Higher Mathematics

Past Papers by Topic

Contents:

Expressions & Functions **Page**

Logarithmic & Exponential Functions…… 1
Addition Formulae…… 14
Wave Function…… 24
Graphs of Functions…… 31
Sets of Functions…… 40
Vectors…… 45

Relationships & Calculus

Polynomials…… 65
Quadratic Functions…… 74
Trigonometry…… 82
Further Calculus…… 88

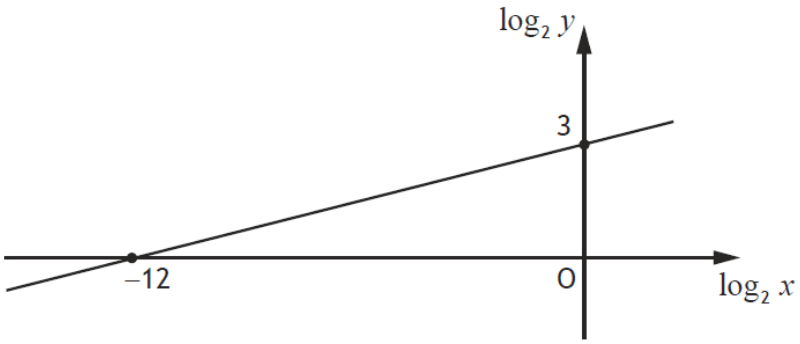
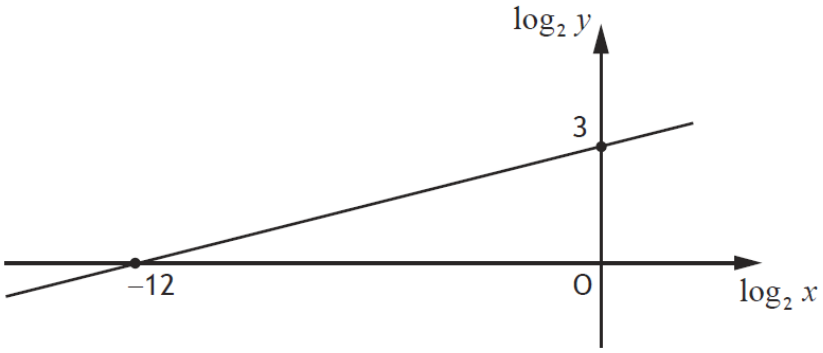
Applications

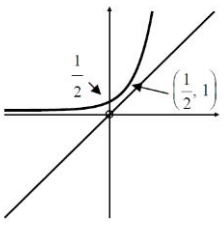
The Straight Line…… 97
The Circle…… 111
Recurrence Relations…… 125
Differentiation…… 131
Integration…… 152

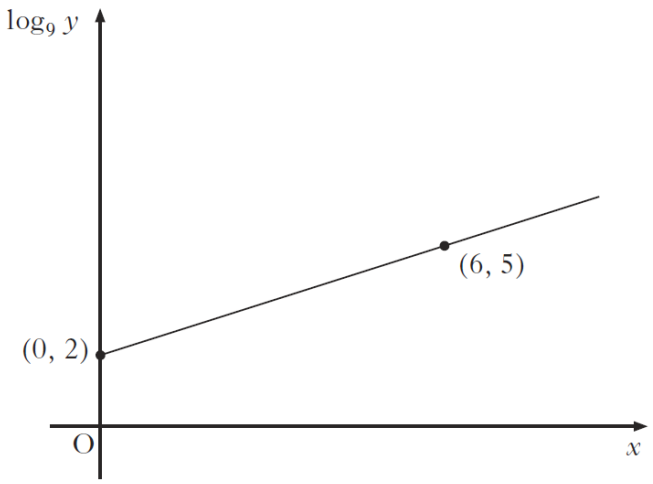
All questions in this document have been made available by and are copyright of the SQA. To obtain past papers by years visit the SQA website at www.sqa.org.uk.

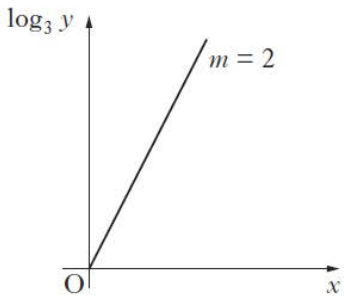
The answers and worked solutions in these documents have been collated from a variety of sources and are not the authorship of Zeta Maths Ltd.

For easy navigation of the document, the contents page is linked to each chapter and the chapter headings at the top of each page are linked back to the contents page.

2017 P1	**12.** Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a.	3
Ans	a = 81	
2017 P2	**9.** Two variables, x and y, are connected by the equation $y = kx^n$. The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.  Find the values of k and n.	5
2017 P2	**9.** Two variables, x and y, are connected by the equation $y = kx^n$. The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.  Find the values of k and n.	
Ans	$k = 8,\ n = \frac{1}{4}$ or $y = 8x^{\frac{1}{4}}$	
2016 P1	**14.** (a) Evaluate $\log_5 25$. (b) Hence solve $\log_4 x + \log_4 (x-6) = \log_5 25$, where $x > 6$.	1 5
Ans	(a) 2 (b) x = 8	

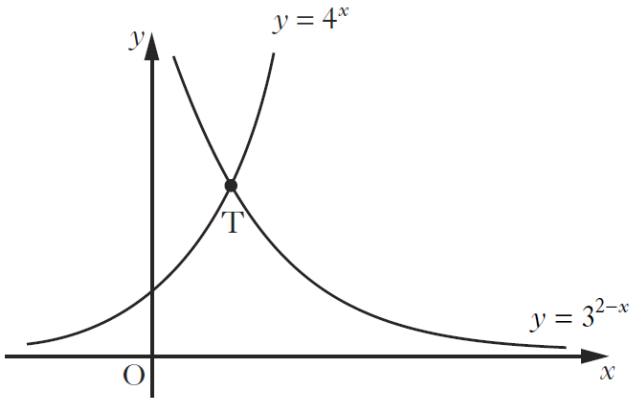
2016 P2	**6.** Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula $$B(t) = 200\,e^{0\cdot107t},$$ where t represents the number of hours since the study began. (a) State the number of bacteria present at the start of the study. (b) Calculate the time taken for the number of bacteria to double.	1 4
Ans	(a) 200 (b) 6.48 hours	
NH 2015 P1	**6.** Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$.	3
Ans	2	
NH SP P2	**7.** Given that $P(t) = 30e^{t-2}$ decide whether each of the statements below is true or false. Justify your answers. Statement A $P(0) = 30$. Statement B When $P(t) = 15$, the only possible value of t is 1·3 to one decimal place.	6
Ans	A False and B True with reason	
NH SP P2	**4.** (a) Express $y = \log_4 2x$ in the form $y = \log_4 x + k$, clearly stating the value of k. (b) Hence, or otherwise, describe the relationship between the graphs of $y = \log_4 2x$ and $y = \log_4 x$. (c) Determine the coordinates of the point where the graph of $y = \log_4 2x$ intersects the x-axis. (d) Sketch and annotate the graph of $y = f^{-1}(x)$, where $f(x) = \log_4 2x$.	2 1 2 3
Ans	(a) $y = \log_4 x + \frac{1}{2}$ (b) Ans: Graph of $y = \log_4 x$ moved up by $\frac{1}{2}$ or graph of $y = \log_4 x$ compressed horizontally by a factor of 2. (c) x = ½ (d) 	

