions	<p><b>e.g.</b> Find <math>H(-1)</math> where <math>H(x) = g(f(x))</math> and <math>f(x) = 3x - 2</math>, <math>g(x) = x^2 - 4</math></p> <p><b>Soln.</b></p> <p><i>Method 1:</i></p> $H(x) = g(f(x)) = \dots = 9x^2 - 12x \text{ (from example above)}$ $H(-1) = 9(-1)^2 - 12(-1) = 21$ <p><i>Method 2:</i></p> $f(-1) = 3(-1) - 2 = -5$ $g(-5) = (-5)^2 - 4 = 21$				
Determine a Suitable Domain of a Function	<p>Restrictions on the domain of a function occur in two instances at Higher Mathematics. A restriction will occur when a denominator is zero, which is undefined or when a square root is negative, which is non-real</p> <p><b>e.g.</b> For <math>f(x) = \frac{12x}{(4-x)^2}</math> and <math>x \in \mathbf{R}</math>, write a restriction on the domain of <math>f(x)</math></p> <p><b>Soln.</b> <math>x \neq 4</math> as this would make the denominator zero</p>				
State the Range of a Function	<p><b>e.g.</b> State the minimum turning point of the function <math>f(x) = x^2 + 5</math> and hence state the range of the function (see <i>prior skills</i>)</p> <p><b>Soln.</b> Minimum turning point is (0, 5) as the y-coordinate of the turning point is 5, the range of the function is <math>f(x) &gt; 5</math></p>				
Find an Inverse Function	<p>For a function <math>f(x)</math> there is an inverse function <math>f^{-1}(x)</math>, such that <math>f(f^{-1}(x)) = x</math></p> <p>To find an inverse function:</p> <ul style="list-style-type: none"> <li>Replace <math>x</math> with <math>y</math> in the function and <math>f(x)</math> with <math>x</math></li> <li>Change the subject to <math>y</math></li> </ul> <p><b>e.g.</b> For the function <math>f(x) = \frac{3}{4-x^2}</math> find the inverse function <math>f^{-1}(x)</math></p> <p><b>Soln.</b></p> $f(x) = \frac{3}{4-x^2}$ $x = \frac{3}{4-y^2}$ $4 - y^2 = \frac{3}{x}$ $4 - \frac{3}{x} = y^2$ $y = \sqrt{4 - \frac{3}{x}}$ $\therefore f^{-1}(x) = \sqrt{4 - \frac{3}{x}}$				
<b>Common Terms</b>					
Domain	The <i>domain</i> of a function is the set of numbers that can be input into the function (see <i>Determine a Suitable Domain</i> above)				
Range	The <i>range</i> of a function is what comes out of the function after the x-values have been put in (see <i>State the Range of a Function</i> )				
Number Sets	<p>There are five standard number sets to consider at Higher Mathematics</p> <ul style="list-style-type: none"> <li>The set of <i>natural numbers</i> <math>\mathbf{N}</math> (counting numbers) <math>\mathbf{N} = \{1, 2, 3, 4, \dots\}</math></li> <li>The set of <i>whole numbers</i> <math>\mathbf{W}</math> (the same as <i>natural numbers</i>, but inclusive of zero) <math>\mathbf{W} = \{0, 1, 2, 3, 4, \dots\}</math></li> <li>The set of <i>integers</i> <math>\mathbf{Z}</math> (the same as <i>whole numbers</i>, but inclusive of negative numbers) <math>\mathbf{Z} = \{\dots, -2, -1, 0, 1, 2, 3, 4, \dots\}</math></li> </ul>				

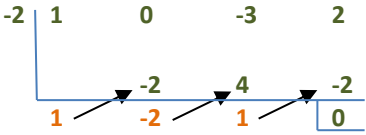
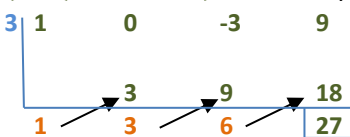
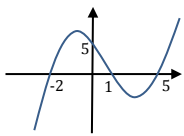


Topic	Skills	Notes			
	<ul style="list-style-type: none"> <li>The set of <i>rational numbers</i> <math>Q</math> (all numbers that can be written as fractions)</li> <li>The set of <i>real numbers</i> <math>R</math> (inclusive of both rational and irrational numbers i.e. <math>\pi, \sqrt{3}</math>, etc.)</li> </ul>				
<b>Vectors</b>					
<b>Prior Skills</b>					
Vector Notation	<p>Vectors can be named in one of two ways. Either by using the letters of the points at the end of the line segment <math>\overrightarrow{AB}</math> or by using a single letter in lower case u. When writing the lower case name, underline the letter</p> 				
2D Line Segments	<p>Add or subtract 2D line Segments</p> <ul style="list-style-type: none"> <li>Vectors end-to-end</li> <li>Arrows in same direction</li> </ul> 				
Finding a Vector from Two Coordinates	<p>Know that to find a vector between two points A and B then <math>\overrightarrow{AB} = b - a</math></p> <p><b>NB:</b> Vector notation for a vector between two points A and B is <math>\overrightarrow{AB}</math></p> 				
3D Vectors	<p>Determine coordinates of a point from a diagram representing a 3D object</p> <p>Look at difference in x, y and z axes individually  <b>e.g.</b> The cuboid is parallel to the x, y and z axes. Find the coordinates of C</p>  <p>C (15, 9, 0)</p>				
Position Vectors	<p>The position vector of a coordinate is the vector from the origin to the coordinate. <b>E.g.</b> A (4, -3) has the position vector <math>a = \begin{pmatrix} 4 \\ -3 \end{pmatrix}</math></p>				
The Zero Vector	<p><math>\begin{pmatrix} 0 \\ 0 \end{pmatrix}</math> is called the zero vector, written <b>0</b></p> $u + (-u) = 0$				
Add and Subtract Vector Components	<p>Add and Subtract 2D and 3D vector components.</p> $a = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} \text{ and } b = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} \quad a + b = \begin{pmatrix} 1+3 \\ 1+2 \\ 4+5 \end{pmatrix}$				
Multiply Vector Components	<p>Multiply vector components by a scalar</p> $2a = 2 \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 8 \end{pmatrix}$				

