Use this booklet to practise working independently like you will have to in course assessments and the Value Added Unit (AVU).

- Get in the habit of turning to this booklet to refresh your memory.
- If you have forgotten how to do a method, examples are given.
- If you have forgotten what a word means, use the index (back page) to look it up.

As you get closer to the final test, you should aim to use this booklet less and less.

This booklet is for:
- Students doing the National 4 Mathematics course
- Students studying one or more of the National 4 mathematics units: Numeracy, Expressions and Formulae or Relationships

This booklet contains:
- The most important facts you need to memorise for National 4 mathematics.
- Examples that take you through the most common routine questions in each topic.
- Definitions of the key words you need to know.

Use this booklet:
- To refresh your memory of the method you were taught in class when you are stuck on a homework question or a practice test question.
- To memorise key facts when revising for assessments and the Value Added Unit.

*The key to revising for a maths test is to do questions, not to read notes.* As well as using this booklet, you should also:
- Revise by working through exercises on topics you need more practice on – such as revision booklets, textbooks, websites, or exercises suggested by your teacher.
- Work through practice tests
- Ask your teacher when you come across a question you cannot answer
- Use resources online (a link that can be scanned with a SmartPhone is on the last page)
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# Formula Sheet

The following formulae are mentioned in these notes and are collected on this page for ease of reference.

**Formulae that are given on the formula sheet in the Added Value Unit** (or in unit assessments)

<table>
<thead>
<tr>
<th>Topic</th>
<th>Formula(e)</th>
<th>Page Reference</th>
</tr>
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<tbody>
<tr>
<td>Circumference of a circle</td>
<td>$C = \pi d$</td>
<td>See page 37</td>
</tr>
<tr>
<td>Area of a circle</td>
<td>$A = \pi r^2$</td>
<td>See page 37</td>
</tr>
<tr>
<td>Curved surface area of a cylinder</td>
<td>$A = 2\pi rh$</td>
<td>See page 44</td>
</tr>
<tr>
<td>Volume of a cylinder</td>
<td>$V = \pi r^3 h$</td>
<td>See page 42</td>
</tr>
<tr>
<td>Volume of a prism</td>
<td>$V = Ah$</td>
<td>See page 41</td>
</tr>
<tr>
<td>Gradient</td>
<td>Gradient = $\frac{\text{Vertical height}}{\text{Horizontal distance}}$</td>
<td>See page 36</td>
</tr>
<tr>
<td>Pythagoras’ Theorem</td>
<td>$a^2 + b^2 = c^2$</td>
<td>See page 60</td>
</tr>
<tr>
<td>Trigonometry in a right-angled triangle</td>
<td>$\sin x^\circ = \frac{\text{Opp}}{\text{Hyp}}$ $\cos x^\circ = \frac{\text{Adj}}{\text{Hyp}}$ $\tan x^\circ = \frac{\text{Opp}}{\text{Adj}}$</td>
<td>See page 62</td>
</tr>
</tbody>
</table>

**Formulae that are not given in the Added Value Unit** (or in unit assessments)

<table>
<thead>
<tr>
<th>Topic</th>
<th>Formula(e)</th>
<th>Page Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of a rectangle</td>
<td>$A = LB$</td>
<td>See page 20</td>
</tr>
<tr>
<td>Area of a square</td>
<td>$A = L^2$</td>
<td>See page 20</td>
</tr>
<tr>
<td>Area of a triangle</td>
<td>$A = \frac{BH}{2}$</td>
<td>See page 20</td>
</tr>
<tr>
<td>Volume of a cuboid</td>
<td>$V = LBH$</td>
<td>See page 20</td>
</tr>
<tr>
<td>Percentage increase and decrease</td>
<td>$\text{\frac{\text{change}}{\text{original amount}}} \times 100$</td>
<td>See page 19</td>
</tr>
<tr>
<td>Range</td>
<td>Range = Highest – Lowest</td>
<td>See page 47</td>
</tr>
<tr>
<td>Mean</td>
<td>Mean = $\frac{\text{Total}}{\text{How many}}$</td>
<td>See page 48</td>
</tr>
<tr>
<td>Equation of a straight line</td>
<td>$y = mx + c$</td>
<td>See page 55</td>
</tr>
<tr>
<td>Speed, Distance, Time</td>
<td>$S = \frac{D}{T}$ $T = \frac{D}{S}$ $D = ST$</td>
<td>See page 23</td>
</tr>
</tbody>
</table>
Assessment Technique

This page outlines some of the key things that you should remember when sitting the Added Value Unit and unit assessments in order to avoid losing marks.

Units

To be fully correct, answers should always contain the correct units. As a general rule, you will lose the mark for the final answer if you do not include the correct units.

There are some occasions where your teacher will still be allowed to give you the mark, even without the correct units, but you do not need to know what these are: the only way to guarantee that you do not lose a mark is to ensure you always include units!

It is especially important to use the correct units in questions relating to area and volume:
- If the question asks you to calculate a volume, the units are ‘cubic’ – e.g. m³, cm³, mm³
- If the question asks you to calculate an area, the units are ‘squared’ – e.g. m², cm², mm²
- If the question asks you to calculate a length (including a perimeter, a circumference or an arc length), the units are ‘normal’ units – e.g. m, cm, mm.

Example

![Rectangle](image)

Calculate the area of this rectangle (1 mark)

Example Solutions

<table>
<thead>
<tr>
<th>Solution One</th>
<th>Solution Two</th>
<th>Solution Three</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ A = LB ]</td>
<td>[ A = LB ]</td>
<td>[ A = LB ]</td>
</tr>
<tr>
<td>= 22 \times 9</td>
<td>= 22 \times 9</td>
<td>= 22 \times 9</td>
</tr>
<tr>
<td>= 198</td>
<td>= 198 cm</td>
<td>= 198 cm²</td>
</tr>
</tbody>
</table>

You would not get the mark for this answer as the answer contains no units

You would not get the mark for this answer as the answer contains incorrect units

This is an area question, so squared units are needed for the answer

You would also have lost the mark if you had used cm³, or if you had written m² instead of cm²

You would get the mark for this answer as the answer contains the correct units

Rounding

If a mark requires you to round your answer, you have to write down your unrounded answer before you then do the rounding. This is because there are often two marks related to the rounding:
The final mark (for the rounding)  
The previous mark (for the actual answer)  

A lot of people get frustrated by this. But whether you like it or not, if you do not write down your unrounded answer first, you will risk losing both marks even if you have the correct answer.

This rule applies in questions where you are explicitly told to round (e.g. ‘give your answers correct to 2 decimal places’), or questions where you have to realise for yourself that you have to round. There are two common examples of this:

- **Money questions**, where you have to know to give decimal answers to 2 decimal places. (e.g. you cannot have an answer of £2.587, you have to round it to £2.59)
- **Real-life situations where decimal answers make no sense**, where you have to know to round up (or down) to the nearest whole number. The example below is an example of this sort.

**Example**

One bus holds 56 people. How many buses must be ordered to take 1294 football fans from Inverness to Glasgow?

**Example Solutions**

In this question, you have to realise that it is impossible to have a decimal number of buses, so you have to round to the nearest whole number. You also have to realise that you always have to round up to the nearest whole number.

<table>
<thead>
<tr>
<th>Solution That Would Get Full Marks</th>
<th>Solution That Would Lose Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1294 ÷ 56 = 23.107...</td>
<td>1294 ÷ 56</td>
</tr>
<tr>
<td>24 buses are needed</td>
<td>24 buses are needed</td>
</tr>
</tbody>
</table>

Notice that you do not have to write down every decimal place from your calculator. It is enough to write a few ‘extra’ decimal places and then to write dots to indicate that they keep on going.

**‘Strategy’ Marks (#2.1)**

The ‘Relationships’ and ‘Expressions and Formulae’ units of the National 4 mathematics course contains an assessment standard 2.1 (often referred to as #2.1 in documentation). The marks for this assessment standards are called ‘reasoning’ marks, and are separate to the ‘process’ marks awarded for getting the calculations correct. This assessment standard is for ‘Interpreting a situation where mathematics can be used and identifying a valid strategy’. This assessment standard will also be tested in the final exam paper.

In everyday language, this assessment standard most commonly requires you to choose a method to answer a question; usually in a question that is slightly unfamiliar to you (a ‘non-routine’ question).
‘Communication’ Marks (#2.2)

The ‘Relationships’ and ‘Expressions and Formulae’ units of the National 4 mathematics course contains an assessment standard 2.2 (often referred to as #2.2 in documentation). The marks for this assessment standards are called ‘reasoning’ marks, and are separate to the ‘process’ marks awarded for getting the calculations correct. This assessment standard is for ‘Explaining a solution and/or relating it to context’. This assessment standard will also be tested in the final exam paper.

Assessment standards 1.5 and 2.2 in the ‘Numeracy’ unit at National 4 also require a reason to be given.

In everyday language, this assessment standard most commonly requires you to write a sentence after performing a calculation.

There are various ways that this assessment standard can be tested. Three possible ways are outlined below:

1. **Writing comments related to statistics** (e.g. the median, mean, mode and range) in the *Expressions and Formulae* unit. A full description of the technique required to answer these questions is given on page 50.

2. **Realising that a final answer needs to be rounded** to the nearest whole number and for giving your answer in a sentence. An example of this type is given on page 6. You have to ensure that you show your unrounded answer as well.

3. Questions that say ‘give a reason for your answer’ or ‘explain your answer’. A valid reason at National 4 must involve comparing two numbers.

**Example** (this example is below National 4 standard, but is included to focus on the reason)

Jack has £17. Ben has £8. They need £30 to buy a computer game. Do they have enough? Give a reason for your answer

**Solution**

You would not get a mark for the following:

- No *(no reason at all)*
- No, because they do not have £30 *(just repeating the number from the question)*
- No, because they only have £25 *(not comparing two numbers)*

You would get a mark for:

- No, because they have £25 and they need £30 *(two numbers compared)*
- No, because they need £5 more. *(writing down the difference between the numbers counts as comparing two numbers)*
Added Value Unit

For the Added Value Unit, you have to be able to answer 12 questions in an exam situation. You are told in advance which topics will be tested in the twelve questions, and these are listed below, along with page references showing examples for each type of question.

In the main notes, you can spot the Added Value content as it will be highlighted by a box looking like this one on page 34.

This topic is assessed in Question 3 of Paper 2 of the ADDED VALUE UNIT. See page 4 for more details

Paper 1 (20 minutes). Non calculator:
- Question 1: Calculate a whole number percentage without a calculator (see page 17 for non-calculator percentages)
- Question 2: Find the mean of a data set using long division and then round the answer to 2 decimal places (see page 48 for the mean, page 11 for written divide sums and page 12 for rounding)
- Question 3: Find a fraction of a quantity (see page 17 for fractions)
- Question 4: adding two decimal numbers and then subtracting from the result (see page 11 for an example)
- Question 5: multiplying a decimal number by a whole number (see page 12 for an example)

Paper 2 (40 minutes). Calculator:
- Question 1: solving an equation with letters on both sides (see page 57 for solving equations)
- Question 2: solving a problem using area or volume (see page 38 for areas of composite shapes, page 41 for volumes and page 42 for surface areas)
- Question 3: creating and then using a formula (see page 34 for creating and using formulae)
- Question 4: using the relationship involving speed, distance and time, where the time is given or calculated as hours and minutes. (see page 22 for Speed Distance and Time, and changing minutes to/from a decimal)
- Question 5: use of Pythagoras’ theorem in a problem (see page 60 for Pythagoras)
- Question 6: use of trigonometry to calculate a side or angle of a right-angled triangle (see page 62 for SOH CAH TOA)
- Question 7: draw a coordinate axis, plot some given points, and plot a fourth point to complete a shape (see page 29 for coordinates and completing shapes)

To pass the Added Value Unit:
1. You must get 50% of the marks (these are known as “operational” marks).
2. There will be 4 ‘reasoning’ questions that require you to choose a method. You must do this correctly in at least two of those four questions.
Numeracy Unit

Numerical Notation and Units

The course notes state that you are expected to be able to work with the following units. You should also know the key facts connecting them.