NH EX P2	**7.** The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation. $$P_t = P_0 e^{-kt}$$ where: • P_0 is the initial concentration; • P_t is the concentration at time t; • t is the time, in days, after the application of the pesticide. Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value. (a) If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. On all *Xpesto* packaging, the manufacturer states that 80 days after application the concentration of *Xpesto* in the soil will have decreased by over 90%. (b) Is this statement correct? Justify your answer.	4 4
Ans	(a) $k \approx 0.028$ (b) No, with reason	
2014 P1	**24.** Two variables, x and y, are related by the equation $$y = ka^x.$$ When $\log_9 y$ is plotted against x, a straight line passing through the points (0, 2) and (6, 5) is obtained, as shown in the diagram.  Find the values of k and a.	5
Ans	$k = 81$, $a = 3$	
2014 P1	**20.** Evaluate $2 - \log_5 \frac{1}{25}$.	
Ans	4	

2014 P1	**3.** If $\log_4 12 - \log_4 x = \log_4 6$, what is the value of x?	2
Ans	2	
2013 P2	**9.** The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation $$P_t = P_0 e^{-kt}$$ where: • P_0 is the initial concentration; • P_t is the concentration at time t; • t is the time, in days, after the application of the pesticide. (*a*) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value. If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. (*b*) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*?	4 3
Ans	(a) $k \approx 0{\cdot}028$ (b) 89%	
2013 P2	**5.** Solve the equation $$\log_5(3-2x) + \log_5(2+x) = 1, \text{ where } x \text{ is a real number.}$$	4
Ans	x= ½ , x = -1	
2013 P1	**20.** The graph of $\log_3 y$ plotted against x is a line through the origin with gradient 2, as shown.  Express y in terms of x.	
Ans	$y = 9^x$	

2012 P2

7. The diagram shows the curves with equations $y = 4^x$ and $y = 3^{2-x}$.



The graphs intersect at the point T.

(*a*) Show that the x – coordinate of T can be written in the form $\frac{\log_a p}{\log_a q}$, for all $a > 1$. — 6

(*b*) Calculate the y – coordinate of T. — 2

Ans

(a) proof required, p = 9, q = 12
(b) T (0.88, 3.39)

2012 P1

20. Simplify $\frac{\log_b 9a^2}{\log_b 3a}$, where $a > 0$ and $b > 0$. — 2

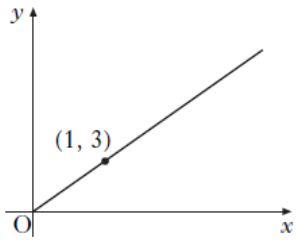
Ans

2

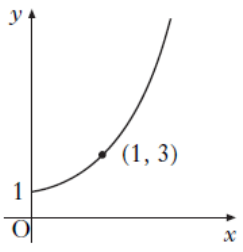
2011 P1

19. Which of the following diagrams represents the graph with equation $\log_3 y = x$?

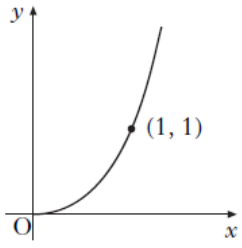
A



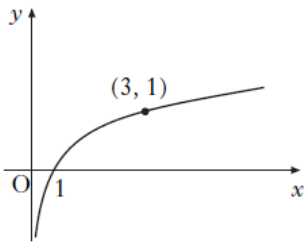
C



B

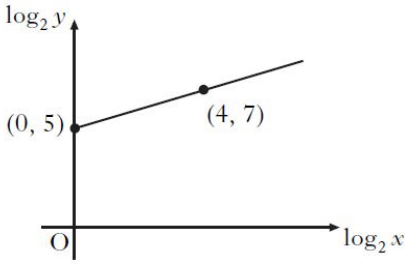
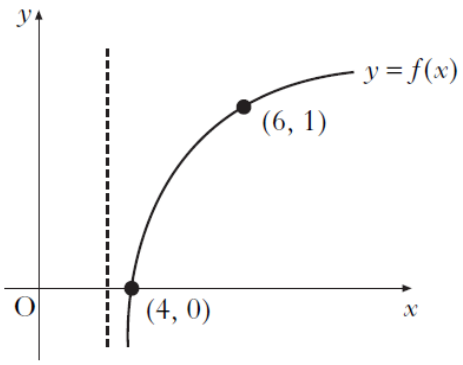
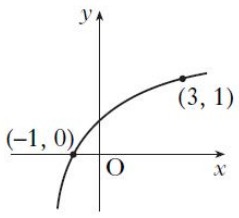
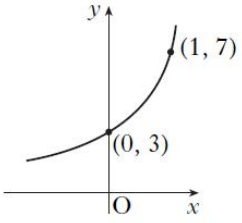
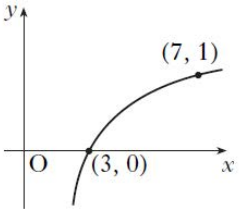
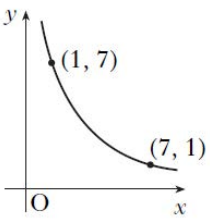