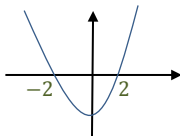

Topic	Skills	Notes			
Find the Magnitude of a vector	Find the magnitude of a 2D or 3D vector: For vector $\mathbf{u} = \begin{pmatrix} x \\ y \end{pmatrix}$ , $ \mathbf{u}  = \sqrt{x^2 + y^2}$ For vector $\mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ , $ \mathbf{v}  = \sqrt{x^2 + y^2 + z^2}$				
<b>New Skills</b>					
Writing Vectors	Vectors can be written in component form i.e. $\mathbf{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or in terms of $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ , where each of these represents the unit vector in the x, y and z direction. <b>e.g.</b> $\overrightarrow{AB} = 4\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ can be written as $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix}$				
Parallel Vectors	Vectors are parallel if one vector is a scalar multiple of the other <b>e.g.</b> $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 4 \\ 16 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ $\mathbf{b} = 4\mathbf{a} \therefore$ vectors are parallel				
The Unit Vector	For any vector, there is a parallel vector $\mathbf{u}$ of magnitude 1. This is called the unit vector <b>e.g.</b> Find the unit vector $\mathbf{u}$ parallel to vector $\mathbf{a} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ $ \mathbf{a}  = \sqrt{5^2 + 12^2} = 13$ $\therefore \mathbf{u} = \frac{1}{13} \begin{pmatrix} 5 \\ 12 \end{pmatrix} = \begin{pmatrix} 5/13 \\ 12/13 \end{pmatrix}$				
Collinearity	Points are said to be collinear if they line on the same line. To show points are collinear using vectors; show (a) they are parallel by demonstrating one vector is a scalar multiple of the other and (b) that they share a common point <b>e.g.</b> Show that A(-3, 4, 7), B(-1, 8, 3) and C(0, 10, 1) are collinear <b>Soln.</b> $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = \begin{pmatrix} 0 \\ 10 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\overrightarrow{AB} = 2\overrightarrow{BC}$ and point B is common $\therefore$ A, B and C are collinear				
Divide Vectors into a Given Ratio to Find an Unknown Point	<b>e.g.</b> P is the point (6, 3, 9) and R is (12, 6, 0). Find the coordinates of Q, such that Q divides PR in the ratio 2:1 <b>Soln.</b> $\frac{\overrightarrow{PQ}}{\overrightarrow{QR}} = \frac{2}{1}$ $\overrightarrow{PQ} = 2\overrightarrow{QR}$ $\mathbf{q} - \mathbf{p} = 2(\mathbf{r} - \mathbf{q})$ $\mathbf{q} - \mathbf{p} = 2\mathbf{r} - 2\mathbf{q}$ $3\mathbf{q} = 2\mathbf{r} + \mathbf{p}$ $3\mathbf{q} = 2 \begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 30 \\ 15 \\ 9 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} 10 \\ 5 \\ 3 \end{pmatrix} \therefore Q(10, 5, 3)$ <b>NB:</b> this could also be calculated using section formula				

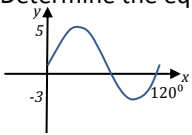
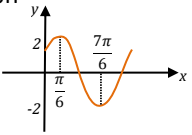
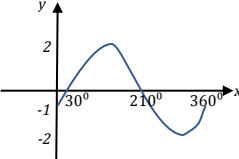
Topic	Skills	Notes			
Find the Ratio in which a Point Divides a Line Segment	<p><b>e.g.</b> A(-2, -1, 4), B(1, 5, 7) and C(7, 17, 13) are collinear. What is the ratio in which B divides AC?</p> <p><b>Soln.</b> <math>\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}</math></p> $\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = \begin{pmatrix} 7 \\ 17 \\ 13 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 6 \\ 12 \\ 6 \end{pmatrix} = 2 \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix} = 2\overrightarrow{AB}$ $2\overrightarrow{AB} = \overrightarrow{BC}$ $\frac{\overrightarrow{AB}}{\overrightarrow{BC}} = \frac{1}{2}$ $\therefore \overrightarrow{AB} : \overrightarrow{BC} = 1:2$				
Scalar Product	<p>When given an angle between two vectors, the scalar product is calculated using</p> $\mathbf{a} \cdot \mathbf{b} =  \mathbf{a}  \mathbf{b}  \cos \theta$  <p><b>NB:</b> To find the angle between the two vectors <math>\theta</math>, the vectors must be pointing away from each other and <math>0 \leq \theta \leq 180^\circ</math></p> <p>When given component form, i.e. if <math>\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}</math> and <math>\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}</math>, the scalar product is calculated using <math>\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3</math></p>				
Angle Between Two Vectors	<p>The angle between two vectors is calculated by rearranging the scalar product formula</p> $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a}  \mathbf{b} } \quad \text{which can be expanded to} \quad \cos \theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{ \mathbf{a}  \mathbf{b} }$ <p><b>NB:</b> to find the angle between two vectors, the vectors must be pointing away from or towards each other. They must not be going in the same direction</p> <p><b>e.g.</b></p>  <p style="text-align: center;"><b>Not</b></p> 				
Perpendicular Vectors	Vectors are perpendicular when $\mathbf{a} \cdot \mathbf{b} = 0$				
<b>Common Terms</b>					
Vector	A vector is a quantity that contains both a size and a direction				