<table>
<thead>
<tr>
<th>Context</th>
<th>Units</th>
<th>Key Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money</td>
<td>Pounds and pence</td>
<td>£1 = 100p</td>
</tr>
<tr>
<td>Time</td>
<td>Months</td>
<td>1 year = 365 days</td>
</tr>
<tr>
<td></td>
<td>Weeks</td>
<td>1 year = 12 months</td>
</tr>
<tr>
<td></td>
<td>Days</td>
<td>1 year = 52 weeks</td>
</tr>
<tr>
<td></td>
<td>Hours</td>
<td>1 day = 24 hours</td>
</tr>
<tr>
<td></td>
<td>Minutes</td>
<td>1 hour = 60 minutes</td>
</tr>
<tr>
<td></td>
<td>Seconds</td>
<td>1 minute = 60 seconds</td>
</tr>
<tr>
<td>Length</td>
<td>Millimetres (mm)</td>
<td>1cm = 10mm</td>
</tr>
<tr>
<td></td>
<td>Centimetres (cm)</td>
<td>1m = 100cm</td>
</tr>
<tr>
<td></td>
<td>Metres (m)</td>
<td>1km = 1000m</td>
</tr>
<tr>
<td></td>
<td>Kilometres (km)</td>
<td>1m = 1000mm</td>
</tr>
<tr>
<td></td>
<td>Miles</td>
<td></td>
</tr>
<tr>
<td>Weight</td>
<td>Grams and Kilograms</td>
<td>1kg = 1000 g</td>
</tr>
<tr>
<td>Volume</td>
<td>Millilitres and Litres</td>
<td>1 litre = 1000ml</td>
</tr>
<tr>
<td>Temperature</td>
<td>Celsius</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fahrenheit</td>
<td></td>
</tr>
</tbody>
</table>

Important
It is essential to give the correct units in your answers. In many questions, you will lost the final mark if you have the ‘correct’ answer without units.

The course notes state that you are expected to be familiar with all of these symbols:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>=</td>
<td>Equal</td>
</tr>
<tr>
<td>+ – × ÷</td>
<td>Add, take away, multiply, divide</td>
</tr>
<tr>
<td>/</td>
<td>An alternative way of showing a divide sum –</td>
</tr>
<tr>
<td></td>
<td>i.e. 15 ÷ 3 could also be written 15/3 or $\frac{15}{3}$</td>
</tr>
<tr>
<td>&lt;</td>
<td>Less than (≤ means less than or equal to)</td>
</tr>
<tr>
<td>&gt;</td>
<td>Greater/bigger than (≥ means greater than or equal to)</td>
</tr>
<tr>
<td>()</td>
<td>Sums in brackets have to be done first</td>
</tr>
<tr>
<td>%</td>
<td>Percentage (see page 16)</td>
</tr>
<tr>
<td>:</td>
<td>Ratio symbol (see page 27)</td>
</tr>
<tr>
<td>•</td>
<td>Decimal point</td>
</tr>
</tbody>
</table>
Add, Subtract, Multiply, Divide and Rounding

Written Sums

You are expected to be able to do written add, take away, multiply and divide sums. You need to know how to do ‘carrying’ and (in subtraction sums) ‘borrowing’ to complete these sums.

Examples 1 – whole numbers
Calculate: (a) 5629 + 3783, (b) 4007 – 2678, (c) 438 × 7, (d) 3875 ÷ 5

Solutions
Worked solutions (with carrying and borrowing as required) are shown below.

\[
\begin{array}{ccc}
5629 & +3783 & \quad 438 \\
\hline
9412 & & 1329 \\
\end{array}
\]

\[
\begin{array}{ccc}
4007 & -2678 & \quad \times 7 \\
\hline
1329 & & 3066 \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{5)3875} & \quad \div 5 \\
\hline
\end{array}
\]

For the Added Value Unit, you need to be able to use these methods with decimals. For example in question 4, you need to be able to add and take away decimals without a calculator.

Example 2 – add and then take away
To make a fruit drink, a father mixes 2.75 litres of water and 0.375 litres of juice. 1.48 litres of the juice is drunk. How much is left?

Solution
We add 2.75 and 0.375, and then we take away 1.48 from the answer

Step one: adding
\[
\begin{array}{c}
2.750 \\
+0.375 \\
\hline
3.125
\end{array}
\]

Answer to step 1: 3.125 litres.

Step two: take away
\[
\begin{array}{c}
2.125 \\
-1.480 \\
\hline
1.645
\end{array}
\]

Answer: There is 1.645 litres of juice left. (in a real life situation we should always give our answer in a sentence, and units are essential)
In question 5 of paper 1 of the Added Value unit, you need to multiply a decimal.

Example 3 – multiplying decimals
One drawing pin weighs 2.074g. What do 7 drawing pins weigh?

Solution
The sum is $2.074 \times 7$.

$$
\begin{array}{c}
2.074 \\
\times 7 \\
\hline
14.518
\end{array}
$$

Answer: 7 drawing pins weigh 14.518g. (in a real life situation we should give our answer in a sentence, and units are essential)

You also need to be able to divide decimal numbers. Sometimes a divide sum still has a remainder left at the end. When this happens, we add zeroes to the end of the number whilst keeping it the same. i.e. if the original number was 85, then we add a zero and a decimal point to make it 85.0. If the original number was 4.7, we add a zero to make it 470.

Example 4 – decimal division that doesn’t stop
Divide 86 by 7, rounding your answer to two decimal places

Solution
To round to two decimal places we actually need to work out the third decimal place as well, so that we know whether to round up or down.

Therefore we rewrite 86 as 86.000 and divide. On this occasion we stop after the third decimal place.

$$
\begin{array}{c}
7 \overline{)86.000} \\
\hline
12.285\ldots
\end{array}
$$

The answer is 12.285… When we round this to two decimal places (see page 13), the answer becomes 12.29.

Rounding

If a question asks you to round your answer, there will be a mark for this. Do not miss it out. The question may say something like:

- “Round your answer to the nearest thousand”
- or “…giving your answer to two decimal places”

Remember that you must write your unrounded answer first before you round.
The instruction “round your answer to the nearest penny” means rounding it to two decimal places (since money always has to have two decimal places).

**Examples 1**

- 4652 rounded to the nearest ten is 4650
- 4652 rounded to the nearest hundred is 4700
- 4652 rounded to the nearest thousand is 5000

- 23.6666666 rounded to one decimal place is 23.7
- 23.6666666 rounded to two decimal places is 23.67
- £23.6666666 rounded to the nearest penny is £23.67

To round to **two decimal places**, we look at the **third** digit after the point (the thousandths).
- If this digit is 0, 1, 2, 3 or 4, we keep the second digit (the hundredths) the same
- If this digit is 5, 6, 7, 8 or 9, we round the second digit up.

This topic is assessed in **Question 2 of Paper 1 of the ADDED VALUE UNIT**. See page 4 for more details

**Examples 2 – rounding to two decimal places**

**Round 75•682 and 57•249 to two decimal places**

**Solutions**

In 75•682, the third digit after the point is 2. Therefore we keep the second digit the same and the answer is **75•68**.

In 57•249, the third digit after the point is 9. Therefore we put the second digit after the point up from 4 to 5 and the answer is **57•25**.

Rounding to the nearest **significant figure** means different things depending on the size of the number we are rounding. If we are rounding a number with four-digits before the decimal point, it means to round to the nearest thousand (i.e. 3745 rounded to one significant figure is 4000), if we are rounding a number with two digits before the decimal point, it means to round to the nearest ten (i.e. 49.5 rounded to one significant figure is 50)

**Adding and Taking Away Negative Numbers (Integers)**

**Definition:** an integer is a whole number that can be either positive or negative. To work with integers, you need to be able to use the rules for doing calculations.

You need to be able to work with integers in everyday situations (e.g. temperature), and in sums.
Adding and taking away integers is all to do with moving up and down a number line. In an exam, you could draw your own number line to count up and down on if it helps you.

Example 1 – temperature

The temperature at midnight was –6°C. By midday it had risen by 23°C. What was the temperature at midday?

Solution

Start at –6°C. Move up the number line by 23. Answer: 17°C.

You also need to be able to complete add and take away sums. Start at the first number, then move up if you are adding, and down if you are taking away.

Example 2 – adding and taking away

–6 + 9 = start at –6 and move up 9. Answer = 3
5 – 7 = start at 5 and move down 7. Answer = –2
(–2) – 8 = start at –2 and move down 8. Answer = –10

Adding a negative number is the same as taking away. When an add and a take away sign are written next to each other, you can “ignore” the add sign.

Example 3– adding a negative

2 + (–6) = 2 – 6 = start at 2 and move down 6. Answer = –4
(–1) + (–7) = (–1) – 7 = start at –1 and move down 7. Answer = –8

Taking away a negative number becomes an add. When two negative signs are written next to each other without a number in between, they become an add sign

“taking away a negative makes an add”

Example 4 – taking away a negative

5 – (–2) = 5 + 2 = 7
(–7) –(–2) = (–7) + 2 = start at –7 and move up 2. Answer = –5

Multiplying and Dividing Negative Numbers (Integers)

Multiplying and dividing integers have completely different rules to adding and taking away. To multiply and divide, you do the sum normally (as if there were no negative signs there), and then you decide whether your answer needs to be negative or positive.

When multiplying and dividing:

- If none of the numbers are negative, then the answer is positive.
- If one of the numbers is negative, then the answer is negative.
- If two of the numbers are negative, then the answer is positive.
- If three of the numbers are negative, then the answer is negative.
- and so on…

In short, the rules are:

- + multiplied by + gives you +  
- - multiplied by + gives you -  
- + multiplied by - gives you -  
- - multiplied by - gives you +

+ divided by + gives you +  
- divided by + gives you -  
+ divided by – gives you –  
- divided by – gives you +

---

Example 1 – multiplication

\(-5\) \times 4 = \(-20\) \hspace{1cm} \text{(one negative number means the answer is negative)}  
60 \times \(-2\) = \(-120\) \hspace{1cm} \text{(one negative number means the answer is negative)}  
\(-3\) \times \(-10\) = +30 \hspace{1cm} \text{(two negative numbers means the answer is positive)}  
\(-2\) \times 3 \times \(-4\) = 24 \hspace{1cm} \text{(two negative numbers means the answer is positive)}  
\(-2\) \times \(-3\) \times \(-4\) = \(-24\) \hspace{1cm} \text{(three negative numbers means the answer is negative)}

In particular, if you square a negative number, the answer always has to be positive, because you are multiplying two negative numbers.

---

Example 2 – squaring

\((-6)^2 = \(-6\) \times \(-6\) = 36 \hspace{1cm} \text{(two negative numbers means the answer is positive)}  
\((-10)^2 = \(-10\) \times \(-10\) = 100 \hspace{1cm} \text{(two negative numbers means the answer is positive)}

---

Example 3 – dividing

\(-28\) ÷ 4 = \(-7\) \hspace{1cm} \text{(one negative number means the answer is negative)}  
50 ÷ \(-5\) = \(-10\) \hspace{1cm} \text{(one negative number means the answer is negative)}  
\(-80\) ÷ \(-10\) = +8 \hspace{1cm} \text{(two negative numbers means the answer is positive)}
Fractions and Percentages

Changing Fractions, Decimals and Percentages

To change a fraction to a decimal, divide the top number by the bottom number.

Example 1 – fractions to decimals

Change the fraction \( \frac{13}{20} \) to a decimal

Solution

\[ 13 \div 20 = 0.65 \]

Answer: 0.65

To change a percentage to a decimal, divide by 100.

To change a decimal to a percentage, multiply by 100.

Example 2 – percentages and decimals

Change the percentage 2.5% to a decimal
Change the decimal 0.8 to a percentage

Solution

(a) \( 2.5 \div 100 = 0.025 \),

Answer: \( 2.5\% = 0.025 \) as a decimal.

(b) \( 0.8 \times 100 = 80 \),

Answer: \( 0.8 = 80\% \) as a percentage.

To change a percentage to a fraction, the number on the top is always 100 and the number on the bottom is the number before the percentage sign.

Example 3 – percentages to fractions

Write the percentage 37% as a fraction

Solution

\( 37\% = \frac{37}{100} \)

To change a decimal to a fraction, the denominator (the number on the bottom) depends on how many digits there are after the decimal point. If there:

- is one digit after the point, the denominator is 10 (e.g. \( 0.3 = \frac{3}{10} \)).
- are two digits after the point, the denominator is 100 (e.g. \( 0.17 = \frac{17}{100} \)).
- are three digits after the point, the denominator is 1000 (e.g. \( 0.451 = \frac{451}{1000} \)).

Example 4 – decimals to fractions

Change the decimal 0.013 to a fraction
Change the decimal 0.57 to a fraction

Solution

(a) \( 0.013 = \frac{13}{1000} \) 
(b) \( 0.57 = \frac{57}{100} \)
To change a fraction to the percentage, there are two steps:

1. Change to a decimal by **dividing**
2. Change to a percentage by **multiplying by 100**

### Example 5 – fractions to percentages

Pete got 24 out of 32 for an exam. What is his mark as a percentage?

**Solution**

As a fraction, Pete got \( \frac{24}{32} \).