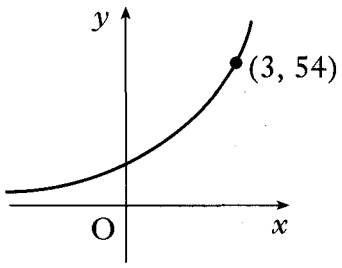
D

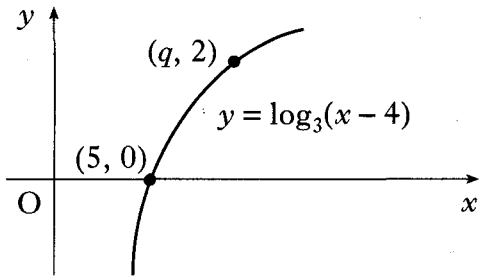


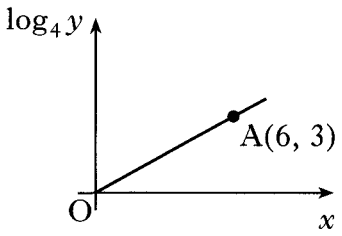
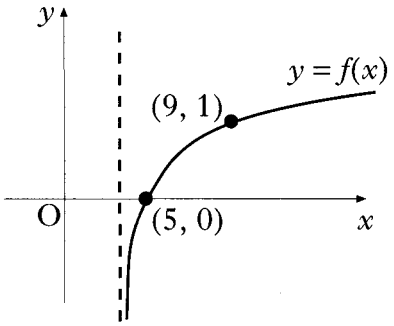
Ans

C

2011 P2	**5.** Variables x and y are related by the equation $y = kx^n$. The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points (0, 5) and (4, 7), as shown in the diagram. Find the values of k and n. 	
Ans	k = 32, n = ½	
2010 P1	**19.** The diagram shows the graph of $y = f(x)$ where f is a logarithmic function.  What is $f(x)$?	
Ans	$f(x) = \log_3(x-3)$	
2010 P2	**7.** (*a*) Given that $\log_4 x = \mathrm{P}$, show that $\log_{16} x = \frac{1}{2}\mathrm{P}$. (*b*) Solve $\log_3 x + \log_9 x = 12$.	**3** **3**
Ans	(b) $x = 3^8$	
2009 P1	**10.** Which of the following graphs has equation $y = \log_5(x-2)$? A  C  B  D 	

Ans	B	
2009 P2	**3.** (*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$. (ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4** (*b*) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$. **5**	
Ans	(a) (ii) $(x - 1)(x + 4)(x + 5)$ (b) $x = 1$	
2009 P2	**6.** The size of the human population, N, can be modelled using the equation $N = N_0e^{rt}$ where N_0 is the population in 2006, t is the time in years since 2006, and r is the annual rate of increase in the population. (*a*) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1·6%. Assuming this growth rate remains constant, what would be the population in 2020? **2** (*b*) In 2006 the population of Scotland was approximately 5·1 million, with an annual rate of increase of 0·43%. Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size? **3**	
Ans	(a) N = 76.3 Million (b) 161 years	
2008 P1	**19.** The diagram shows part of the graph whose equation is of the form $y = 2m^x$. What is the value of m?  2	
Ans	3	

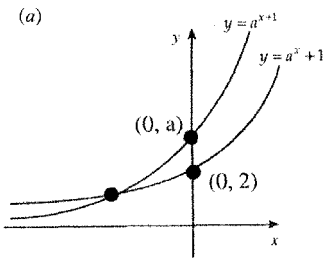
2008 P1	**20.** The diagram shows part of the graph of $y = \log_3(x - 4)$. The point $(q, 2)$ lies on the graph.  What is the value of q?	2
Ans	13	
2008 P1	**23.** Functions f, g and h are defined on suitable domains by $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$. (*a*) Find expressions for $h(f(x))$ and $h(g(x))$. (*b*) Hence solve $h(f(x)) - h(g(x)) = 3$.	3 5
Ans	(*a*) $h(f(x)) = \log_2(x^2 - x + 10)$ $h(g(x)) = \log_2(5 - x)$ (*b*) $x = 3, -10$	
2007 P2	**8.** The curve with equation $y = \log_3(x - 1) - 2{\cdot}2$, where $x > 1$, cuts the x-axis at the point $(a, 0)$. Find the value of a.	4
Ans	12.2	
2007 P2	**11.** Two variables x and y satisfy the equation $y = 3 \times 4^x$. (*a*) Find the value of a if $(a, 6)$ lies on the graph with equation $y = 3 \times 4^x$. (*b*) If $(-\frac{1}{2}, b)$ also lies on the graph, find b. (*c*) A graph is drawn of $\log_{10} y$ against x. Show that its equation will be of the form $\log_{10} y = Px + Q$ and state the gradient of this line.	1 1 4
Ans	(*a*) $a = \frac{1}{2}$ (*b*) $b = \frac{3}{2}$ (*c*) $y = 3 \times 4^x$ $\log_{10} y = \log_{10} 3 + \log_{10}(4^x)$ $= \log_{10} 3 + x \log_{10}(4)$ So gradient of line $= \log_{10}(4)$	

2006 P1	**10.** Two variables, x and y, are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a. 	4
Ans	$a = 2$	
2006 P2	**11.** It is claimed that a wheel is made from wood which is over 1000 years old. To test this claim, carbon dating is used. The formula $A(t) = A_0 e^{-0.000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years. For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree. Is the claim true?	5
Ans	$t = 1031$ years so claim valid	
2005 P1	**7.** The function f is of the form $f(x) = \log_b (x - a)$. The graph of $y = f(x)$ is shown in the diagram. (*a*) Write down the values of a and b. (*b*) State the domain of f. 	2 1
Ans	(*a*) $a = 4$ $b = 5$ (*b*) domain is $x > 4$	
2005 P2	**7.** Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$.	4
Ans	$x = \frac{43}{15}$	

2005 P2	**9.** The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0{\cdot}06335t}$. (*a*) What was its value when launched? (*b*) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold?	1 4
Ans	(*a*) 252(£m) (*b*) $t = 40$	
2004 P1	**9.** Solve the equation $\log_2(x + 1) - 2\log_2(3) = 3$.	4
Ans	$x = 71$	
2004 P2	**10.** The amount A_t micrograms of a certain radioactive substance remaining after t years decreases according to the formula $A_t = A_0e^{-0{\cdot}002t}$, where A_0 is the amount present initially. (*a*) If 600 micrograms are left after 1000 years, how many micrograms were present initially? (*b*) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance?	3 4
Ans	(*a*) 4433 (*b*) 347 years	
2003 P1	**12.** Simplify $3\log_e(2e) - 2\log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers.	4
Ans	$1 + \log_e 8 - \log_e 9$	
2003 P2	**11.** (*a*) (i) Sketch the graph of $y = a^x + 1$, $a > 2$. (ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$. (*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$.	2 3

Ans

(a)



(b) $a^{x+1} = a^x + 1$

$a \times a^x - a^x = 1$

$(a-1) \times a^x = 1$

$a^x = \frac{1}{a-1}$

$x = \log_a \left(\frac{1}{a-1}\right)$

2002W P1

12. If $\log_a p = \cos^2 x$ and $\log_a r = \sin^2 x$, show that $pr = a$. 3

Ans

- $\log_a p + \log_a r = \cos^2 x + \sin^2 x$
- $\log_a p + \log_a r = \log_a pr$
- $\log_a pr = 1$ and so $pr = a$

Alternative

- $p = a^{\cos^2 x}$ $r = a^{\sin^2 x}$
- $pr = a^{\cos^2 x + \sin^2 x}$
- $pr = a^1 = a$

2002W P2

9. A researcher modelled the size N of a colony of bacteria t hours after the beginning of her observations by $N(t) = 950 \times (2{\cdot}6)^{0{\cdot}2t}$.