# Relationships & Calculus

Topic	Skills	Notes			
<b>Polynomials</b>					
<b>Prior Skills</b>					
Factorise	Common Factor, Difference of Two Square and Trinomial factorising				
Solving Quadratics	Algebraically or using quadratic formula				
Factors and Roots	<ul style="list-style-type: none"> <li>A root is a value for which a polynomial <math>f(x) = 0</math>. These are the x-coordinates at the point of intersection with the x-axis</li> <li>A factor is the form from which a root is derived e.g. for <math>x(x - 4) = 0</math>, <math>x</math> and <math>(x - 4)</math> are <b>factors</b>, <math>x = 0</math> and <math>x = 4</math> are <b>roots</b></li> </ul>				
<b>New Skills</b>					
Fully Factorise a Polynomial	<p>Use synthetic division e.g. Factorise <math>f(x) = x^3 - 3x + 2</math> Soln. Set up synthetic division using coefficients from polynomial.</p> <ul style="list-style-type: none"> <li>If there is no term, use 0</li> <li>The value outside the division is derived from factors of the last term (in this case factors of 2)</li> <li>If the remainder of the division is 0 then the value outside the division is a root</li> </ul>  <p><math>\therefore (x + 2)</math> is a factor and <math>x = -2</math> is a root  <math>(x + 2)(x^2 - 2x + 1) = 0</math>  <math>(x + 2)(x - 1)(x - 1) = 0</math>,  <math>\therefore x = -1</math> (twice) and <math>x = 2</math></p>				
Remainder Theorem	<p>Find the Quotient and Remainder of a Function</p> <ul style="list-style-type: none"> <li>Use synthetic division with the given value</li> <li>The value at the end is the remainder</li> </ul> <p>e.g. Find the quotient and remainder when <math>f(x) = (x^3 - 3x + 9)</math> is divided by <math>(x - 3)</math>  Soln. </p> <p><math>\therefore f(x) = (x^3 - 3x + 9) = (x - 3)(x^2 + 3x + 6) + 27</math>  Quotient is <math>(x^2 + 3x + 6)</math> and remainder 27</p>				
Identify the Equation of a Polynomial from a Graph	<ul style="list-style-type: none"> <li>Determine the factors from the roots on the graph</li> <li>Set up a polynomial with a coefficient of <math>k</math> outside the brackets</li> <li>Substitute the y-intercept or given point to determine value of <math>k</math></li> </ul> <p>e.g. </p> <p>Soln. <math>y = k(x + 2)(x - 1)(x - 5)</math>,  When <math>x = 0, y = 5</math>  <math>5 = k(0 + 2)(0 - 1)(0 - 5)</math>  <math>5 = 10k</math>, <math>k = \frac{1}{2}</math>  <math>\therefore y = \frac{1}{2}(x + 2)(x - 1)(x - 5)</math></p>				