To change this to a decimal, divide:

\[
24 \div 32 = 0.75
\]

To change 0.75 to a percentage, multiply by 100:

\[
0.75 \times 100 = 75\%
\]

A quick way of remembering this is to do \( \text{top ÷ bottom} \times 100 \)

### Calculating Fractions

This topic is assessed in **Question 3 of Paper 1 of the ADDED VALUE UNIT**. See page 4 for more details.

To calculate a fraction, you divide by the number on the bottom (the denominator) and multiply by the number on the top (the numerator).

**Sometimes we say this as ‘Divide by the bottom and times by the top’**

### Example

A washing machine costs £385. \( \frac{3}{5} \) of the cost is for the materials. What is the cost of the materials?

**Solution**

\[
\frac{3}{5} \text{ of } 385 = 385 \div 5 \times 3
\]

\[
= 77 \times 3
\]

\[
= £231
\]

### Percentages without a calculator

You will be asked to calculate a percentage in the non calculator paper. You should know the following:

<table>
<thead>
<tr>
<th>Percentage</th>
<th>Fraction</th>
<th>Percentage</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>50%</td>
<td>( \frac{1}{2} )</td>
<td>10%</td>
<td>( \frac{1}{10} )</td>
</tr>
<tr>
<td>25%</td>
<td>( \frac{1}{4} )</td>
<td>1%</td>
<td>( \frac{1}{100} )</td>
</tr>
<tr>
<td>75%</td>
<td>( \frac{3}{4} )</td>
<td>33 ( \frac{1}{3} )%</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>20%</td>
<td>( \frac{1}{5} )</td>
<td>66 ( \frac{2}{3} )%</td>
<td>( \frac{2}{3} )</td>
</tr>
</tbody>
</table>
This topic is assessed in **Question 1 of Paper 1 of the ADDED VALUE UNIT**. See page 4 for more details.

### Examples 1 and 2

What is 75% of 480cm?  
What is $33\frac{1}{3}$% of £330?

#### Solution

<table>
<thead>
<tr>
<th>Percentage</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>75% of 480cm</td>
<td>$\frac{3}{4} \times 480 = 360$ cm</td>
<td></td>
</tr>
<tr>
<td>$33\frac{1}{3}$% of £330</td>
<td>$\frac{1}{3} \times 330 = 110$ £</td>
<td></td>
</tr>
</tbody>
</table>

Other percentages can be worked out without a calculator by finding 1% or 10% first.

For example to find 30%, find 10% first, then multiply the answer by 3.

To find 4%, find 1% first then multiply the answer by 4.

### Examples 3 and 4

What is 40% of £120?  
What is 7% of 3000kg?

#### Solution

<table>
<thead>
<tr>
<th>Percentage</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>40% of £120</td>
<td>$12 \times 4 = £48$</td>
<td></td>
</tr>
<tr>
<td>7% of 3000kg</td>
<td>$30 \times 7 = 210$ kg</td>
<td></td>
</tr>
</tbody>
</table>

### Percentages with a calculator

For every question, there are two ways of doing it. Use the one you are happiest with.

<table>
<thead>
<tr>
<th>Question</th>
<th>Method 1</th>
<th>Method 2</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>27% of £360</td>
<td>$360 \div 100 \times 27$</td>
<td>$0.27 \times 360$</td>
<td>£97.20</td>
</tr>
<tr>
<td>3% of £250</td>
<td>$250 \div 100 \times 3$</td>
<td>$0.03 \times 250$</td>
<td>£7.50</td>
</tr>
<tr>
<td>17.5% of £4200</td>
<td>$4200 \div 100 \times 17.5$</td>
<td>$0.175 \times 4200$</td>
<td>£735</td>
</tr>
<tr>
<td>4.2% of £360</td>
<td>$360 \div 100 \times 4.2$</td>
<td>$0.042 \times 360$</td>
<td>£15.12</td>
</tr>
</tbody>
</table>

### Example 1

A car is normally priced at £8800. In a sale, the price has been reduced by 12%. What is the new price of the car?

#### Solution

<table>
<thead>
<tr>
<th>Percentage</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>12% of £8800</td>
<td>$0.12 \times 8800 = 1056$</td>
<td>£7744</td>
</tr>
</tbody>
</table>

New price = £8800 – £1056 = £7744

You need to be able to work out percentages when interest is paid on money in a bank account. Interest is expressed as a percentage per annum (p.a.) **Per annum** means “per year”.

---

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**Page 18**
Example 2 (Bank Account Interest)

Vanessa puts £4800 in the bank at an interest rate of 4% per annum. How much interest would she receive after:

One year?
Five months?

Solution

a) In one year: 4% of £4800 = 0.04 × 4800 = £192

b) There are twelve months in a year, so in one month, Vanessa will get 192 ÷ 12 = £16

This means that in five months, she will get 16 × 5 = £80

Calculating the Percentage Increase or Decrease

To find the percentage increase or decrease, we have to use the method for changing fractions to percentages from page 17.

In these questions, we have to work out the percentage of the original amount. The steps are:

1) Work out the increase (or decrease) by taking away
2) Dividing by the original amount
3) Multiplying by 100

**Formula:** this formula is not given on the formula sheet in assessments

\[
\text{Percentage increase/decrease} = \frac{\text{change}}{\text{original amount}} \times 100
\]

Example

The temperature in an oven was 180°C. It went up to 207°C. What was the percentage increase?

Solution

Step one – what is the increase? 207 – 180 = 27°C

Step two – write as a fraction of the original amount

Original amount was 180°C, so as a fraction this is \( \frac{27}{180} \).

Step three – divide and multiply by 100 to change to a percentage:

\[ 27 ÷ 180 \times 100 = 15\% \]
Length, Area and Volume

Area of a Rectangle and Triangle

**Formula:** these formula are not given on the formula sheet in assessments

**Area of a rectangle:** \( A = \text{Length} \times \text{Breadth}, \) or \( A = LB \)

**Area of a square:** \( A = L^2 \)

**Area of a triangle:** \( A = \frac{1}{2} \times \text{Base} \times \text{Height}, \) or \( A = \frac{BH}{2} \)

In any area question, the units are ‘squared’ units (e.g. cm², m², mm²). You would always lose the final mark if you did not include these units in your answer.

**Examples**

Find the areas of these shapes

![Diagram showing a triangle, square, and rectangle with dimensions 10cm, 13cm, 14cm, 25m, 3m]

**Solution**

\[
A = \frac{BH}{2} = \frac{13 \times 10}{2} = \frac{130}{2} = 65 \text{ cm}^2
\]

\[
A = L^2 = 14^2 = 196 \text{ cm}^2
\]

\[
A = LB = 25 \times 3 = 75 \text{ m}^2
\]

Volumes of Cubes and Cuboids

**Definition:** the volume of a 3d shape is a measure of the amount of space inside it.

**Formula:** this formula is not given on the formula sheet in assessments

**Volume of a cuboid:** \( V = \text{Length} \times \text{Breadth} \times \text{Height}, \) or \( V = LBH \)

In any volume question, the units are ‘cubed’ units (e.g. cm³, m³, mm³). You would always lose the final mark if you did not include these units in your answer.

**Example 1 – cuboid**

Calculate the volume of this cuboid

**Solution**

\[
V = LBH = 5 \times 4 \times 3 = 60 \text{ cm}^3
\]
Example 2 – cube

**Calculate the volume of this cube**

**Solution**

In a cube, all the sides are the same length.

\[ V = L \times B \times H \]

\[ = 40 \times 40 \times 40 \quad \text{(NOT} \ 40 \times 3) \]

\[ = 64000 \text{cm}^3 \]

Some volume questions refer to litres and millilitres. The key facts are:

| 1cm³ = 1 millilitre | 1000 millilitres = 1 litre |

A millilitre and a centimetre cubed are essentially the ‘same thing’. One millilitre of water takes up 1cm³ of space.

Example 3 – litres

**A tank of water is shaped as shown. How many litres of water can it hold?**

**Solution**

\[ V = L \times B \times H \]

\[ = 60 \times 25 \times 10 \]

\[ = 15000 \text{cm}^3 \]

15000cm³ = 15000ml because millilitres and cm³ are the same thing.

There are 1000ml in a litre, so in this tank there are \( 15000 \div 1000 = 15 \) litres.
Speed, Distance and Time

You need to know how to work out how much time an event takes from beginning to end. The best way is to split each question up into smaller steps:

**Example 1**

How long is it from 10.45am to 2.20pm?

**Solution**

\[\text{Total time} = 15 \text{ minutes} + 3 \text{ hours} + 20 \text{ minutes} = 3 \text{ hours 35 minutes} \]

**Example 2**

A plane leaves Paris at 9.45pm and arrives in Los Angeles the next morning at 7.10am. How long was the flight?

**Solution**

\[\text{Total time} = 15 \text{ min} + 2 \text{ hours} + 7 \text{ hours} + 10 \text{ min} = 9 \text{ hours 25 min} \]

**Changing time to a decimal**

This topic is assessed in **Question 4 of Paper 2 of the ADDED VALUE UNIT**. See page 4 for more details

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{4}$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Remember basic fractions and decimals

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Decimal (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 hour 30 minutes</td>
<td>1 $\frac{1}{2}$ hours</td>
</tr>
<tr>
<td>5 hours 15 minutes</td>
<td>5 $\frac{1}{4}$ hours</td>
</tr>
<tr>
<td>45 minutes</td>
<td>$\frac{3}{4}$ hour</td>
</tr>
</tbody>
</table>
We can change other fractions of an hour using our knowledge of decimals and tenths. In particular, it is worth noting the fact that \(0.1\text{ hour} = 6\text{ minutes}\).

**Example 1**

Change 7.2 hours into hours and minutes

**Solution**

\[
0.2 \text{ hours} = \frac{2}{10} \text{ of an hour} = \frac{2}{10} \text{ of 60 minutes}
\]

\[
\frac{2}{10} \text{ of 60} = 60 \div 10 \times 2 = 12
\]

**Answer:** 7.2 hours = 7 hours 12 minutes

**Example 2**

Change 5 hours 18 minutes into a decimal

**Solution**

18 minutes is \(\frac{18}{60}\) as a fraction.

If we simplify this fraction, we get \(\frac{18}{60} = \frac{3}{10}\)

As a decimal, \(\frac{3}{10}\) is written 0.3

**Answer:** 5 hours 18 minutes = 5.3 hours.

**Speed, Distance and Time Calculations**

This topic is assessed in **Question 4 of Paper 2 of the ADDED VALUE UNIT.** See page 4 for more details

**Formula:** these formulae are **not** given on the formula sheet in assessments

\[
\begin{align*}
\text{Speed} & = \frac{\text{Distance}}{\text{Time}} \\
\text{Time} & = \frac{\text{Distance}}{\text{Speed}} \\
\text{Distance} & = \text{Speed} \times \text{Time}
\end{align*}
\]

If you have been taught it, you might also like to use the Speed, Distance and Time triangle

**Units:** the units for speed, distance and time are interlinked:

- If distance is in **km** and time is in **hours** (h), speed is measured in **km/hour**
- If distance is in **metres** (m) and time is in **seconds** (s), speed is measured in **m/s**
- If the speed is in **cm/min**, then the distance is in **cm** and the time in **minutes**
Sometimes speed is referred to as **average speed** or **mean speed** to reflect the fact that it can vary during a journey. This makes no difference to how you answer a question.

---

**Example 1 – find speed**

I drive 90km in 2 hours and 15 minutes. Calculate my average speed

**Solution**

We are working out speed, so we use the formula \( \text{Speed} = \frac{\text{Distance}}{\text{Time}} \)

2 hours and 15 minutes is not 2.15 hours. It is 2 ¼ hours = 2.25 hours.

\[
\text{Speed} = \frac{90}{2.25} = 40 \text{km/h}
\]

---

**Example 2 – find distance**

A bird flies 3½ hours at an average speed of 42km/h. How far does it fly?

**Solution**

We are working out distance, so we use the formula \( \text{Distance} = \text{Speed} \times \text{Time} \)

3 ½ hours is not 3.30 hours. It is 3.5 hours.

\[
\text{Distance} = 42 \times 3.5 = 147 \text{km}
\]

---

**Example 3 – find time**

A driver travels 129 miles at an average speed of 30mph. How long does it take her? Give your answer in hours and minutes.