(a) What was the size of the colony when observations began? 1

(b) How long does it take for the size of the colony to be multiplied by 10? 4

Ans

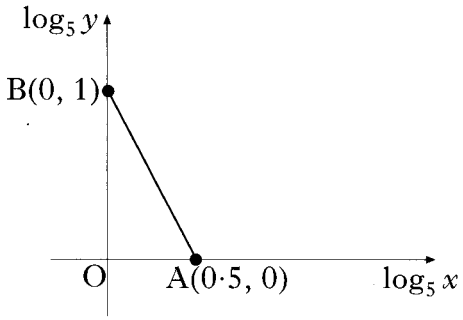
(a) 950

(b) approx 12 hours

2002 P1

11. The graph illustrates the law $y = kx^n$.

If the straight line passes through A(0·5, 0) and B(0, 1), find the values of k and n. 4



Ans

$\log_5 y = -2 (\log_5 x) + 1$
$\log_5 y = \log_5 x^{-2} + \log_5 5$
$\log_5 y = \log_5 5x^{-2}$
$k = 5$, $n = -2$
$(y = 5x^{-2})$

2002 P2

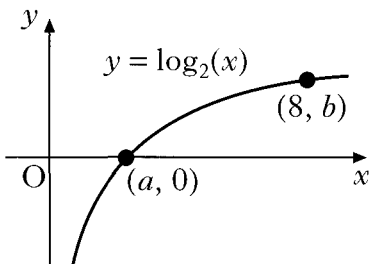
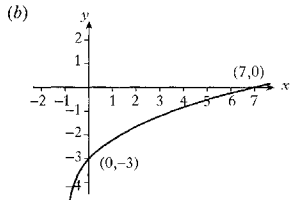
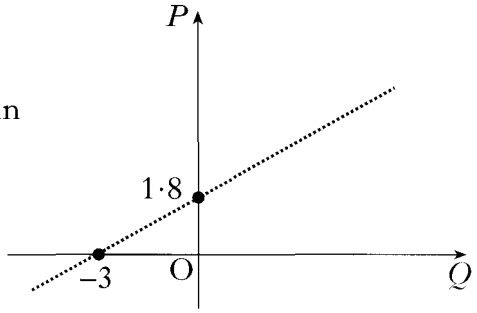
7. Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3 (x - 2) + 1$ intersects the x-axis. 3

Ans

$\log_3 (x - 2) = -1$
$x = 2\frac{1}{3}$

2001 P1

8. Find x if $4 \log_x 6 - 2 \log_x 4 = 1$. 3

Ans	$x = 81$	
2001 P1	**10.** The diagram shows a sketch of part of the graph of $y = \log_2(x)$. (*a*) State the values of a and b. (*b*) Sketch the graph of $y = \log_2(x + 1) - 3$. 	1 3
Ans	(*a*) $a = 1$, $b = 3$ (*b*) 	
2001 P2	**9.** Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later. If it takes one and half hours for the area of the forest fire to double, find the value of the constant k.	3
Ans	$k = 0{\cdot}46$	
2000 P1	**9.** Evaluate $\log_5 2 + \log_5 50 - \log_5 4$.	3
Ans	2	
2000 P2	**11.** The results of an experiment give rise to the graph shown. (*a*) Write down the equation of the line in terms of P and Q.  It is given that $P = \log_e p$ and $Q = \log_e q$. (*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b.	2 4
Ans	(*a*) $P = 0{\cdot}6Q + 1{\cdot}8$ (*b*) $a = 6{\cdot}05$, $b = 0{\cdot}6$	

Specimen 2 P2

7. The intensity I_t of light is reduced as it passes through a filter according to the law $I_t = I_0 e^{-kt}$ where I_0 is the initial intensity and I_t is the intensity after passing through a filter of thickness t cm. k is a constant.

(*a*) A filter of thickness 4 cm reduces the intensity from 120 candle-power to 90 candle-power. Find the value of k. — 4

(*b*) The light is passed through a filter of thickness 10 cm. Find the percentage reduction in its intensity. — 3

Ans

(*a*) $90 = 120e^{-4k} \Rightarrow e^{-4k} = 0{\cdot}75$

$-4k = \ln 0{\cdot}75 \Rightarrow k = 0{\cdot}0719$

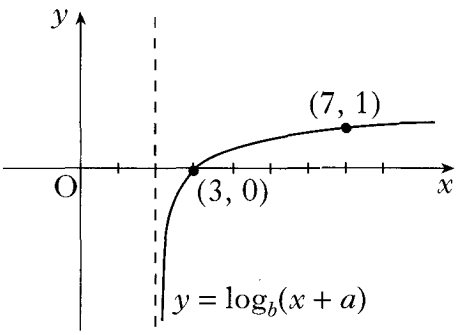
(*b*) $I_{10} = I_0 e^{-10\times 0{\cdot}0719}$