Topic	Skills	Notes			
Show that a Term is a Factor of a Polynomial	<ul style="list-style-type: none"> <li>From the factor, determine the root (<i>see Factors and Roots above</i>)</li> <li>Use synthetic division with the given value If the remainder is 0 the term is a factor</li> </ul>				
Find Unknown Coefficients of a Polynomial	<ul style="list-style-type: none"> <li>Substitute roots into equation and solve simultaneously <b>e.g.</b> Find the values of <math>p</math> and <math>q</math> if <math>(x + 2)</math> and <math>(x - 1)</math> are factors of <math>f(x) = x^3 + 4x^2 + px + q</math> <b>Soln.</b> <math>(x + 2)</math> is a factor <math>\therefore x = -2</math> is a root  <math>(-2)^3 + 4(-2)^2 - 2p + q = 0</math>  <math>8 - 2p + q = 0</math>  <math>q = 2p - 8</math>  <math>(x - 1)</math> is a factor <math>\therefore x = 1</math> is a root  <math>(1)^3 + 4(1)^2 + p + q = 0</math>  <math>5 + p + q = 0</math>  <math>q = -p - 5</math>  Solve two equations simultaneously  <math>2p - 8 = -p - 5</math>  ...  <math>\therefore p = 1, q = -6</math> and <math>f(x) = x^3 + 4x^2 + x - 6</math> </li> </ul>				
Sketch the Graph of a Polynomial Function	<ul style="list-style-type: none"> <li>Find the <math>x</math>-intercepts (roots, when <math>y = 0</math>) using synthetic division</li> <li>Find the <math>y</math>-intercept (when <math>x = 0</math>)</li> <li>Find stationary points and their nature (<i>see differentiation</i>)</li> <li>Find large negative and positive <math>x</math></li> </ul>				
Common Terms					
Polynomial	A function containing multiple terms of different powers <b>e.g.</b> $x^4 - 2x^3 + 3$				
Quadratic Functions					
Prior Skills ( <i>see Relationships in National 5 Checklist</i> )					
Solve a Quadratic	Graphically, Algebraically or using Quadratic Formula				
Sketch a Quadratic	<ul style="list-style-type: none"> <li>In completed square form</li> <li>In root form</li> </ul>				
Identify the Equation of a Quadratic from its Graph	See <i>Identify the equation of a Polynomial from a Graph</i> (above)				
Complete the Square	$x^2 + ax + b = (x + \frac{a}{2})^2 + b - \frac{a^2}{4}$  <b>Ex 1.</b> $x^2 + 8x - 13 = (x + 4)^2 - 13 - 16 = (x + 4)^2 - 29$  <b>Ex 2.</b> $x^2 + 3x + 10 = (x + \frac{3}{2})^2 + 10 - \frac{9}{4} = (x + \frac{3}{2})^2 + \frac{40}{4} - \frac{9}{4} = (x + \frac{3}{2})^2 + \frac{31}{4}$				
Discriminant	$b^2 - 4ac$ where $y = ax^2 + bx + c$ The discriminant describes the nature of the roots $b^2 - 4ac > 0$ two real roots $b^2 - 4ac = 0$ equal roots (i.e. tangent) $b^2 - 4ac < 0$ no real roots				

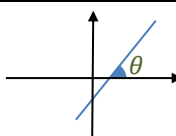
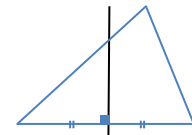
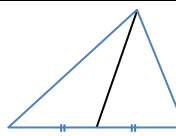
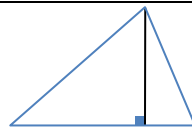
Topic	Skills	Notes			
<b>New Skills</b>					
Show a Line is a Tangent to a Quadratic Function	<ul style="list-style-type: none"> <li>Equate the line and quadratic</li> <li>Bring to one side</li> <li>Use the discriminant <i>or</i> factorise to show repeated root</li> </ul> <p><b>e.g.</b> Show that the line <math>y = x + 5</math> is a tangent to the curve <math>y = x^2 + 5x + 9</math></p> <p><b>Soln.</b> <math>x^2 + 5x + 9 = x + 5</math>  <math>x^2 + 4x + 4 = 0</math>  <math>(x + 2)(x + 2) = 0</math>  <math>x = -2</math> twice <math>\therefore</math> line is a tangent to quadratic.</p>				
Determine Nature of Intersection Between a Line and a Quadratic Function	<ul style="list-style-type: none"> <li>Equate line and quadratic</li> <li>Bring to one side</li> <li>Use the discriminant to determine nature of intersection</li> </ul>				
Determine Points of Intersection between a Line and a Quadratic Function	<ul style="list-style-type: none"> <li>Equate line and quadratic</li> <li>Bring to one side</li> <li>Solve for <math>x</math></li> <li>Substitute <math>x</math> values into line to find <math>y</math> values</li> </ul>				
Use Discriminant to Find Unknown Coefficients of a Quadratic Function	<ul style="list-style-type: none"> <li>Identify coefficients <math>a</math>, <math>b</math> and <math>c</math></li> <li>Use discriminant</li> </ul> <p><b>Ex 1.</b> Find <math>p</math> given that <math>x^2 + x + p = 0</math> has real roots</p> <p><b>Soln.</b> <math>a = 1, b = 1, c = p</math></p> $b^2 - 4ac \geq 0$ $1^2 - 4(1)(p) \geq 0$ $1 - 4p \geq 0$ $1 \geq 4p$ $p \leq \frac{1}{4}$ <p><b>Ex 2.</b> Find <math>p</math> given that <math>4x^2 + 2px + 1 = 0</math> has no real roots</p> <p><b>Soln.</b> <math>a = 4, b = 2p, c = 1</math></p> $b^2 - 4ac < 0$ $(2p)^2 - 4(4)(1) < 0$ $4p^2 - 16 < 0$ $4(p^2 - 4) < 0$ $4(p + 2)(p - 2) < 0 \quad (\text{Sketch a graph})$ <p>For no real roots <math>-2 &lt; p &lt; 2</math></p> 				
<b>Common Terms</b>					
Parabola	The graph of a quadratic function				
<b>Trigonometry</b>					
<b>Prior Skills</b>					
Exact Values	<p>Know exact values from table or using triangles and SOHCAHTOA</p>  <p>Know exact values of <math>0^\circ, 90^\circ, 180^\circ, 270^\circ</math> and <math>360^\circ</math> from trig graphs</p>				
Solve Trig Equations	Use the CAST diagram or graphical method to solve equations (see <i>Relationships</i> in National 5 checklist)				