**Solution**

We are working out time, so we use the formula \( \text{Time} = \frac{\text{Distance}}{\text{Speed}} \)

\[
\text{Time} = \frac{129}{30} = 4.3
\]

4.3 hours is not 4 hours 3 minutes.

\[
0.3 \times 60 = 18, \text{ so } 4.3 \text{ hours} = 4 \text{ hours} \ 18 \text{ minutes.}
\]
Graphs, Charts and Tables

Bar Graphs, Line Graphs and Tables
In an assessment, you might be asked to draw a bar graph or a line graph. This means that you need to remember the difference between them:

Questions will usually ask you to use the graph to make a comment. They often ask you to describe the trend of a graph. The trend is an overall description of what the graph is showing. For example in the two pictures above, the trend in the bar graph is that the figures are getting higher. The trend in the line graph is that the figures are getting lower.

For the National 4 numeracy assessment, we are expected to be able to read information from tables with at least four rows or columns.

Pie Charts

A pie chart shows how something is split up into different categories. When told the angle of a particular slice, you need to be able to work out the original frequency.

E.g. this pie chart shows the results of a survey into newspapers that people buy:

The slice for The Herald is 120° out of 360°. As a fraction this is $\frac{120}{360}$ (we could simplify this if we chose to, but we don’t need to).

The angles in a pie chart always add up to 360°, so the fraction will always be out of 360.

Example
1800 people were asked what newspaper they bought. The pie chart above shows the results. How many people bought The Mirror?

Solution
The slice for The Mirror is 50°, so the fraction of people who chose The Mirror is $\frac{50}{360}$.
There were 1800 people in total, so we calculate $\frac{50}{360}$ of 1800.
1800 ÷ 360 × 50 = 250, so 250 people said they bought The Mirror.
For the Expressions and Formulae unit, you also need to know how to draw a pie chart, which is covered on page 50.

**Stem and Leaf Diagrams**

A Stem-and-leaf diagram is another way of showing data. The easiest way to make one is to make an unordered one first, and then to make a second, ordered diagram.

It must always have a **key**. The key in this diagram is indicated by the arrow.

### Example

Make a stem and leaf diagram to show class 5C’s test results:

93 44 37 57 82 92 36 39
54 65 67 30 48 51 59 73
86 91 58 64 37 62 56 45

**Solution**

**Step one** – produce an unordered diagram

3 | 7 6 9 0 7
4 | 4 8 5
5 | 7 4 1 9 8 6
6 | 5 7 4 2
7 | 3
8 | 2 6
9 | 3 2 1

**Step two** – rewrite each row in order, and give the diagram a **key** and a **title**

### Probability

To pass the Numeracy unit, you also need to be able to use probability skills. This is also covered in the Expressions and Formulae unit. See the notes starting on page 51.
**Ratio and Proportion**

A **rate** is a way of comparing numbers when the figures are different. A rate is often expressed using the word ‘per’ which means ‘for each one’:

- Texts per month (how many texts did you send in one month?)
- Miles per hour (how many miles did you travel in one hour?)
- Pence per kilogram (how many pence does one kilogram cost?)

To calculate a rate, we divide. Order is important: the word before *per* is divided by the word after *per*. e.g. for pence per kilogram, our sum is pence ÷ kilograms.

---

**Example**

**A car travels 300 miles on 20 gallons of fuel. How many miles per gallon is this?**

**Solution**

For miles per gallon, we do the sum miles ÷ gallons: 300 ÷ 20 = 15.

**Answer:** 15 miles per gallon. *(in a real life situation units are essential)*

---

**Ratio**

Another way to describe the proportions in which quantities are split up is with **ratio**. Ratios consist of numbers separated by a colon symbol e.g, 2:3, 4:1, 3:2:4 .

For example, it might be said that a particular shade of purple paint is made by mixing red paint and blue paint in the ratio 4:5. This means that for every 4 litres (or spoonfuls, tins, gallons) of red paint, you must add 5 litres (or spoonfuls, tins, gallons…) of blue paint to get the correct shade of purple.

---

**Example**

**Claire and David share £4500 in the ratio 4:5. How much money does each get?**

**Solution**

Claire gets 4 shares of the money, David gets 5 shares.
This means that there are 9 shares in total.

Claire gets 4 out of 9 shares:

\[
\frac{4}{9} \text{ of } £4500 = 4500 \div 9 \times 4 = £2000
\]

David gets 5 out of 9 shares:

\[
\frac{5}{9} \text{ of } £4500 = 4500 \div 9 \times 5 = £2500
\]

Therefore Claire gets £2000 and David gets £2500
Direct Proportion

A direct proportion question is one where you have to use the fact that numbers change at the same rate. The method for one of these questions is usually to find the cost for one first.

Example 1
John hires a car for 4 days. It costs him £90. How much will it cost for his friend Sam to hire it for 7 days?

Solution
Step One – How much does it cost to hire the car for one day?
Divide by 4: 90 ÷ 4 = 22.5, so it costs £22.50 per day.

Step Two – How much does it cost to hire the car for seven days?
Multiply by 7: 22.50 × 7 = £157.50.

Another way to consider this problem is by considering equivalent ratios (by dividing or multiplying both numbers to create a new, equivalent ratio). A table usually helps to lay these calculations out:

<table>
<thead>
<tr>
<th>Days</th>
<th>Pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>90</td>
</tr>
<tr>
<td>1</td>
<td>22.50</td>
</tr>
<tr>
<td>7</td>
<td>157.50</td>
</tr>
</tbody>
</table>

The last entry in the final row shows us the answer is that 7 days hire costs £157.50 (remembering units and two decimal places for money).

Example 2
Jo is an electrician. She charges customers £27 for every 15 minutes she has to work. How much will they pay Jo for a job that lasts 3 hours?

Solution
Step One – How much does it cost for one hour?
15 minutes cost £27. There are four lots of 15 minutes in an hour, so for one hour it costs 27 × 4 = £108.

Step Two – How much does it cost for three hours?
Multiply: 108 × 3 = £324.

Using the table approach gives us a way of considering the mixture of units in the question. The right-hand column can be expressed in either minutes or hours, but either way we get the same answer of £324.
Coordinates

The basic rule for coordinates is go along and then up (often said as “along the corridor and up the stairs”). The first part of the coordinate tells you how far to count along, the second number tells you how far to count up (or down).

Example 1

The coordinates of the points on the grid are:

A(4, 2)   B(8, –6)   C(–9, –3)   D(–5, 10)
E(6, 0)   F(–7, 0)   G(0, –7)   H(0, 8)

The point (0,0) is usually referred to as the origin and is in the centre of this diagram.

In the Added Value Unit, you will be asked to plot three coordinates on a grid, and then to plot a fourth point to make a particular shape.

The shape you will be asked to complete might be a rectangle, rhombus, parallelogram or kite. You can find out more about parallelograms and kites on page 38. A rhombus is a shape with two lines of symmetry, and is used in the example below.

Example 2 – completing a shape

Draw coordinate axes and plot A(–5, 1) B(3, 1) and C(–1, 3)
Plot a fourth point D to form a rhombus ABCD
Write down the coordinates of the point D
Solution

Part (a)

Step one – draw axes on to the blank grid

Step two – plot and label the three points

(a) A rhombus has two lines of symmetry, so we need to plot point D so that
D is the reflection of C in line AB.

(b) Point D is (–1, –1)

Note: rectangles, parallelograms and rhombuses all have two pairs of parallel sides.
This is another fact that can help plot the fourth point.
Expressions and Formulae Unit

Algebra

Using a Formula

Read the question carefully and substitute in the numbers you are given.

**Definition:** Evaluate means “do the sum”

---

**Example 1**

Evaluate $2a - 3c$ when $a = 12$ and $c = 1.5$

**Solution**

\[
2a - 3c \\
= 2 \times 12 - 3 \times 1.5 \\
= 24 - 4.5 \\
= 19.5
\]

---

**Example 2**

Evaluate $3x^2$ when $x = 5$

**Solution**

\[
3x^2 \\
= 3 \times 5^2 \\
= 3 \times 25 \\
= 75
\]

---

**Example 3**

\[
S = 3bc - a \\
\text{Evaluate $S$ when $a = 10$, $b = 2$ and $c = 7$}
\]

**Solution**

\[
S = 3bc - a \\
= 3 \times 2 \times 7 - 10 \\
= 42 - 10 \\
= 32
\]

You would also be expected to understand a formula in a real-life situation, where the letters would be explained to you.

---

**Example 4 – real life situation**

The cost of hiring a car is given by the formula $C = 25d + 2p$, where $C$ is the cost of hiring the car, $d$ is the number of days hired for, and $p$ is the number of litres of petrol used.

Find the cost of hiring the car for 3 days and using 40 litres of petrol.
Solution

$d$ is the days. We are asked for the cost of hiring for 3 days, so $d = 3$

$p$ is the number of litres of petrol. We are asked for the cost of using 40 litres of petrol, so $p = 40$.

We now substitute $d = 3$ and $p = 40$ into the equation given:

\[ C = 25d + 2p \]
\[ = 25 \times 3 + 2 \times 40 \]
\[ = 75 + 80 \]
\[ = 155 \]

**Answer:** the total cost of hiring the car is £155. (In a real life situation we should give our answer in a sentence, and units are essential)

---

### Simplifying

You would be expected to simplify a basic algebraic expression by collecting like terms.

You can only add or take away letters that are the same. Constants (numbers with no letters attached) must be kept separate.

**Examples**

Simplify:

(a) $4a + 5b - 2a + b$

(b) $3x + 5y + 4 - x + 4y - 2$

**Solutions**

(a) $2a + 6b$

(b) $2x + 9y + 2$

---

### Multiplying out Brackets

Multiply everything inside the bracket by the number outside the bracket.

**Examples 1**

Multiply out the brackets: (may also be called “expand the brackets”)

- $3(x + 5)$
- $2(3y + 4)$

**Solution**

- $3(x + 5) = 3x + 15$
- $2(3y + 4) = 6y + 8$

Sometimes you need to simplify if there are other letters or numbers outside the brackets. You always have to simplify if you are able to, even if you are not explicitly told to. *Only* numbers inside the bracket are multiplied. Anything else that is not inside the bracket should remain unchanged until you start simplifying.
Examples 2
Multiply out the brackets and simplify:

\[4(m + 5) - 18 \quad 4(x + 5) + 3(x - 2)\]

Solution

\[
4(m + 5) - 18 = 4m + 20 - 18 = 4m + 2 \\
4(x + 5) + 3(x - 2) = 4x + 20 + 3x - 6 = 7x + 14
\]

However, be careful the only numbers (or letters) that you multiply by are ones that are right next to the bracket. Anything else that is not inside the bracket should remain unchanged until you start simplifying.

Examples 3
Multiply out the brackets and simplify:

\[4 + 7(a + 2) \quad 2x + 3(x + 1)\]

Solution

\[
4 + 7(a + 2) = 4 + 7a + 14 \quad (NOT \ 11(a + 2)) = 2x + 3x + 3 \quad (NOT \ 5(x + 1)) \\
= 18 + 7a \quad (or \ 7a + 18) = 5x + 3 \quad (or \ 3 + 5x)
\]

Factorising

Definition: Factorise means “put the brackets back in”. Think of it as the opposite of multiplying out the brackets.

Example 1
Factorise \(6a + 9b\) \quad Factorise \(15x + 25y\)

Solution

The highest common factor is 3 \quad The highest common factor is 5
Write 3 in front of the bracket \quad Write 5 in front of the brackets
\[3(\quad) \quad 5(\quad)\]

Work out what goes inside the brackets (it may help to think of dividing)

Answer: \(3(2a + 3b)\) \quad Answer: \(5(3x + 5y)\)

You always need to take the largest possible number (and/or letter) outside the brackets. You can spot these questions as they will say factorise fully instead of just factorise.

Example 2
Factorise fully: \(18x + 24\)

Solution

You could answer \(2(9x + 12)\) or \(3(6x + 8)\). However you would not get full marks as the biggest number that goes into both 18 and 24 is 6. This means
that 6 needs to be outside the bracket.

**Correct answer:** $6(3x + 4)$

---

**Formulae and Patterns**

You need to be able to write a formula that represents a number pattern in a real-life situation.

This topic is assessed in **Question 3 of Paper 2 of the ADDED VALUE UNIT**. See page 4 for more details

At National 4 level, your formula with always be **linear**. This means it will have a multiplication of a letter followed by an add or take away. For example your formula could be $y = 2x + 3$ or $C = 10p - 4$ or $H = 7p - 3$.

**Rules for writing a formula:**

1) The letter from the bottom line of the table is the one *in front of* the equals sign
2) The letter from the top line of the table is the one *after* the equals sign.
3) The number we multiply by is the number that we are ‘going up in’ along the bottom row.
4) To get the number we add or take away, we compare to the relevant times table (see the examples below for an explanation of what is meant by this).

**Example 1**

Here is a table showing how many chairs are needed to surround restaurant tables of different lengths. Write a formula for calculating the number of chairs ($C$) when you know the number of tables ($T$).