$\frac{I_{10}}{I_0} = 0{\cdot}487$ so a 51·3% reduction

Specimen 2 P2

8. The diagram shows part of the graph of $y = \log_b(x + a)$.

Determine the values of a and b. — 3



Ans

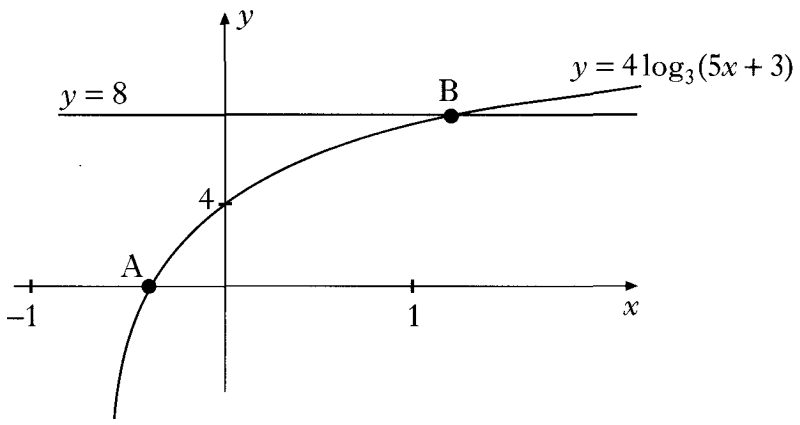
translation $\Rightarrow a = -2$

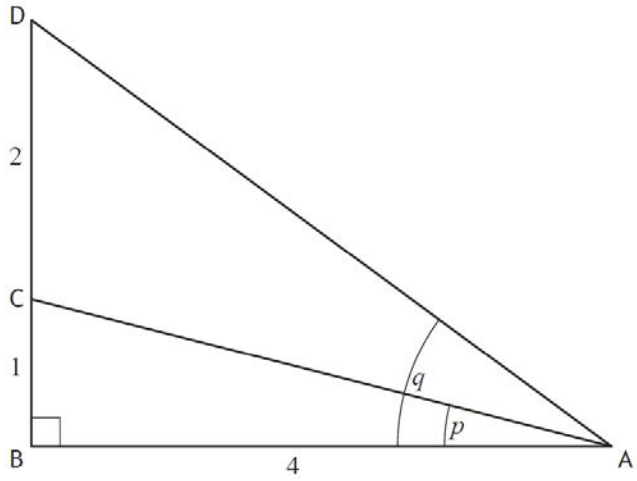
$(7,1) \Rightarrow 1 = \log_b(7 - 2) \Rightarrow b = 5$

Specimen 1 P1

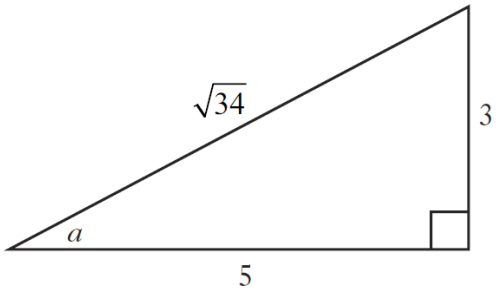
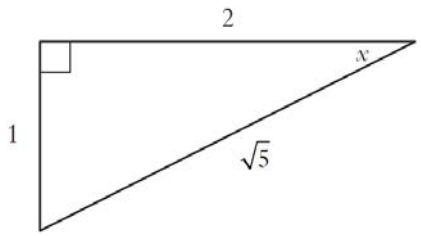
10. Part of the graph of $y = 4\log_3(5x + 3)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B.

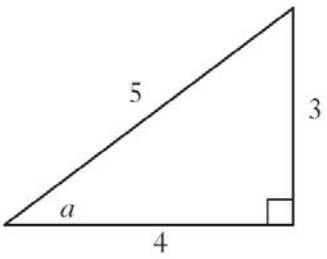
Find the x-coordinate of B. — 3

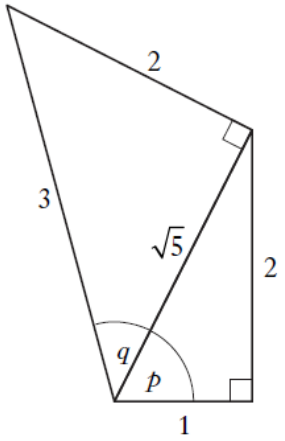


2017 P2	**6.** Solve $5\sin x - 4 = 2\cos 2x$ for $0 \leq x < 2\pi$.	5
Ans	x = 0.253, x = 2.89	
2017 P2	**11.** (a) Show that $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$.	3
Ans	Proof	
2016 P1	**13.** Triangle ABD is right-angled at B with angles BAC $= p$ and BAD $= q$ and lengths as shown in the diagram below.  Show that the exact value of $\cos(q-p)$ is $\frac{19\sqrt{17}}{85}$.	5
Ans	Proof	
NH 2015 P1	**10.** Given that $\tan 2x = \frac{3}{4}$, $0 < x < \frac{\pi}{4}$, find the exact value of (a) $\cos 2x$ (b) $\cos x$.	1 2
Ans	(a) 4/5 (b) $\frac{3}{\sqrt{10}}$	
HN SP P1	**6.** (a) Find an equivalent expression for $\sin(x + 60)°$. (b) Hence, or otherwise, determine the exact value of $\sin 105°$.	1 3
Ans	$\frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$	

Higher: Addition Formulae

NH Ex P1	**7.** (a) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$.	5
	(b) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$.	2
Ans	(a) 0, 60, 300 (b) 0, 30, 150, 180, 210 and 300	
2014 P1	**18.** What is the value of $1 - 2\sin^2 15°$?	2
Ans	$\frac{\sqrt{3}}{2}$	
2014 P1	**7.** A right-angled triangle has sides and angles as shown in the diagram.  What is the value of $\sin 2a$?	
Ans	15/17	
2013 P2	**8.** Solve algebraically the equation $\sin 2x = 2\cos^2 x \qquad \text{for } 0 \leq x < 2\pi$	6
Ans	$x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$	
2013 P1	**10.** If $0 < a < 90$, ~~which of the following is equivalent~~ to $\cos(270 - a)°$? (simplify)	
Ans	$-\sin a°$	
2013 P1	**9.** The diagram shows a right-angled triangle with sides and angles as marked.  Find the value of $\sin 2x$.	

Ans	4/5	
2012 P2	**6.** For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation $u_{n+1} = (\sin x)u_n + \cos 2x$, with $u_0 = 1$. (*a*) Why do these sequences have a limit? (*b*) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$. Find the value(s) of x.	2 7
Ans	(a) $0 < x < \frac{\pi}{2}$ so $0 < \sin x < 1$ (b) 0.73 radians, 2.41 radians, 4.71 radians	
2012 P1	**5.** The diagram shows a right-angled triangle with sides and angles as marked.  What is the value of $\cos 2a$?	2
Ans	$\frac{7}{25}$	
2011 P1	**23.** (*a*) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. (*b*) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$.	5 2
Ans	(a) x = 0, 60, 300 (b) x = 0, 30, 150, 180, 210, 330	

2011 P1	**12.** The diagram shows two right-angled triangles with sides and angles as given.  What is the value of $\sin(p+q)$?	2
Ans	$\frac{2}{3}+\frac{2}{3\sqrt{5}}$	
2010 P2	**4.** Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \leq x < 2\pi$.	5
Ans	x = 2.42 and 3.86 radians	

2010 P1

23. (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$. **4**

(ii) Find the value of $\sin a$.