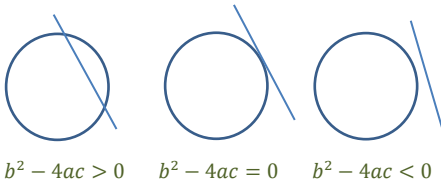

Topic	Skills	Notes			
Determine Period, Shape and Max and Min Values from a Trig Equation	$y = a \cos bx$ Amplitude = $a$ Period = $\frac{360}{b}$ <b>NB:</b> in the graph $y = a \tan bx$ the amplitude cannot be measured				
Adding Fractions	For solutions in radians, the ability to add fractions is required <b>e.g.</b> $\pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$ , $2\pi + \frac{\pi}{3} = \frac{6\pi}{3} + \frac{\pi}{3} = \frac{7\pi}{3}$				
<b>New Skills</b>					
Convert from Degrees to Radians	<ul style="list-style-type: none"> <li>Multiply degrees by <math>\pi</math> and divide by 180 then simplify</li> </ul> <b>e.g.</b> Change $35^\circ$ to radians <b>Soln.</b> $\frac{35\pi}{180} = \frac{7\pi}{36}$				
Convert from Radians to Degrees	<ul style="list-style-type: none"> <li>Multiply radians by 180 and divide by <math>\pi</math> then simplify</li> </ul> <b>e.g.</b> Change $\frac{5\pi}{6}$ to degrees <b>Soln.</b> $\frac{5\pi}{6} = \frac{5 \times 180\pi}{6\pi} = \frac{5 \times 30}{1} = 150^\circ$				
Solve Trig Equations with multiple solutions	<ul style="list-style-type: none"> <li>Identify how many solutions from the question</li> <li>Solve the equation</li> </ul> <b>e.g.</b> Solve $2 \cos 3x = 1$ , for $0 \leq x \leq \pi$ <b>Soln.</b> $2 \cos 3x = 1$ $\cos 3x = \frac{1}{2} \quad (\text{As } \cos 3x \text{ has 6 solutions for } 0 \leq x \leq 2\pi, \therefore 3 \text{ solutions for } 0 \leq x \leq \pi)$ $3x = \frac{\pi}{3}, \pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3}$ $3x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}$ $x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}$				
Solve Trig Equations by Factorising	Factorise the equation in the same way as an algebraic equation, by looking for <i>common factors</i> , <i>difference of two square</i> or a <i>trinomial</i> <b>e.g.</b> Solve $2 \sin x \cos x + \sin x = 0$ , for $0 \leq x \leq 180^\circ$ <b>Soln.</b> Factorise $\sin x (2 \cos x + 1) = 0$ $\sin x = 0 \qquad 2 \cos x + 1 = 0$ $x = 0^\circ, 180^\circ, 360^\circ \qquad \cos x = -\frac{1}{2} \quad (\text{using CAST})$ $x_A = 60^\circ$ $x = 120^\circ, 240^\circ$ $\therefore x = 0^\circ, 120^\circ, 180^\circ$				
Identify the Equation of a Trig function from its Graph	<b>e.g.</b> Determine the equation of the graph <b>1.</b>  <b>Ans:</b> $y = 4 \sin 3x + 1$ <b>2.</b>  <b>Ans:</b> $y = 2 \cos(x - \frac{\pi}{6})$				
Sketch a Trig Graph from its Equation	<b>e.g.</b> Sketch the graph of $y = 2 \sin(x - 30^\circ)$ for $0 \leq x \leq 360$ showing clearly where the graph cuts the x-axis and the y-axis <b>Ans.</b> Amplitude of 2. Graph moves $30^\circ$ to the right. Find x-intercepts when $y = 0$ . Find y-intercept when $x = 0$ 				

Topic	Skills	Notes			
<b>Common Terms</b>					
<b>Radians</b>	Radians are an alternative unit for measuring angles. $\pi$ radians = $180^\circ$ Common Conversions: $\frac{\pi}{2} = 90^\circ, \frac{\pi}{3} = 60^\circ, \frac{\pi}{4} = 45^\circ, \frac{\pi}{6} = 30^\circ, \frac{2\pi}{3} = 120^\circ, \frac{3\pi}{2} = 270^\circ$				
<b>Further Calculus</b>					
<b>Prior Skills</b>					
Differentiate	See <i>Differentiation</i> (in Applications)				
Integrate	See <i>Integration</i> (in Applications)				
Trig Equations	See <i>Trigonometry</i> (above)				
Radians	See <i>Trigonometry</i> (above)				
<b>New Skills</b>					
Differentiate Trig Functions	$y = \sin x$ $\frac{dy}{dx} = \cos x$ $y = \cos x$ $\frac{dy}{dx} = -\sin x$				
Integrate Trig Functions	$\int \sin x \, dx = -\cos x + C$ $\int \cos x \, dx = \sin x + C$				
Chain Rule	Used for differentiating composite functions <ul style="list-style-type: none"> <li>Differentiate the outer function</li> <li>Multiply by the derivative of the inner function</li> </ul> e.g. If $h(x) = f(g(x))$ then $h'(x) = f'(g(x)) \times g'(x)$  <b>Ex. 1.</b> Find $\frac{dy}{dx}$ when $y = \sqrt{2x-5}$ <b>Soln.</b> Prepare function for differentiation $y = (2x-5)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(2x-5)^{-\frac{1}{2}} \times 2$ $\frac{dy}{dx} = \frac{1}{\sqrt{2x-5}}$  <b>Ex 2.</b> Find $\frac{dy}{dx}$ when $y = 3 \cos^2 x$ <b>Soln.</b> Prepare function for differentiation $y = 3(\cos x)^2$ $\frac{dy}{dx} = 6(\cos x)^1 \times \sin x$ $\frac{dy}{dx} = 6 \cos x \sin x$				
Integration of Composite Functions	When integrating composite functions <ul style="list-style-type: none"> <li>Integrate the outer function</li> <li>Divide by the <u>derivative</u> of the inner function</li> </ul> e.g. $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{n+1 \times a} + C$  <b>Ex 1.</b> $\int (2x^3 + 5)^4 \, dx$ <b>Soln.</b> $\int (2x^3 + 5)^4 \, dx = \frac{(2x^3+5)^5}{5 \times 6x^2} + C = \frac{(2x^3+5)^5}{30x^2} + C$  <b>Ex 1.</b> $\int \sin(4x-3) \, dx$ <b>Soln.</b> $\int \sin(4x-3) \, dx = \frac{-\cos(4x-3)}{4} + C$				