<table>
<thead>
<tr>
<th>Tables ($T$)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chairs ($C$)</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

**Solution**

The letter from the bottom line of the table ($C$) must be the one in front of the equals sign, and the letter from the top line ($T$) is the one after the equals sign:

$$C = ?T + ?$$

From looking at the bottom row of the table, the numbers are going up in 2s, so the number we multiply by is 2:

$$C = 2T + ?$$

The 2 times table is 2, 4, 6, 8, 10, .... However the sequence we *actually* have is 3, 5, 7, 9, 11, ... This means we have added 1 to all the numbers. So our final formula is:

$$C = 2T + 1$$
In National 4 assessments, you are expected to work with number patterns from diagrams. Each question will be set in a different 'real-life' situation, but the basic structure of the question tends to be the same.

Part (a): complete a table with numbers obtained from the diagram.
Part (b): write a formula in using the method outlined above.
Part (c): use the formula to go backwards.

Example 2 – 2011 Standard Grade General Past Exam Question

Margaret is working on the design for a gold bracelet. She is using gold lengths to make each section.

Complete the table below:

<table>
<thead>
<tr>
<th>Number of sections (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of gold lengths (g)</td>
<td>6</td>
<td>10</td>
<td></td>
<td></td>
<td>42</td>
</tr>
</tbody>
</table>

Write down a formula for calculating the number of gold lengths (g) when you know the number of sections (s)
Margaret uses 66 gold lengths to make a bracelet. How many sections does this bracelet contain?

Solution

(a) The completed table is:

<table>
<thead>
<tr>
<th>Number of sections (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of gold lengths (g)</td>
<td>6</td>
<td>10</td>
<td>14</td>
<td>18</td>
<td>42</td>
</tr>
</tbody>
</table>

(b) Using the same method as in example 1:
Step one – get the letters the correct way about \( g = ?s + ? \)
Step two – the bottom row is going up by 4, so we must multiply by 4:
\[ g = 4s + ? \]
Step three – the 4 times table is 4, 8, 12, … We have 6, 10, 14, … so we are adding 2 each time

**Final Answer:** \( g = 4s + 2 \)

(c) There are a few methods to do this part. One way is to add more columns to the table until you get to 66 on the bottom row. This would work, but it takes a while. Another way is to solve an equation. However whatever method you use, you must show some working.

To solve an equation for 66 gold lengths, we take our formula from part (b) \((4s + 2)\) and write ‘= 66’ on the end of it.

\[ 4s + 2 = 66 \]

\[ 4s = 66 - 2 \] (moving the + 2 over to become – 2)
\[ 4s = 64 \] (simplifying 66 – 2)
\[ s = \frac{64}{4} \] (dividing by 4)
\[ s = 16 \]
Gradient

The **gradient** of a slope is its steepness. The higher the gradient is, the steeper the slope is.

- A gradient of 2 means ‘for every 1 you go along, you go up 2’.
- A gradient of 0.68 means ‘for every 1 you go along, you go up 0.68’.

Sometimes gradient is referred to as **average gradient** or **mean gradient** to reflect the fact that it can vary as you go up a slope. This makes no difference to how you answer a question.

**Formula:** this formula is given on the formula sheet in assessments

\[ \text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}} \]

**Example 1**

**Calculate the gradient of this ramp.**

**Solution**

\[ \text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}} = \frac{4.2}{28.5} = 0.147 \text{ (3d.p.)} \]

**Example 2 – from coordinates**

**Find the gradient between the coordinate points (–1, 2) and (3, 7)**

**Solution**

From counting squares in the diagram, the vertical height is 5 and the horizontal distance is 4.

\[ \text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}} = \frac{5}{4} = 1.25 \]

In some situations, such as on coordinate grids, direction is important. In these situations, a **positive** gradient (e.g. gradients such as 2, 10 or \( \frac{1}{2} \)) means the line slopes **upwards**. A **negative** gradient (e.g. gradients such as –2, –10 or \( –\frac{1}{2} \)) means the line slopes **downwards**.
Circumference, Areas and Volume

Circumference of a Circle

Definitions:
- the diameter of a circle is the distance across a circle (passing through the centre).
- the radius is half of the diameter.
- the circumference is the curved length around the outside of a circle. It is a special name for the perimeter of a circle.

Formula: this formula is given on the formula sheet in assessments

Circumference of a circle: 

\[ C = \pi d \]

The units for circumference are just ‘normal’ units (not squared or cubic). If the diameter is measured in centimetres or metres, then so is the circumference.

Examples

Calculate the circumferences of these two circles:

\[ C = \pi d \]

1. The diameter is 12cm, so \( d = 12 \)
   \[ C = \pi \times 12 \quad \text{(or } 3.14 \times 12) \]
   \[ = 37.6991184... \]
   \[ = 37.7 \text{ cm (1d.p.)} \]

2. In this circle, the radius is 13m so the diameter is 26m, i.e. \( d = 26 \)
   \[ C = \pi \times 26 \quad \text{(or } 3.14 \times 26) \]
   \[ = 81.68140899... \]
   \[ = 81.68 \text{ m (2d.p.)} \]

Area of a Circle

Formula: this formula is given on the formula sheet in assessments

Area of a circle: 

\[ A = \pi r^2 \]

The units for area are ‘squared’ units. If the radius is measured in cm, then the area is measured in cm\(^2\). If the radius is measured in miles, the area is measured in miles\(^2\).

Example 1 – radius

Find the area of this circle

Solution

The radius of this circle is 5cm, so \( r = 5 \).
\[ A = \pi r^2 \]
\[ = \pi \times 5^2 \quad \text{(or } 3.14 \times 5^2) \]
\[ = \pi \times 5 \times 5 \quad \text{(or } 3.14 \times 5 \times 5) \]
\[ = 78.53981634... = 78.5 \text{cm}^2 \quad \text{(1d.p.)} \]

**Example 2 – diameter**

**Find the area of this circle**

**Solution**

8 mm is the diameter, so the radius is 4 mm, or \( r = 4 \).

\[ A = \pi r^2 \]
\[ = \pi \times 4^2 \quad \text{(or } 3.14 \times 4^2) \]
\[ = \pi \times 4 \times 4 \quad \text{(or } 3.14 \times 4 \times 4) \]
\[ = 50.26548... = 50.3 \text{mm}^2 \quad \text{(1d.p.)} \]

You may come across more difficult examples that involve quarter and half circles.

**Definition:** a semicircle is half of a circle.

**Example 3 – semicircle**

**Find the area of this semicircle**

**Solution**

22 cm in this diagram is the _diameter_. This means that the radius is 11 cm or \( r = 11 \text{ cm} \).

\[ A = \pi r^2 \div 2 \]
\[ = \pi \times 11^2 \div 2 \quad \text{(or } 3.14 \times 11^2 \div 2) \]
\[ = \pi \times 11 \times 11 \div 2 \quad \text{(or } 3.14 \times 11 \times 11 \div 2) \]
\[ = 190.0663555... \]
\[ = 190.1 \text{cm}^2 \quad \text{(1d.p.)} \]

**Areas of composite shapes made of rectangles and triangles**

This topic could be assessed as part of **Question 2 of Paper 2 of the ADDED VALUE UNIT**. See page 4 for more details

In the Numeracy unit, you learnt how to work out the area of a rectangle, square or triangle (see page 20). Knowing this means that you can also work out the area of more complex shapes by splitting them up into rectangles and triangles.

**Definition:** a _composite shape_ is one made by joining two or more other shapes together. In National 4 assessments, areas will always be of composite shapes, usually made up of rectangles, squares, triangles or semi-circles (for semicircles see page 37).

The method to work out the area of a composite shape is always the same:
Step one – split the shape up into smaller, simpler shapes.

Step two – work out the area of each smaller shape separately.

Step three – either add or take away the areas:
- If the two shapes are joined together, you add the areas.
- If one shape is ‘cut out of’ the other, you take away its area.

You need to be able to work out the area of a trapezium, or a parallelogram or a kite by splitting the shapes up into rectangles and triangles.

Definitions:
- A quadrilateral is a shape with four straight sides.
- A parallelogram is a quadrilateral with two pairs of parallel sides. It can be split up to be a rectangle and two identical right-angled triangles.
- A kite is a quadrilateral with one line of symmetry. It can be split up to be four triangles (two pairs of equal triangles).
- A trapezium is a quadrilateral with one pair of parallel sides. It can be split up to be a rectangle and one or two right-angled triangles.

Example 1

The shape shown is a trapezium, made up of a rectangle and a right-angled triangle. Calculate the area of the shape.

Solution

Step one – start by splitting the shape up into a rectangle and a triangle.
We don’t know the height of the triangle yet (the length marked ‘?’) – but from looking at the diagram, we can see that $9+? = 15$, so $? = 6$cm.

Step two – Calculate the area of each shape.

Area of triangle $= \frac{BH}{2}$

$= \frac{18 \times 6}{2} = \frac{108}{2} = 54 \text{cm}^2$

Area of rectangle $= LB$

$= 18 \times 9 = 162 \text{cm}^2$

Step three – the two shapes are joined together, so add the areas

Total Area $= 54 + 162 = 216 \text{cm}^2$

Example 2 (Standard Grade General paper 2001)

The base of a lift is in the shape of a rectangle with a semicircle end as shown. Calculate the area of the base of the lift.

Round your answer to 1 decimal place

Solution

Step one – start by splitting the shape up into a rectangle and a semicircle.

Step two – Calculate the area of each shape.

1.2 is the diameter of the circle, so 0.6 is the radius.

Area of semicircle $= \frac{\pi r^2}{2}$

$= \frac{\pi \times 0.6^2}{2} = 0.565... \text{m}^2$

Area of rectangle $= LB$

$= 1.4 \times 1.2 = 1.68 \text{m}^2$

Step three – the two shapes are joined together, so add the areas

Total Area $= 1.68 + 0.565... = 2.245... = 2.2 \text{m}^2$ (1d.p.)
Volume of 3-d Shapes

This topic could be assessed as part of Question 2 of Paper 2 of the ADDED VALUE UNIT. See page 4 for more details

Definition: A **prism** is a 3d solid with a uniform cross-section. In everyday language, we could say that it is the ‘same shape all the way along’.

Definition: The **cross-section** is the shape at either end (and throughout the middle) of a prism.

Formula: this formula is given on the formula sheet in assessments

<table>
<thead>
<tr>
<th>Volume of a prism:</th>
<th>$V = \text{Area of cross-section} \times \text{height}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V = Ah$</td>
</tr>
</tbody>
</table>

In volume questions, the units are always in ‘cubic’ units. If all the lengths in the question are in centimetres (cm), the volume is in ‘cubic centimetres’ (cm³). If all the lengths in the question are in miles, then the volume is measured in miles³.

Example 1 – area of cross-section is given

The shape shown in the diagram on the right is a prism with length 8.9cm. The cross-section (shaded) is a pentagon with area 42cm². Calculate the volume of the prism.

**Solution**

\[ V = Ah \]

\[ = 42 \times 8.9 \]

\[ = 373.8 \text{cm}^3 \]

Example 2 – triangular prism

Find the volume of this prism, whose cross section is a triangle.

**Solution:**

The length of this prism is the distance from one (triangular) end to the other. In this shape, the length is 20cm.

Step 1: Work out the area of the cross-section

In this shape, the cross-section is a triangle. The formula for the area of a triangle is $\frac{1}{2}bh$ (see page 20).
**Important:** you will use a different formula in each question, depending on whether the cross section is a rectangle, square, triangle, circle, semicircle etc.

\[ A_{\text{triangle}} = \frac{1}{2}bh \]
\[ = 10 \times 12 \div 2 \]
\[ = 60 \text{cm}^2 \]

**Step 2:** Use the formula to find the volume

\[ V = Ah \]
\[ = 60 \times 20 \]
\[ = 1200 \text{cm}^3 \]

A special type of prism with a circular cross-section is a cylinder. To find the area of the cross-section we use the usual formula for a circle (\( A = \pi r^2 \)), which gives rise to a special formula.

**Formula:** this formula is given on the formula sheet in assessments

Volume of a Cylinder: \( V = \pi r^2 h \)

---

**Example 3 – cylinder**

**Calculate the volume of this cylinder.**

**Solution**

Diameter is 10cm so radius is 5cm

\[ V = \pi r^2 h \]
\[ = \pi \times 5^2 \times 20 \quad (\text{or } \pi \times 5 \times 5 \times 20) \]
\[ = 1570.796327... \]
\[ = 1600 \text{cm}^3 \text{ (2 s.f.)} \]

---

**Surface Areas of 3-d Shapes**

This topic could be assessed as part of **Question 2 of Paper 2 of the ADDED VALUE UNIT.** See page 4 for more details

A three-dimensional shape has faces, edges and vertices. Given a simple 3-d shape (most likely a cuboid or prism), you are expected to be able to:

1. state the number of faces, edges and vertices
2. draw a possible net of the shape
3. calculate out the surface area of the shape (using its net or otherwise) by splitting it up into individual rectangles, triangles and/or circles.