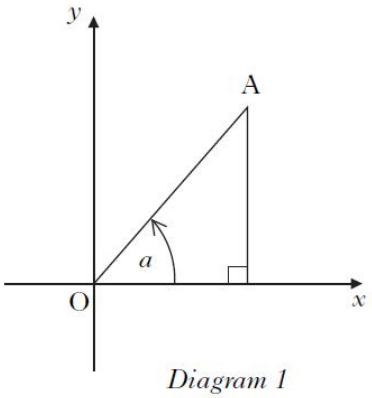


Diagram 1

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$. **4**

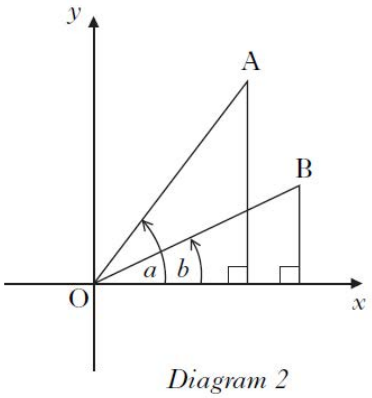


Diagram 2

(*c*) (i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$. **4**

Ans

(a) (ii) $\sin a = \frac{3}{\sqrt{13}}$ (b) $\sin b = \frac{3}{5}$ $\cos b = \frac{4}{5}$ (c) $\sin(a-b) = \frac{6}{5\sqrt{13}}$ $\sin(b-a) = -\frac{6}{5\sqrt{13}}$

2009 P1

24. (*a*) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. **3**

(*b*) Show that $\sin(A + B) + \sin(A - B) = 2\sin A \cos B$. **2**

(*c*) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$. **4**

Ans

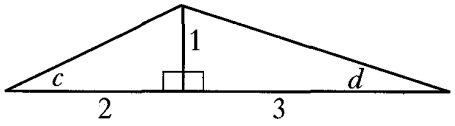
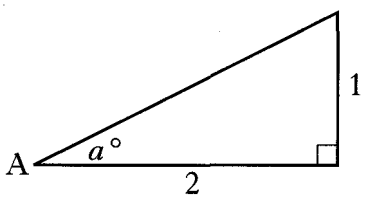
(a) $\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$ (c) (i) $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ (ii) $\frac{\sqrt{6}}{2}$

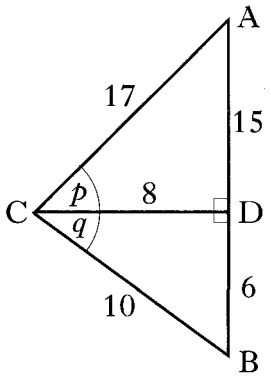
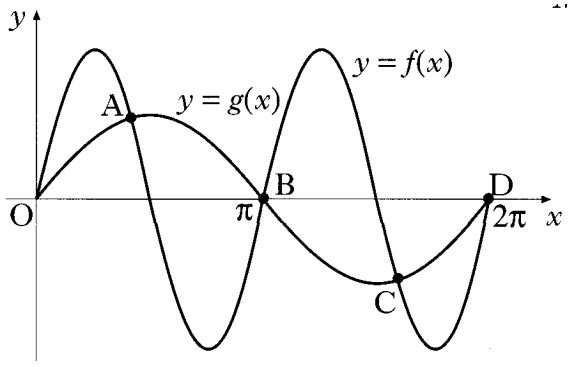
2009 P1

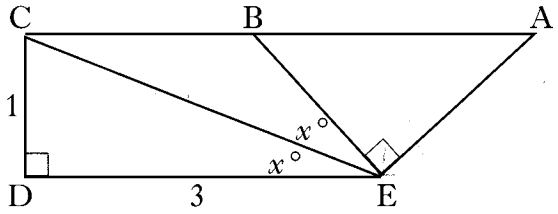
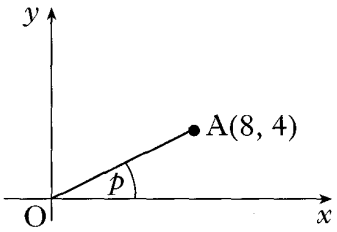
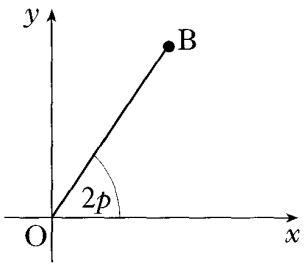
7. If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, find the exact value of $\cos 2x$. **2**

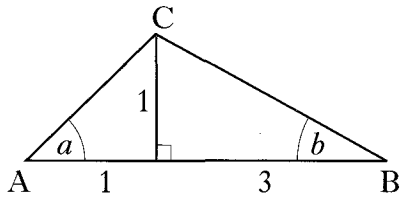
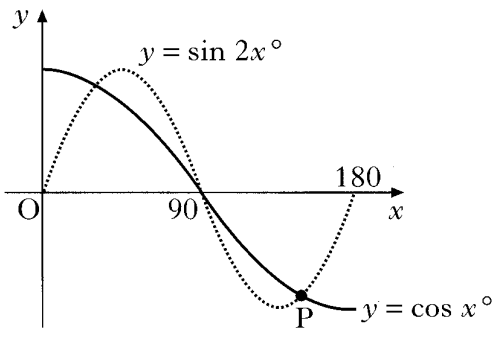
Ans

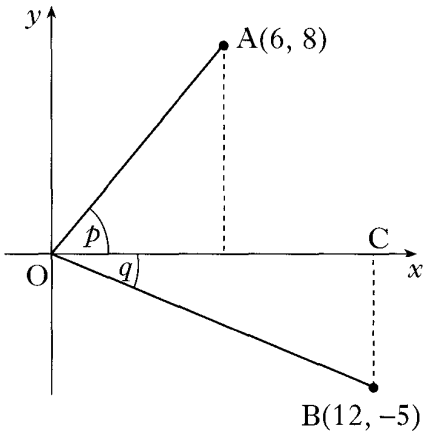
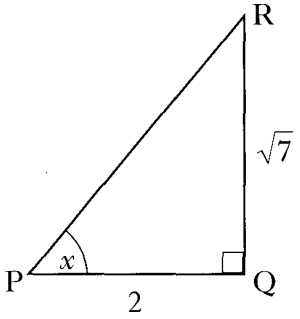
$-\frac{3}{5}$