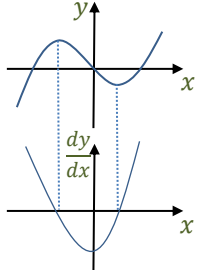


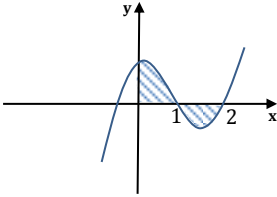
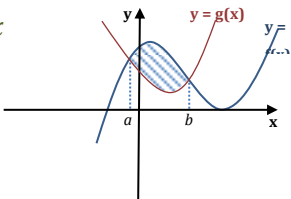
# Applications

Topic	Skills	Notes			
<b>The Straight Line</b>					
<b>Prior Skills</b>					
Distance Between Two Points	Distance formula or alternative: $Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$				
Gradient of a line	<ul style="list-style-type: none"> <li><math>m = \frac{y_2 - y_1}{x_2 - x_1}</math></li> <li><math>m = \tan \theta</math></li> <li>Perpendicular gradients: <math>m_1 \times m_2 = -1</math></li> </ul> <p>e.g. if <math>m_1 = 2</math>, the perpendicular gradient <math>m_2 = -\frac{1}{2}</math></p> 				
Equation of a Line From Two Points	For every equation of line a <b>point</b> and <b>gradient</b> is required <ul style="list-style-type: none"> <li>Calculate gradient and substitute point <math>(a, b)</math> into equation: <math>y - b = m(x - a)</math></li> <li>Expand bracket and simplify</li> </ul>				
Midpoint	$Midpoint = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$				
Point of Intersection	<ul style="list-style-type: none"> <li>Solve using simultaneous equations</li> </ul>				
<b>New Skills</b>					
Equation of a Perpendicular Bisector	<ul style="list-style-type: none"> <li>Find the midpoint of the line joining the two points</li> <li>Find gradient using perpendicular gradients</li> <li>Substitute midpoint and inverted gradient into <math>y - b = m(x - a)</math></li> </ul> 				
Equation of a Median	<ul style="list-style-type: none"> <li>Find the midpoint of the line joining the two points</li> <li>Find gradient of the median</li> <li>Substitute into <math>y - b = m(x - a)</math></li> </ul> 				
Equation of an Altitude	<ul style="list-style-type: none"> <li>Find gradient of the altitude using perpendicular gradients</li> <li>Substitute into <math>y - b = m(x - a)</math> with point from vertex</li> </ul> 				
Collinearity	<ul style="list-style-type: none"> <li>Show that three points are collinear (i.e. on the same line)</li> <li>Find gradients (the same if parallel) and point in common</li> <li>Statement: Points A, B and C are collinear as <math>m_{AB} = m_{BC}</math> and point B is common to both</li> </ul>				
<b>Common Terms</b>					
Collinear	Points on the same line				
Congruent	The same size				
Concurrent	Lines that intersect at the same point <b>NB:</b> In a triangle, altitudes are concurrent (intersect at <i>orthocentre</i> ), medians are concurrent (intersect at <i>centroid</i> ) and perpendicular bisectors are concurrent (intersect at <i>circumcentre</i> )				
Centroid	The point of intersection of the three medians of a triangle				
Circumcentre	The point of intersection of three perpendicular bisectors in a triangle				
Orthocentre	The point of intersection of three altitudes of a triangle				