A **cuboid** is made up of six rectangular faces, made up of three pairs of equal faces.
Example 1 – Cuboid

Draw a possible net for the cuboid shown in the diagram
Calculate the total surface area of the cuboid

Solution

(a) A possible net, made up of six rectangles, is drawn below.

(b) Using the labels A, B and C from the diagram of the net:

Area of rectangle labelled A = Length × Breadth = 10 × 3 = 30cm²
Area of rectangle labelled B = Length × Breadth = 10 × 6 = 60cm²
Area of rectangle labelled C = Length × Breadth = 3 × 6 = 18cm²

Total surface area = A + A + B + B + C + C (or 2A + 2B + 2C)
= 30 + 30 + 60 + 60 + 18 + 18
= 216cm²

A triangular prism is made up of three rectangular faces and two identical triangular faces. To find the surface area of the whole shape, you have to find the area of each one, and add them together.

Example 2 – Triangular prism

For the shape pictured, which is a triangular prism:
Draw a possible net
Calculate the surface area

Solution

(a) A possible net, made up of three rectangles and two identical triangles, is drawn below.
(b) Area of one triangle $= \frac{1}{2} \text{Base} \times \text{Height}$

$= 18 \times 24 \div 2$

$= 216 \text{cm}^2$

Area of rectangle labelled A $= \text{Length} \times \text{Breadth} = 18 \times 50 = 900 \text{cm}^2$

Area of rectangle labelled B $= \text{Length} \times \text{Breadth} = 24 \times 50 = 1200 \text{cm}^2$

Area of rectangle labelled C $= \text{Length} \times \text{Breadth} = 30 \times 50 = 1500 \text{cm}^2$

Total surface area $= \text{Two triangles} + \text{three rectangles}$

$= 216 + 216 + 900 + 1200 + 1500$

$= 4032 \text{cm}^2$

A **cylinder** is made up of two circular faces and the **curved surface area (CSA)**, which is a rectangle curved around. To find the surface area of the whole shape, you have to find the area of each one, and add them together.

**Formula:** these formulae are given on the formula sheet in assessments

- Area of a circle: $A = \pi r^2$
- Curved Surface Area of a cylinder: $A = 2\pi rh$

where $r$ is the radius of the cylinder, and $h$ is the height

---

**Example 3 – cylinder**

**Find the total surface area of this cylinder**

**Solution**

- The radius is 5cm, so $r = 5$
- The height is 3cm, so $h = 3$
Area of circle:
\[
A = \pi r^2
\]
\[
= \pi \times 5 \times 5 \quad \text{(or } 3.14 \times 5 \times 5) \]
\[
= 78.5\text{cm}^2
\]

Curved surface area:
\[
A = 2\pi rh
\]
\[
= 2 \times \pi \times 5 \times 3 \quad \text{(or } 2 \times 3.14 \times 5 \times 3) \]
\[
= 94.2\text{cm}^2
\]

Total = Circle + Circle + Curved Surface Area
\[
= 78.5 + 78.5 + 94.2
\]
\[
= 251.2\text{cm}^2
\]

Rotational Symmetry

**Definitions**
The order of rotational symmetry of a shape is the number of times it will fit back onto itself (exactly) when the shape is rotated 360°

**Half-turn symmetry** is a special name for rotational symmetry of order 2. It means that if you rotated the shape by half a turn, it would still look the same.

**Quarter-turn symmetry** is a special name for rotational symmetry of order 4. If the shape was rotated by 90° in any direction, it would still look the same.

In National 4 assessments, you would be expected to draw the rest of a shape so that it has rotational symmetry about a particular point (usually called O). You need to be careful to count squares as well as getting direction correct.
Example (Standard Grade General 2007 Past Exam Question)

Complete this shape so that it has quarter turn symmetry about O

Solution

The completed diagram would look like this:
Statistics and Probability

Grouped Frequency Tables

A grouped frequency table is used for spread out data where there are a lot of different, but close, values – e.g. heights. Rather than having one number per row (which would result in a massive table), we group the values into rows (e.g. 5s, 10s, 100s) as appropriate.

When discrete data is displayed in a grouped frequency table, we write rows as 0–9, 10–19 etc. Notice that they are not written 0–10, 10–20, as then it would be unclear which row ‘10’ would go in.

Example
(adapted from 2002 Intermediate 2 exam question) The results from a census of a small Scottish village are analysed to find the age of the inhabitants. The results are shown below.

<table>
<thead>
<tr>
<th>Age (to the nearest year)</th>
<th>Number of residents</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 9</td>
<td>4</td>
</tr>
<tr>
<td>10 – 19</td>
<td>9</td>
</tr>
<tr>
<td>20 – 29</td>
<td>11</td>
</tr>
<tr>
<td>30 – 39</td>
<td>16</td>
</tr>
<tr>
<td>40 – 49</td>
<td>21</td>
</tr>
<tr>
<td>50 – 59</td>
<td>18</td>
</tr>
<tr>
<td>60 – 69</td>
<td>17</td>
</tr>
<tr>
<td>70 – 79</td>
<td>4</td>
</tr>
<tr>
<td>TOTAL</td>
<td>100</td>
</tr>
</tbody>
</table>

Mean, Mode and Range

Definition: The range is the difference between the highest and the lowest numbers. It shows how varied (or consistent) a list of numbers is.

Formula: this formula is not given on the formula sheet in assessments
Range = Highest – Lowest

Example 1 – Range
Each day a shop records how much money it takes:
19, 42, 47, 45, 18, 36, 68, 22, 27, 35. What is the range?

Solution
Highest = 68, Lowest = 18
Range = 68 – 18 = 50
Definition: the mean, median and mode are all methods for coming up with an “average value”. They each have advantages and disadvantages in different situations.

If people are talking about “the average” in everyday life, they are probably referring to the mean.

**Formula:** this formula is not given on the formula sheet in assessments

\[
\text{Mean} = \frac{\text{Total}}{\text{How many}}
\]

To find the mean:
1. add all the numbers together
2. divide by how many numbers there are

**Example 2 – Mean**

For the list of numbers in Example 1, find the mean.

**Solution**

Step one – add all the numbers together: 19 + 42 + 47 + .... + 35 = 359

Step two – there are ten numbers, so we divide by 10. 359 ÷ 10 = 35.9

Answer: the mean is 35.9

**IMPORTANT** – if you type 19+42+47+45+18+36+22+27+35÷10 straight into a calculator, you will get the wrong answer. You have to either press equals before you divide, or you need to use brackets: \((19+42+47+45+18+36+22+27+35) \div 10\).

Always check your final answer sounds reasonable.

**Definition:** The mode is the most frequent number (the number that appears the most).

**Example 3 – Mode**

Find the mode: 6 6 6 6 7 7 7 8 8 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9 0 1 0 1 1 1 1 1 2

**Solution**

The mode is 9 because there are more 9s than any other number.

**Median**

**Definition:** The median is the middle number in the list once all the numbers have been written in order.

You can spot the median in a list, because it will have the same number of numbers before it and after it. Drawing a line in the middle of the list can sometimes make this clearer. It is essential to put the list of numbers in order first.
Example 1
Find the median of 8, 6, 4, 2, 2, 5, 8

Solution:
Putting the numbers in order: 2, 2, 4, 5, 6, 8, 8

The arrow divides the list into two equal parts – because there are 3 numbers to the left and three numbers to the right.

The arrow goes straight through the number 5, so the median is 5.

Example 2
Find the median of 8, 6, 4, 3, 2, 6, 7, 8

Solution:
Putting the numbers in order: 2, 3, 4, 6, 6, 7, 8, 8

This time the arrow above is NOT dividing the list into two equal halves, as there are four numbers before the arrow, and only 3 after it. To make the two halves equal, the arrow has to go midway between two numbers:

2, 3, 4, 6, 6, 7, 8, 8

This time the arrow DOES divide the list into two equal halves, as there are four numbers either side of the arrow. The arrow is between 6 and 6. This means that the median is 6. (it is NOT 6.5)

Example 3
Find the median of 8, 10, 16, 19, 23, 12, 14, 16

Solution:
Putting the numbers in order: 8, 10, 12, 14, 16, 16, 19, 23

The arrow is midway between 14 and 16, because that leaves four numbers on either side. This means that the median is 15 because 15 is midway between 14 and 16.

Comparing Statistics

The mean, median and mode are averages. They say whether a list of numbers is higher or lower on average.

The range is NOT an average. Instead it is a measure of spread. It says whether a list of numbers is more or less varied/consistent:

- A lower range means the numbers are more consistent.
- A higher range means the numbers are more varied.
Example
The temperature in Aberdeen has a mean of 3°C and a range of 5. In London it has a mean of 9°C and a range of 3. Compare the temperatures in London and Aberdeen.

Solution
You would get NO MARKS (as you are just stating the obvious) for:
“Aberdeen has a lower mean”, “London has a higher mean”, “Aberdeen has a higher range”, “London has a lower range”.

You WOULD get marks (as you say what the numbers are saying) for:
“The temperature in Aberdeen is lower and the temperature is less consistent”
“The temperature in London is higher and more consistent” or similar

Pie Charts

You need to be able to work out the angles you would use to draw the slices in a pie chart. This is another example of a circles question – you have to work out what fraction of the circle each slice has to be.

Example
In a school survey 72 pupils were asked what their favourite takeaway food was. The results were:

<table>
<thead>
<tr>
<th>Food</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pizza</td>
<td>39</td>
</tr>
<tr>
<td>Fish and Chips</td>
<td>12</td>
</tr>
<tr>
<td>Chinese</td>
<td>21</td>
</tr>
</tbody>
</table>

How many degrees would you need for each sector to represent this data on a pie chart?

Solution
Step One – Work out how many degrees are needed for one person.

The whole pie chart is 360°
The total number of people surveyed was 72

So one person’s share of the pie chart 360 ÷ 72 = 5°

Step Two – multiply each frequency by that number

<table>
<thead>
<tr>
<th>Food</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pizza</td>
<td>39 × 5 = 195°</td>
</tr>
<tr>
<td>Fish and Chips</td>
<td>12 × 5 = 60°</td>
</tr>
<tr>
<td>Chinese</td>
<td>21 × 5 = 105°</td>
</tr>
</tbody>
</table>

Step Three – (optional but highly advisable) check that your answer adds up to 360°. You may then be asked to draw the pie chart.
Probability

**Definition:** **Probability** is a measure of how likely an event is:

- If the event is **impossible**, the probability is 0
- If the event is **certain** to happen, the probability is 1
- If the event is between certain and uncertain, the probability is given as a fraction or decimal.
  - Probabilities closer to zero indicate an event is unlikely to happen.
  - Probabilities closer to one indicate an event is likely to happen.
  - Probabilities close to ½ mean that it is just as likely to happen as not happen.

**Example 1**

In a bag, there are 20 white balls, 30 black balls and 9 blue ones. Taz picks a ball at random. What is the probability that she picks a white ball?

**Solution**

In total there are 20 + 30 + 9 = 59 balls.

20 of these are white. So the probability is \( \frac{20}{59} \).

**Example 2**

In a lottery, the bonus number is picked at random from the numbers 1 to 7. What is the probability that a number smaller than 3 is picked?

**Solution**

Firstly note that smaller than 3 does not include 3.

In total there are 7 numbers.

2 of these are smaller than 3.

So the probability is \( \frac{2}{7} \).

You may be asked to give advice or make a decision based on a probability calculation. To compare two probabilities, it is usually easiest to convert them to decimals.

If you want to have the best chance of something happening (e.g. winning a prize, passing a test), choose the one with the biggest probability.

If you want to have the best chance of something not happening (e.g. getting injured, weather being wet), chose the one with the smallest probability to minimise risk.

**Example 3**

Coco’s Cake Shop are running a competition. If you buy a cake containing a lucky charm, then you win a prize.

In their Mayfield shop, they bake 200 cakes and put lucky charms in 6 of them at random. In their Dalkeith shop, they bake 160 cakes and put lucky charms in 4 of them.
Sharon is going to buy a cake today. Which shop should she buy from to have the best chance of winning a prize?

Solution

The probability of finding a lucky charm in a cake bought from the Dalkeith shop is $\frac{6}{200}$. As a decimal, this is $6 \div 200$, which is 0.03.