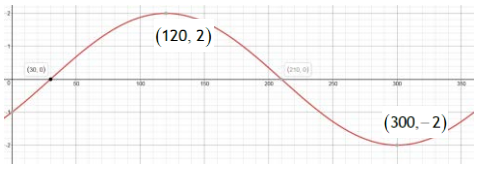
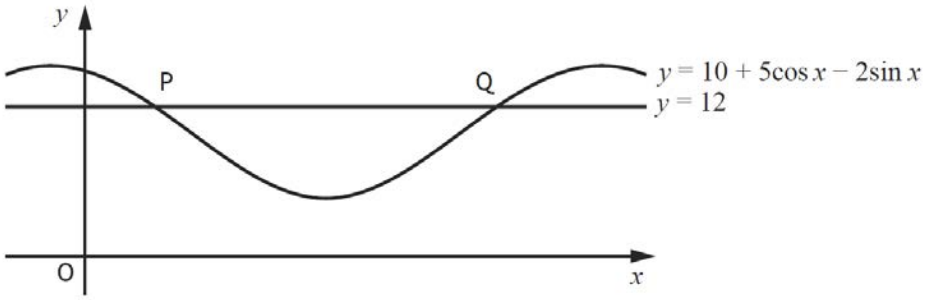
2008 P1	**9.** Given that $0 \le a \le \frac{\pi}{2}$ and $\sin a = \frac{3}{5}$, find an expression for $\sin(x + a)$.	2
Ans	$\frac{4}{5}\sin x + \frac{3}{5}\cos x$	
2008 P2	**5.** Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \le x < 360$.	5
Ans	*90 °, 199.5 °, 340.5 °*	
2007 P1	**6.** Solve the equation $\sin 2x° = 6\cos x°$ for $0 \le x \le 360$.	4
Ans	*x = 90, 270*	
2007 P2	**2.** The diagram shows two right-angled triangles with angles c and d marked as shown.  (*a*) Find the exact value of $\sin(c + d)$. (*b*) (i) Find the exact value of $\sin 2c$. (ii) Show that $\cos 2d$ has the same exact value.	4 4 4
Ans	(*a*) $\sqrt{\frac{1}{2}}$ (*b*) (i) $\frac{4}{5}$	
2006 P1	**7.** Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \le x \le 360$.	4
Ans	$x = 0, 180, 360, \quad 60, 300$	
2006 P2	**8.** The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a°$ at A.  (*a*) Find the exact values of: (i) $\sin a°$; (ii) $\sin 2a°$. (*b*) By expressing $\sin 3a°$ as $\sin(2a + a)°$, find the exact value of $\sin 3a°$.	4 4

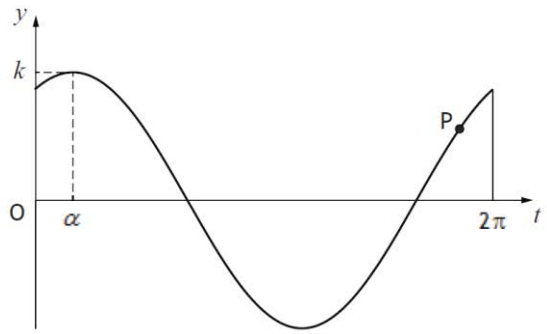
Ans	(*a*) (i) $\sin a^\circ = \frac{1}{\sqrt{5}}$ (ii) $\sin 2a^\circ = \frac{4}{5}$ (*b*) $\sin 3a^\circ = \frac{11}{5\sqrt{5}}$	
2005 P1	**9.** If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$.	4
Ans	$\cos(x) = \frac{4}{5}$ $\sin(x) = \frac{3}{5}$	
2005 P2	**2.** Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram. (*a*) Show that the exact value of $\sin(p+q)$ is $\frac{84}{85}$. (*b*) Calculate the exact values of: (i) $\cos(p+q)$; (ii) $\tan(p+q)$. 	4 3
Ans	(*a*) $\cos(p) = \frac{8}{17}$, $\sin(p) = \frac{15}{17}$ $\cos(q) = \frac{8}{10}$, $\sin(q) = \frac{6}{10}$ From sin(A+B) = sin(A)cos(B) + cos(A)sin(B) $\frac{15}{17} \times \frac{8}{10} + \frac{8}{17} \times \frac{6}{10} = \frac{84}{85}$ (*b*) (i) $-\frac{13}{85}$ (ii) $-\frac{84}{13}$	
2005 P2	**8.** Two functions, f and g, are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$. The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D. Show that, at A and C, $\cos x = \frac{1}{2k}$. 	5

Ans	$k\sin(2x)=\sin(x)$ $k \times 2\sin(x)\cos(x)$ $\sin(x) \times (2k\cos(x) - 1) = 0$ $\sin(x) = 0$ **and** $\cos(x) = \frac{1}{2k}$ $\sin(x) = 0 \Rightarrow x = 0, \pi, 2\pi$ at (O), B and D **and** $\cos(x) = \frac{1}{2k}$ at A and C	
2004 P1	**10.** In the diagram angle DEC = angle CEB = $x°$ and angle CDE = angle BEA = 90°. CD = 1 unit; DE = 3 units. By writing angle DEA in terms of $x°$, find the exact value of cos(DÊA). 	7
Ans	$\cos \text{DÊA} = -\frac{6}{10}$	
2003 P1	**10.** A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis. (*a*) Find the exact values of: (i) $\sin(2p)$; (ii) $\cos(2p)$.  The line OB is inclined at an angle $2p$ radians to the x-axis. (*b*) Write down the exact value of the gradient of OB. 	5 1
Ans	(*a*) (i) $\frac{4}{5}$ (ii) $\frac{3}{5}$ (*b*) $\frac{4}{3}$	
2003 P2	**10.** Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \le x \le \pi$, correct to 2 decimal places.	5
Ans	1·23 radians only	

2002W P2	**5.** Solve the equation $\cos 2x - 2\sin^2 x = 0$ in the interval $0 \le x < 2\pi$.	4
Ans	$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	
2002 P1	**3.** Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$. (*a*) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$. (*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \le x \le 360$.	2 5
Ans	(*a*) (i) $\sin(2x°)$ (ii) $2\sin(x°)$ (*b*) $4\sin(x°)\cos(x°) - 2\sin(x°) = 0$ $2\sin(x°)(2\cos(x°) - 1) = 0$ $\sin(x°) = 0$, $\cos(x°) =$ 0°, 60°, 180°, 300°, 360°	
2002 P1	**5.** In triangle ABC, show that the exact value of $\sin(a+b)$ is $\frac{2}{\sqrt{5}}$. 	4
Ans	• $AC = \sqrt{2}$ **and** $BC = \sqrt{10}$ • $\sin(a+b) = \sin a \cos b + \cos a \sin b$ • $= \frac{1}{\sqrt{2}} \cdot \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{10}}$ • $= \frac{4}{\sqrt{20}} = \frac{4}{\sqrt{4} \times \sqrt{5}} = \frac{2}{\sqrt{5}}$	
2001 P1	**5.** (*a*) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \le x \le 180$. (*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$. Use your solutions in (*a*) to write down the coordinates of the point P. 	4 1