Topic	Skills	Notes			
<b>The Circle</b>					
<b>Prior Skills</b>					
Distance between two points	Distance formula or alternative: $Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$				
Gradient	<ul style="list-style-type: none"> <li><math>m = \frac{y_2 - y_1}{x_2 - x_1}</math></li> <li>Perpendicular gradients: <math>m_1 \times m_2 = -1</math></li> </ul>				
Midpoint	$Midpoint = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$				
Discriminant	<ul style="list-style-type: none"> <li><math>b^2 - 4ac</math> where <math>ax^2 + bx + c = 0</math></li> </ul>  <p><math>b^2 - 4ac &gt; 0</math>    <math>b^2 - 4ac = 0</math>    <math>b^2 - 4ac &lt; 0</math></p>				
<b>New Skills</b>					
Equation of Circle with Centre the Origin and Radius $r$	<ul style="list-style-type: none"> <li><math>x^2 + y^2 = r^2</math></li> </ul>				
Equation of a Circle with Centre $(a, b)$ and Radius $r$	<ul style="list-style-type: none"> <li>Determine the centre and radius <b>NB:</b> Finding the centre often involves finding a midpoint of a diameter, or using a coordinate diagram and symmetry</li> <li>Substitute into equation <math>(x - a)^2 + (y - b)^2 = r^2</math></li> </ul>				
Centre and Radius of a Circle from its Equation	<ul style="list-style-type: none"> <li>Use the equation <math>x^2 + y^2 + 2gx + 2fy + c = 0</math></li> <li>Centre from <math>(-g, -f)</math></li> <li>Radius: <math>r = \sqrt{g^2 + f^2 - c}</math> <b>NB:</b> if <math>g^2 + f^2 - c &lt; 0</math> the equation is not a circle</li> </ul>				
Equation of a Tangent to a Circle	<ul style="list-style-type: none"> <li>Determine the gradient of the radius from centre and the point of contact of the tangent</li> <li>Find gradient using perpendicular gradients</li> <li>Substitute perpendicular gradient and point into equation of a line <math>y - b = m(x - a)</math></li> </ul> 				
Points of Intersection of a Line and a Circle	<ul style="list-style-type: none"> <li>Rearrange the line to <math>y = mx + c</math> then substitute into the equation of a circle</li> <li>Solve the quadratic to find <math>x</math></li> <li>Substitute <math>x</math> value into <math>y = mx + c</math> to find <math>y</math></li> </ul>				
Use Discriminant to Determine whether a Line and Circle Intersect	<ul style="list-style-type: none"> <li>Rearrange the line to <math>y = mx + c</math> then substitute into the equation of a circle</li> <li>Simplify to quadratic form</li> <li>Use discriminant to determine intersection</li> </ul>				
<b>Common Terms</b>					
Concentric	Circles that have the same centre				
<b>Recurrence Relations</b>					
<b>Prior Skills</b>					
Finding Percentage Multipliers	<ul style="list-style-type: none"> <li>Determine whether question is percentage increase or decrease</li> <li>Add or subtract from 100%</li> <li>Divide by 100</li> </ul> <p><b>e.g.</b> 4.3% increase  <math>100\% + 4.3\% = 104.3\% = 1.043</math></p>				

Topic	Skills	Notes															
New Skills																	
Form Linear Recurrence Relations	<ul style="list-style-type: none"><li>Find values of <math>a</math> and <math>b</math> for relation <math>u_{n+1} = au_n + b</math> where <math>a</math> is the percentage multiplier and <math>b</math> is the increase</li></ul>																
Use Linear Recurrence Relations to Find Values	<ul style="list-style-type: none"><li>Start with <math>u_0</math> (initial value) and substitute into relation <b>NB:</b> Set up calculator by inputting <math>u_0</math> and pressing '=' then <math>ANS \times a + b</math> and continue pressing '=' until answer is reached. Write down all answers</li></ul>																
Find the Limit of a Linear Recurrence Relation	<ul style="list-style-type: none"><li>Determine values of <math>a</math> and <math>b</math></li><li>Ensure <math>-1 &lt; a &lt; 1</math></li><li>Use limit formula <math>L = \frac{b}{1-a}</math></li><li>Interpret what the limit means in a specific context</li></ul>																
Differentiation																	
Prior Skills																	
Laws of indices	<ul style="list-style-type: none"><li>Know and use each of the laws of indices to manipulate algebraic fractions; <b>e.g.</b> <math>\frac{x^3 + \sqrt{x}}{x^2} = \frac{x^3}{x^2} + \frac{x^{\frac{1}{2}}}{x^2} = x + x^{-\frac{3}{2}}</math></li></ul>																
New Skills																	
Differentiate a Function	<ul style="list-style-type: none"><li>To differentiate <math>f(x) = ax^n</math>, <math>f'(x) = anx^{n-1}</math></li><li>For <math>f(x) = g(x) + h(x)</math> <math>f'(x) = g'(x) + h'(x)</math></li></ul> <b>NB:</b> For differentiation questions, algebraic fractions need to be broken down into individual fractions (see <i>Laws of Indices above</i> )																
Find the Gradient or Rate of Change of a Function at a Given Point	<ul style="list-style-type: none"><li>Know that <math>f'(x) = m = \text{rate of change}</math></li><li>Differentiate the function</li><li>Substitute x-coordinate into derivative <b>e.g.</b> Find the gradient of <math>f(x) = 2x^3</math> when <math>x = 1</math> <b>Soln.</b> <math>f'(x) = 6x^2</math> <math>f'(4) = 6(1)^2 = 6</math></li></ul>																
Find the Equation of a Tangent to a Curve	<ul style="list-style-type: none"><li>Differentiate function</li><li>Find gradient from derivative (see above)</li><li>Substitute point and gradient into equation of a line <math>y - b = m(x - a)</math></li></ul>																
Find Stationary Points and Determine their Nature	<p>Find Stationary Points</p> <ul style="list-style-type: none"><li>Differentiate function</li><li>Know that stationary points occur when <math>f'(x) = 0</math></li><li>Solve <math>f'(x) = 0</math> to find x-coordinates of stationary points</li><li>Substitute x-coordinates into <math>f(x)</math> to find y-coordinates</li></ul> <p>Determine Nature</p> <ul style="list-style-type: none"><li>Draw nature table with x values slightly above and below the stationary points (make sure they are not lower or higher than any other stationary points)</li><li>Sketch the nature from positive or negative values</li><li>Answer question, <b>e.g.</b> Max turning point at (x, y)</li></ul> <table><tr><td><math>x</math></td><td><math>3^-</math></td><td><math>3</math></td><td><math>3^+</math></td></tr><tr><td><math>f'(x)</math></td><td><math>-</math></td><td><math>0</math></td><td><math>+</math></td></tr><tr><td>slope</td><td><math>\backslash</math></td><td><math>-</math></td><td><math>/</math></td></tr></table>	$x$	$3^-$	$3$	$3^+$	$f'(x)$	$-$	$0$	$+$	slope	$\backslash$	$-$	$/$				
$x$	$3^-$	$3$	$3^+$														
$f'(x)$	$-$	$0$	$+$														
slope	$\backslash$	$-$	$/$														