The probability of finding a lucky charm in a cake bought from the Mayfield shop is $\frac{4}{160}$. As a decimal, this is $4 \div 160$, which is 0.025.

0.03 is bigger than 0.025 so Sharon should buy a cake from the Dalkeith shop for a (very slightly) increased chance of winning.
Relationships Unit

Algebra

Vertical and Horizontal Lines

If we are given an equation (such as $y = 3x - 5$ or $y = 10 - \frac{1}{2}x$), we can use algebra to come up with coordinates of points that match that equation. These points will always lie on a straight line. If we join the points together, we say we have drawn “the graph of the equation”.

The simplest lines are vertical and horizontal ones. They are very easy to draw. However because we spend most of the time talking about the more complicated lines, people tend to forget these ones.

**Vertical Lines**

Vertical lines have equations such as $x = 2$ or $x = -4$ (we say that these equations are “of the form $x = a$” where $a$ can be any number). The diagram on the right shows that:
- the line $x = 2$ is a line going vertically through 2 on the $x$ axis.
- the line $x = -4$ is a line going vertically through -4 on the $x$ axis.

**Horizontal lines**

Horizontal lines have equations such as $y = 1$ or $y = -3$ (we say that these equations are “of the form $y = b$” where $b$ can be any number). The diagram on the left shows that:
- the line $y = 1$ is a line going horizontally through 1 on the $y$ axis.
- the line $y = -3$ is a line going horizontally through -3 on the $y$ axis.
Example 1

**Draw the line with equation** \( y = 3 \)

**Solution**

\( y = 3 \) has to go through number 3 on the \( y \)-axis, so we can mark that point \((3, 0)\) straight away.

To go through that point, the line has to be horizontal, so we can draw the line and complete the graph.

We must remember to label the graph \( y = 3 \).

---

Example 2

The diagram below shows the line \( AB \). What is the equation of \( AB \)?

**Solution**

The line is vertical and goes through 10 on the \( x \) axis. Therefore its equation is \( x = 10 \).

---

**Drawing a Straight line from its equation**

Other straight line equations that have both \( x \) and \( y \) in are a little more complicated to do. We have to use a **table of values** to work out what \( y \) is for different values of \( x \).

When doing a table of values, we can choose any values of \( x \). However it makes sense to choose simple numbers, and 0, 1, 2, 3 is a common choice.

**Example**

**Draw the straight line with equation** \( y = 3x - 4 \)

**Solution**

**Step one** – draw up a table of values

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

**Step two** – use the equation to work out what \( y \) is for each value of \( x \).

Showing your working is useful.

\[
\begin{align*}
  y &= 3x - 4 \\
  a) \quad \text{when } x = 0, \quad y &= 3 \times 0 - 4 \\
  &= -4
\end{align*}
\]
b) when \( x = 1 \), \( y = 3 \times 1 - 4 = -1 \)

\[
\begin{array}{|c|c|c|c|}
\hline
\text{ } & x & 0 & 1 & 2 & 3 \\
\text{ } & y & -4 & -1 & & \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{ } & x & 0 & 1 & 2 & 3 \\
\text{ } & y & -4 & -1 & 2 & 5 \\
\hline
\end{array}
\]

\[\text{c)}\text{ also work out when } x = 2 \text{ and } x = 3:\]

**Step three** – every column in this table of values now gives you a coordinate. Write these down

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{ } & x & 0 & 1 & 2 & 3 \\
\text{ } & y & -4 & -1 & 2 & 5 \\
\hline
\end{array}
\]

**Step four** – plot these points on a coordinate grid. They should lie in a straight line. *If they don’t, you have made a mistake – go back and check step two carefully.*

**Step five** – draw a line through the points, making sure the line goes all the way from the top to the bottom of the grid.

Label your line with its equation.

---

**The Equation of a Straight Line**

**Definition:** the gradient of a line is the gradient between any two coordinates on the line. See page 36 for instructions on how to find the gradient.

**Definition:** the \( y \)-intercept of a straight line is the number on the \( y \)-axis that the line passes through. For the graph in the last example, the \( y \) intercept is \(-4\).
The equation of any straight line can be written \( y = mx + c \)

where \( m \) is the gradient of the line and \( c \) is the \( y \)-intercept of the line.

In everyday language, this means that:

- the gradient is “the number before \( x \)’’
- the \( y \)-intercept is “the number that is not before \( x \)’’

**Examples**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Gradient</th>
<th>( y )-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 2x - 5 )</td>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>( y = 1.5x + 4 )</td>
<td>1.5</td>
<td>4</td>
</tr>
<tr>
<td>( y = 8 - x )</td>
<td>-1</td>
<td>8</td>
</tr>
<tr>
<td>( y = 4 - 3x )</td>
<td>-3</td>
<td>4</td>
</tr>
</tbody>
</table>

**Solving Basic Equations**

You need to be able to solve simple equations. The method used in these notes is to *change side and do the opposite*, though another common method is ‘balancing’. You have to use a method to get the answer – *if you just write the answer down (even if you think it is obvious) you will get no marks.*

You should always check your final answer so that you know it is correct.

**Example 1**

**Solve algebraically the equation** \( 7m - 1 = 41 \)

**Solution**

**Step one** – move the ‘–1’ over to the other side to become ‘+1’
**Step two** – divide by the 7
**Step three** – write down the answer
**Step four** (check your answer) –
    - double check that \( 7 \times 6 - 1 \) does equal 41.

\[
7m - 1 = 41 \\
7m = 41 + 1 \\
7m = 42 \\
m = \frac{42}{7} \\
m = 6
\]

**Example 2**

**Solve algebraically the equation** \( 8a + 4 = 34 \)

**Solution**

**Step one** – move the ‘+4’ over to the other side to become ‘–4’
**Step two** – divide by the 8
**Step three** – write down the answer
**Step four** (check your answer) –
    - double check that \( 8 \times 3.75 + 4 \) does equal 34.

\[
8a + 4 = 34 \\
8a = 34 - 4 \\
8a = 30 \\
a = \frac{30}{8} \\
a = 3.75
\]
Equations with letters on both sides

This topic is assessed in Question 1 of Paper 2 of the ADDED VALUE UNIT. See page 4 for more details

In National 4 assessments, you will have to solve an equation that has letters on both sides.

[Before you start (optional) – write the “invisible plus signs” in, in front of anything that does not have a sign in front of it already, to remind you it is positive.]

- **Step one** – move everything with a letter in to the left-hand side, and all the numbers to the right-hand side, remembering to “change side and do the opposite”.
- **Step two** – simplify each side
- **Step three** – solve the resulting equation
- **Final step** – double check your answer works by substituting it back in to both sides of the original equation and checking both sides give the same answer.

**Example 1**

Solve algebraically the equation $2a + 5 = 15 - 2a$

**Solution**

- **Optional step** – write in “invisible plus signs” in front of anything that does not already have a sign

  \[
  \begin{align*}
  2a + 5 &= 15 - 2a \\
  +2a + 5 &= +15 - 2a \\
  +2a + 2a &= +15 - 5 \\
  4a &= 10 \\
  a &= \frac{10}{4} \\
  a &= 10 \div 4 = 2.5
  \end{align*}
  \]

- **Step one** – move the ‘$-2a$’ over to the left-hand side where it becomes ‘$+2a$’. Move ‘$+5$’ to the right-hand side where it becomes ‘$-5$’.

- **Step two** – simplify both sides
- **Step three** – divide to get the final answer

**Final step**: check, by substituting $a = 2.5$ into the original equation
The left-hand side is $2a + 5$. If we substitute $a = 2.5$, we get $2 \times 2.5 + 5$, which equals 10.
The left-hand side is $15 - 2a$. If we substitute $a = 2.5$, we get $15 - 2 \times 2.5$, which equals 10. These are the same, so our answer has to be correct.

If the equation contains brackets, we must multiply them out before continuing.

**Example 2 – equation with brackets**

Solve algebraically the equation $5(y - 1) = 3(y + 3)$

**Solution**

- **Optional first step** – write in “invisible plus signs” in front of anything that does not already have a sign
Step one – multiply out both brackets

Step two – move the ‘+ 3y’ over to the left-hand side where it becomes ‘−3y’. Move ‘−5’ to the right-hand side where it becomes ‘+5’.

Step three – simplify both sides

Step four – divide to get the final answer

Final step: check, by substituting \( y = 7 \) into the original equation
- The left-hand side is \( 5(y - 1) \). If we replace \( y \) with 7, we get \( 5(7 - 1) = 5 \times 6 \), which makes 30.
- The left-hand side is \( 3(y + 3) \). If we replace \( y \) with 7, we get \( 3(7 + 3) \), which also makes 30. These are the same, so our answer is correct.

### Changing the subject of a formula

The **subject** of a formula is the letter at the start of a formula. For example in speed-distance-time, in the formula \( S = \frac{D}{T} \), the letter \( S \) is the subject. The formula \( D = ST \) is the same formula but with \( D \) as the subject. We say we have **changed the subject** of the formula from \( S \) to \( D \).

Changing the subject of a formula is just like rearranging an equation: you move things from one side to the other and ‘do the opposite’.

A useful (but optional) tip for changing the subject questions is to switch the left-hand side and the right-hand side before you begin moving things, so that the letter that will be the subject is already on the left-hand side.

When adding and subtracting the letter that moves always ends up at the **end** of the other side.

---

**Example 1 – add/take away**

**Change the subject of the formula** \( g = h + 3 \) **to ‘**\( h \)**’

**Solution**

**Step One** – flip the left and right hand sides: \( h + 3 = g \)

**Step Two** – move the ‘+ 3’ over to the other side, where it becomes ‘− 3’

**Final Answer:** \( h = g - 3 \)
When multiplying, the letter that moves goes to the *bottom* of a dividing formula.

**Example 2 – multiply/divide**

*Change the subject of the formula \( C = \pi d \) to ‘\( d \)’*

**Solution**

**Step One** – flip the left and right hand sides: \( \pi d = C \)

**Step Two** – move the ‘\( \pi \)’ over to the other side and divide. The \( \pi \) ends up on the bottom.

**Final Answer:** \( d = \frac{C}{\pi} \)

**Example 3**

*Change the subject of the formula \( y = ab + d \) to ‘\( b \)’*

**Solution**

**Step One** – flip the left and right hand sides: \( ab + d = y \)

**Step Two** – rearrange; dealing with the adding/subtracting first, then multiplying/dividing

\[
\begin{align*}
ab + d &= y \\
ab &= y - d \quad (+d \text{ moves over and becomes } -d) \\
b &= \frac{y - d}{a} \quad (\times a \text{ moves over and becomes } \div a)
\end{align*}
\]
**Trigonometry**

**Pythagoras’ Theorem**

This topic is assessed in *Question 5 of Paper 2 of the ADDED VALUE UNIT*. See page 4 for more details.

When you know how long any two of the sides in a right-angled triangle are, you can use Pythagoras’ Theorem (usually just known as *Pythagoras*) to find the length of the third side without measuring.

**Formula:** this formula is given on the formula sheet in assessments

Theorem of Pythagoras

![Pythagoras Diagram]

**Definition:** the hypotenuse is the longest side in a right-angled triangle. In the diagram above, the hypotenuse is \( c \). The hypotenuse is *always* opposite the right angle.

There are three steps to any Pythagoras question:

- **Step One** – square the length of the two sides
- **Step Two** – either add or take away (see below)
- **Step Three** – square root

**Choosing whether to add or take away:**

- If you are finding the length of the longest side (the hypotenuse), you **add** the squared numbers.
- If you are finding the length of a shorter side, you **take away** the squared numbers.

**Example 1 – finding the length of the hypotenuse**

*Calculate the length of \( x \) in this triangle. Do not use a scale drawing.*

**Solution**

We are finding the length of \( x \). \( x \) is the hypotenuse, so we **add**.

\[
\begin{align*}
x^2 &= 5^2 + 6^2 \\
x^2 &= 61 \\
x &= \sqrt{61} \\
x &\approx 7.81024.... \\
x &\approx 7.81 \text{ cm (2d.p.)}
\end{align*}
\]
Example 2 – finding the length of a shorter side

Calculate $x$, correct to 1 decimal place. Do not use a scale drawing.

Solution

We are finding the length of $x$. $x$ is a smaller side, so we take away.

\[ \begin{align*}
x^2 & = 12.3^2 - 8.5^2 \\
x^2 & = 79.04 \\
x & = \sqrt{79.04} \\
x & = 8.8904... \\
x & = 8.9 \text{ cm} \quad (1 \text{ d.p.})
\end{align*} \]

In National 4 assessments, the Pythagoras question may well be ‘hidden’. On this occasion, there would be a reasoning mark (#2.1) available to you for choosing to use Pythagoras (see page 6). One way to spot them is to look out for the words “do not use a scale drawing”.