Topic	Skills	Notes			
Sketch a curve	<ul style="list-style-type: none"> <li>Find stationary points and nature (<i>see above</i>)</li> <li>Find roots by solving function (when <math>y = 0</math>)</li> <li>Find y-intercept (when <math>x=0</math>)</li> <li>Find large positive and large negative x. e.g. as <math>f(x) \rightarrow -\infty, x \rightarrow -\infty</math> and as <math>f(x) \rightarrow +\infty, x \rightarrow +\infty</math></li> <li>Sketch information on graph</li> </ul>				
Sketch the derived function	<ul style="list-style-type: none"> <li>Sketch function</li> <li>Extend stationary points to other coordinate axis</li> <li>Determine where the gradient <math>m</math> is +ve and -ve. (the gradient is +ve where the graph of <math>\frac{dy}{dx}</math> is above the x-axis and negative where it is below)</li> </ul> 				
Closed Intervals	Find the maximum and minimum value in a closed interval <ul style="list-style-type: none"> <li>Find the stationary points and determine their nature (<i>see above</i>)</li> <li>Find the y-coordinates at the extents of the interval</li> <li>Examine to see where the maximum and minimum values are</li> </ul>				
Increasing and Decreasing Functions	<ul style="list-style-type: none"> <li>Differentiate the function</li> <li>Determine where gradient is positive or negative from the derivative or a sketch of the derivative</li> </ul> <b>NB:</b> A function is increasing where the gradient $m$ is positive and decreasing where the gradient is negative				
<b>Common Terms</b>					
Leibniz Notation	$y$ and $\frac{dy}{dx}$				
Function Notation	$f(x)$ and $f'(x)$				
Rate of Change	The rate at which one variable changes in relation to another. To find rate of change, differentiate function				
<b>Integration</b>					
<b>Prior Skills</b>					
Laws of indices	See <i>Differentiation</i>				
<b>New Skills</b>					
Integrate a Function	<ul style="list-style-type: none"> <li>To integrate: <math>\int ax^n dx = \frac{ax^{n+1}}{n+1} + C</math>,</li> </ul> <b>NB:</b> When the integral is indefinite (i.e. there are no limits), remember 'C' the constant of integration				
Evaluate a Definite Integral	<ul style="list-style-type: none"> <li>Integrate function</li> <li>Evaluate between two limits</li> </ul> e.g. Evaluate $\int_1^3 4x dx$ <b>Soln.</b> $\int_1^3 4x dx = [2x^2]_1^3 = (2(3)^2) - (2(1)^2) = 16$				

Topic	Skills	Notes			
Area Between Curve and x-axis	<ul style="list-style-type: none"> <li>Find the area above the x-axis</li> <li>Find the area below the x-axis (ignore the negative)</li> <li>Add them together</li> </ul> $\int_0^1 f(x) dx \text{ and } \int_1^2 f(x) dx$ <p><b>NB:</b> The area below the x-axis will give a negative answer. Ignore the negative</p> 				
Area Between Two Curves	<ul style="list-style-type: none"> <li>Set the curves equal to each other and solve to find the limits</li> <li>Set up integral with:  <math display="block">\int_a^b [\text{upper curve} - \text{lower curve}] dx</math> <math display="block">\int_a^b [f(x) - g(x)] dx</math> </li> <li>Evaluate answer</li> </ul> 				
Differential Equations	<p>Equations of the form <math>\frac{dy}{dx} = ax + b</math> are called <i>differential equations</i>. They are solved by integration</p> <p><b>e.g.</b> The curve <math>y = f(x)</math> is such that <math>\frac{dy}{dx} = 9x^2</math>, the curve passes through (1, 5). Express <math>y</math> in terms of <math>x</math></p> <p><b>Soln.</b> <math>y = \int 9x^2 dx = 3x^3 + C</math>  at (1, 5), <math>5 = 3(1)^3 + C</math>  <math>5 = 3 + C</math>  <math>C = 2 \therefore y = 3x^3 + 2</math></p>				