The main way to spot a Pythagoras question is to look for right-angled triangles. However they are not always obvious. The question below is a Pythagoras question although it may not appear to have any right-angled triangle at first.

Example 3 – from coordinates

Calculate the length of the line $AB$. Do not use a scale drawing.

Solution

Can you see the right-angled triangle? Draw lines to complete the triangle.

The triangle has sides 3 squares and 4 squares.

Side $AB$ is the hypotenuse, so we add

\[ \begin{align*}
AB^2 & = 3^2 + 4^2 \\
AB^2 & = 25 \\
AB & = 5
\end{align*} \]
Trigonometry (SOH CAH TOA)

Trigonometry is the study of triangles, commonly referred to as SOH CAH TOA. At National 4, you may need to work out the length of a side or the size of an angle.

This topic is assessed in Question 6 of Paper 2 of the ADDED VALUE UNIT. See page 4 for more details

Trigonometric ratios in a right angled triangle:
these formulae are given on the formula sheet in assessments

\[
\begin{align*}
\sin x^\circ &= \frac{\text{Opposite}}{\text{Hypotenuse}} \\
\cos x^\circ &= \frac{\text{Adjacent}}{\text{Hypotenuse}} \\
\tan x^\circ &= \frac{\text{Opposite}}{\text{Adjacent}}
\end{align*}
\]

Example 1 – calculating an angle

Find the size of angle \(x^\circ\) in this right-angled triangle.

Solution

Step one – label the sides O, H or A. Do not bother labelling the side that we do not know the length of (in this case it is H), as we don’t need it.

Step two – here we have the opposite and the adjacent. Now use SOH CAH TOA to tick off the sides we have:

\[
\begin{align*}
\sqrt{} & \sqrt{} & \sqrt{} \\
\text{SOH CAH TOA}
\end{align*}
\]

TOA has two ticks above it, telling us to use \(\tan\) in this question.

Step three – copy out the formula carefully

Step four – substitute the numbers 3 and 7 in the correct places

Step five – use \(\tan^{-1}\) (or \(\sin^{-1}\) or \(\cos^{-1}\)) to find the angle

\[
\begin{align*}
\tan x &= \frac{\text{opposite}}{\text{adjacent}} \\
\tan x &= \frac{3}{7} \\
x &= \tan^{-1}\left(\frac{3}{7}\right) \\
x &= 23.198\ldots = 23.2^\circ \text{ (1d.p.)}
\end{align*}
\]
Example 2 – calculating a length

Find the length \( x \) in this right-angled triangle.

Solution

Step one – label the sides O, H or A. One side will have nothing written on it – do not bother labelling that side (in this case it is A) as we do not need it.

Step two – we have the opposite and the hypotenuse. Now use SOH CAH TOA to tick off the sides you have:

\[
\sqrt{\_} \quad \sqrt{\_} \quad \sqrt{\_} \quad \text{SOH CAH TOA}
\]

SOH has two ticks above it, telling us to use \( \sin \) in this question.

Step three – copy out the formula carefully

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}
\]

Step four – substitute the numbers 29 and 8 in the correct places. The angle always has to go straight after sin (or cos or tan)

\[
x = 8 \times \sin 29
\]

\[
x = 3.878...
\]

\[
x = 3.88\text{ cm (2d.p.)}
\]

Step five – multiply to get the answer

If the unknown length is on the bottom of the fraction, we have to do a little more rearranging to solve the equation, using the rules of changing the subject.

Example 3 – unknown in the denominator

Find the length \( x \) in this right-angled triangle.

Solution

Step one – label the sides O, H or A. One side will have nothing written on it – do not bother labelling that side (in this case it is O) as we do not need it.
Step two – we have the adjacent and the hypotenuse. Now use SOH CAH TOA to tick off the sides you have:

\[ \sqrt{\text{adjacent}} \quad \sqrt{\text{opposite}} \quad \sqrt{\text{hypotenuse}} \]

SOH CAH TOA

CAH has two ticks above it, telling us to use \( \cos \) in this question.

Step three – copy out the formula carefully

Step four – substitute the numbers 35 and 12 in the correct places. The angle always has to go straight after sin (or cos or tan)

Step five – move the \( x \) over to the left hand side and multiply

Step six – move the \( \cos 35 \) over to the other side and divide

\[
\cos x = \frac{\text{adjacent}}{\text{hypotenuse}} \\
\cos 35 = \frac{12}{x} \\
x \cos 35 = 12 \\
x = \frac{12}{\cos 35} \\
x = 14.65\text{mm (2d.p.)}
\]

In National 4 assessments, you will be expected to use SOH CAH TOA and Pythagoras. You will have to know which is the correct one in each question:

- If there is any angle (other than the right angle) in the question, then use SOH CAH TOA
- If there are no angles involved in the question (only lengths), then use Pythagoras
Angles and Scale

Angles in Triangles and Quadrilaterals

Facts
The three angles in every triangle always add to make 180°.
The four angles in every quadrilateral always add to make 360°.

Example 1
How big are angles $x^\circ$ and $y^\circ$ in these shapes?

Solution
In the triangle, the three angles add up to make 180°.
Therefore $x = 180 - 80 - 30 = 70^\circ$

In the quadrilateral, the four angles add up to make 360°.
Therefore $y = 360 - 90 - 65 - 55 = 150^\circ$

A special type of triangle is an isosceles triangle.

Isosceles triangles have two equal sides and two equal angles. They can usually be identified by dashes, which represent the two equal sides, as shown in the diagram on the left.

If a diagram contains an isosceles triangle, it means we know that the two angles opposite the equal sides are identical.

Example 2 – isosceles triangle

The diagram on the right shows an isosceles triangle. Calculate the size of the other two angles.

Solution
The three angles in a triangle add to make 180°. The angle we are given is 40°, so the other two angles must make 140°.

The other two angles are equal, so each angle must be $140 \div 2 = 70^\circ$. 
Angles and Parallel Lines

We know two lines are parallel if they both have an arrow on them, such as in the diagram on the right.

When two parallel lines are crossed by a third straight line, the angles at each X shape are identical. In the diagram on the right, the four angles that are shaded darker are all the same size; and the four angles shaded lighter are also all the same size.

<table>
<thead>
<tr>
<th>Facts</th>
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<tr>
<td>• Two angles on a straight line add up to make 180°.</td>
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<tr>
<td>• Opposite angles in X-shapes are equal.</td>
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<tr>
<td>• Angles in Z shapes (made by parallel lines) are the same.</td>
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Angles and Circles

If there is a circle in a diagram containing angles, then there are two extra rules that can help us calculate unknown angles.

One rule relates to a triangle that fills half a circle (i.e. its longest side is a diameter of the circle, and the other side is on the circumference). We refer to the angle opposite the diameter as the angle in the semicircle.

<table>
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<th>Fact</th>
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<tr>
<td>The angle in a semicircle is always a right angle.</td>
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Example 1

The diagram shows a circle with diameter BC. Angle ABC is 32°. Calculate the size of angle ACB.

Solution

Angle CAB is an angle in a semicircle, so CAB = 90°.

Then we know ABC is a triangle with angles 90°, 32°. The third angle (ACB) is 180 – 90 – 32 = 58°.

Final answer: angle ACB is 58°.
**Definition:** A tangent to a circle is a line that just touches the edge of the circle.

![Diagram of a tangent to a circle]

**Fact**
A tangent always makes a right-angle with a radius.

A harder example is given below – however every question is different. The best way to get used to them is to practice them from past Standard Grade General exam papers, past Intermediate 2 NABs and textbooks.

**Example 2 – harder**

The diagram shows a circle centre O. DE is a tangent to the circle at point C. Angle OAB is 35°.

Calculate the size of angle BCE

**Solution**

A tangent and a radius meet at right angles, so ACD and ACE are 90°.

The angle in a semi-circle is a right angle, so ABC is also 90°.

We know two of the angles in triangle ABC. Since we know the angles in a triangle must add to make 180°, angle ACB must be 55°.

Finally since we already knew ACE is 90°, this tells us that ACB and BCE must add to make 90°. Therefore angle BCE is 35°.

**Final answer:** clearly state that angle BCE is 35° (just marking it on the diagram isn’t enough as it doesn’t make it clear that you know which angle is angle BCE).

---

**Enlargement and Reduction by a Scale Factor**

An enlargement of a diagram is another diagram which is the exact same shape (same shape, same angles) except all the lengths have been scaled.

The amount of the enlargement is called the **scale factor**. For example:
• an enlargement with a scale factor 2 is double the size of the original. You have to draw the same shape, making every line exactly twice as long.

• an enlargement with a scale factor \(\frac{1}{2}\) is half the size of the original. You have to draw the same shape, making every line exactly half as long.

In National 4 assessments, you are expected to draw an enlarged copy of a shape when told a scale factor. The scale factor you will be given will be a fraction, such as \(\frac{3}{2}\) or \(\frac{5}{4}\).

Deal with a fractional scale factor in the same way you would deal with any fraction: divide by the number on the bottom and times by the number on the top.

**Example**

Shape ABCDE is shown in the diagram on the right.

**Draw an enlargement of the given shape using a scale factor of \(\frac{3}{2}\)**

**Solution**

For a scale factor of \(\frac{3}{2}\), we divide lengths by 2 and multiply by 3 \((\div 2 \times 3)\).

For example in the original diagram, line AB is 4 squares long. If we divide by 2 and multiply by 3, we get 6. So AB will be 6 squares long in the answer.

Similarly, CD will be 6 squares long as well, DE will be 3 squares long, and AE will be 9 squares long.

BC will go through the diagonal corners of 3 squares.

The diagram below shows the original diagram and the enlargement side by side. The letters have been left out to make the diagram clearer.
**Statistics**

**Scatter Graphs**

A scatter graph is a way of displaying information and looking for a connection between two things.

**Definition:** the correlation between two sets of numbers refers to the relationship (if any) between the numbers. A scatter graph is good for showing correlation. Correlation can be **positive** (going up), **negative** (going down), or **none**.

**Definition:** a line of best fit is line drawn on to a scatter graph that shows the correlation of the graph. The straight lines drawn above for positive and negative correlation are examples of lines of best fit.

The line of best fit should go:
- go through the middle of the points, with roughly the same number of points above and below the line
- in the same direction that the points are laid out on the page. **Do not “join the dots”!**

You will *always* be asked to draw the line of best fit in a scatter graph question in a maths assessment. Once you have drawn the line, you will always be asked to *use it*.

**Example 1**

A gift shop records the temperature each day for 13 days. They also record how many scarves they sell each day. Show this information in a scatter graph.
Solution

The graph has been drawn and is shown on the right

From this graph, we can see there is **negative correlation** between temperature and scarf sales.

Example 2

**Draw a line of best fit on the scattergraph above**

**Solution**

These lines of best fit would be marked **wrong**.

- Joining the dots - **WRONG**
- Not going in the same direction as the points - **WRONG**
- Not going through the middle of the points (too low) - **WRONG**

Any of these answers would be marked **correct** as they go roughly through the middle of the points, and roughly in the same direction as the points.
Example 3

**On the next day, the temperature is 6°C. Using your line of best fit, estimate how many scarves the shop will sell.**

**Solution**

The key words here are *using your line of best fit*. If your answer matches with your line, you get the mark. If it doesn’t match with your line, you don’t get any marks. Simple as that.

Incorrect answers:

- The answer is not 7 even though there is a point there. This is because the point is far away from the line, meaning it was unusual and so not a good guess.

Possible Correct answers:

The correct answer will depend on your graph. You need to draw lines on your graph at 6°C, and to see where they meet the line of best fit. For the first two examples above, this would look like this:

![Graphs showing line of best fit](image)

If your line of best fit was the one on the left, your answer would be 2 scarves. If your line of best fit was the one on the right, your answer would be 3 scarves.

It does not matter that these answers are different – remember the question only asked for an *estimate*. The key thing is that it matches your line of best fit.
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www.newbattle.org.uk/Departments/Maths/n4.html

All information in this revision guide has been prepared in best faith, with thorough reference to the documents provided by the SQA, including the course arrangements, course and unit support notes, exam specification, specimen question paper and unit assessments.

These notes will be updated as and when new information become available. Schools or individuals who purchase a site licence before May 2014 will be eligible to receive a free copy of any revised notes. To enquire further about this, email david@dynamicmaths.co.uk.

We try our hardest to ensure these notes are accurate, but mistakes sometimes appear. If you discover any mistakes in these notes, please email us at david@dynamicmaths.co.uk. A corrected replacement copy of the notes will be provided free of charge! We would also like to hear of any suggestions you have for improvement.

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