Ans	(*a*) 30, 90, 150 (*b*) $\left(150, -\frac{\sqrt{3}}{2}\right)$	
2001 P1	**7.** Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers. (*a*) Find expressions for: (i) $f(h(x))$; (ii) $g(h(x))$. (*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$. (ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \le x \le 2\pi$.	2 5
Ans	(*a*) (i) $\sin\left(x + \frac{\pi}{4}\right)$; (ii) $\cos\left(x + \frac{\pi}{4}\right)$ (*b*) (i) $\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$ (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$	
2000 P1	**1.** On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, −5). Angle AOC = p and angle COB = q. Find the exact value of $\sin(p + q)$. 	4
Ans	$\frac{63}{65}$	
Specimen 2 P1	**7.** Using triangle PQR, as shown, find the exact value of $\cos 2x$. 	3
Ans	$\cos x = \frac{2}{\sqrt{11}}$ $\cos 2x = 2 \times \left(\frac{2}{\sqrt{11}}\right)^2 - 1 = -\frac{3}{11}$	

2017 P1	**14.** (a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x-a)°$, where $k > 0$ and $0 < a < 360$. (b) Hence, or otherwise, sketch the graph with equation $y = \sqrt{3}\sin x° - \cos x°$, $0 \le x \le 360$. Use the diagram provided in the answer booklet.	4 3
Ans	(a) $2\sin(x-30)°$ (b) 	
2016 P2	**8.** (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x+a)$, where $k > 0$ and $0 < a < 2\pi$. (b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$. The line cuts the curve at the points P and Q.  Find the x-coordinates of P and Q.	4 4
Ans	(a) $\sqrt{29}\cos(x+0{\cdot}38)$ (b) P(0.8097, 12), Q(4.712, 12)	
NH 2015 P2	**9.** The blades of a wind turbine are turning at a steady rate. The height, h metres, of the tip of one of the blades above the ground at time, t seconds, is given by the formula $h = 36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t) + 65$. Express $36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t)$ in the form $k\sin(1{\cdot}5t - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$, and hence find the **two** values of t for which the tip of this blade is at a height of 100 metres above the ground during the first turn.	8
Ans	t = 1.006 and 1.615	

NH SP P2	**10.** Two sound sources produce the waves $y = \sin t$ and $y = \sqrt{3}\cos t$. An investigation into the addition of these two waves produces the graph shown, with equation $y = k\cos(t - \alpha)$ for $0 \le t \le 2\pi$.  (a) Calculate the values of k and α. The point P has a y-coordinate of 1·2. (c) Hence calculate the value of the t-coordinate of point P.	4 4
Ans	(a) $K = 2.$, $\alpha = \frac{\pi}{6}$ (b) 5.9	
NH Ex P1	**11.** Functions f and g are defined on suitable domains by $f(x) = x^3 - 1$ and $g(x) = 3x + 1$. (a) Find an expression for $k(x)$, where $k(x) = g(f(x))$. (b) If $h(k(x)) = x$, find an expression for $h(x)$.	2 3
Ans	(a) $3(x^3 - 1) + 1$ (b) $h(x) = \sqrt[3]{\frac{x+2}{3}}$	
NH EX P1	**9.** The expression $\cos 4x - \sqrt{3}\sin 4x$ can be written in the form $k\cos(4x + a)$ where $k > 0$ and $0 \le a \le 2\pi$. (a) Calculate the values of k and a. (b) Find the points of intersection of the graph of $y = \cos 4x - \sqrt{3}\sin 4x$ with the x axis, in the interval $0 \le x \le \frac{\pi}{2}$.	4 3
Ans	(a) $k = 2$, $a = \frac{\pi}{3}$ (b) $\left(\frac{\pi}{24}, 0\right)$, $\left(\frac{7\pi}{24}, 0\right)$	
2014 P1	**12.** A function f, defined on a suitable domain, is given by $f(x) = \frac{6x}{x^2 + 6x - 16}$. What restrictions are there on the domain of f?	2
Ans	$x \neq -8, x \neq 2$	
2014 P1	**4.** If $3\sin x - 4\cos x$ is written in the form $k\cos(x - a)$, what are the values of $k\cos a$ and $k\sin a$?	

Ans	$k\cos a = -4$, $k\sin a = 3$	
2013 P1	**23.** (*a*) The expression $\sqrt{3}\sin x° - \cos x°$ can be written in the form $k\sin(x - a)°$, where $k > 0$ and $0 \le a < 360$. Calculate the values of k and a. (*b*) Determine the maximum value of $4 + 5\cos x° - 5\sqrt{3}\sin x°$, where $0 \le x < 360$.	4 2
Ans	(a) $k = 2$, $a = 30$ (b) 14	
2012 P1	**22.** (*a*) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos(x + a)$ where $k > 0$ and $0 \le a < 2\pi$. Calculate the values of k and a. (*b*) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x and y axes, in the interval $0 \le x \le 2\pi$.	4 3
Ans	(a) $k = 2, a = \frac{\pi}{3}$ (c) $(0, 1), \left(\frac{\pi}{6}, 0\right), (\frac{7\pi}{6}, 0)$	
2011 P2	**6.** (*a*) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x + a)$ where $R > 0$ and $0 \le a < 2\pi$. Calculate the values of R and a. (*b*) Hence find the value of t, where $0 \le t \le 2$, for which $\int_0^t (3\cos x + 5\sin x)\, dx = 3.$	4 7
Ans	(a) R = $\sqrt{34}$ a = 5.3 radians (b) t = 0.7	
2010 P2	**2.** (*a*) $12\cos x° - 5\sin x°$ can be expressed in the form $k\cos(x + a)°$, where $k > 0$ and $0 \le a < 360$. Calculate the values of k and a. (*b*) (i) Hence state the maximum and minimum values of $12\cos x° - 5\sin x°$. (ii) Determine the values of x, in the interval $0 \le x < 360$, at which these maximum and minimum values occur.	4 3
Ans	(a) k = 13 and a = 22.6 degrees (b) (i) Max 13, Min -13 (ii) max at 337.4 and Min at 157.4	

2009 P1

13. k and a are given by

$$k \sin a^\circ = 1$$
$$k \cos a^\circ = \sqrt{3}$$

where $k > 0$ and $0 \leq a < 90$.

What are the values of k and a?

2

Ans

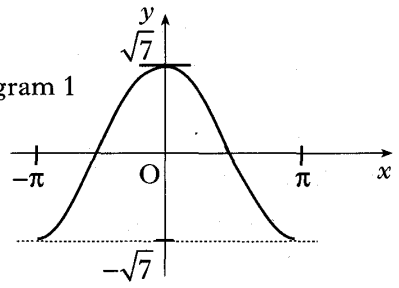
$k = 2$, $a = 30$

2008 P2

3. (a) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

Diagram 1

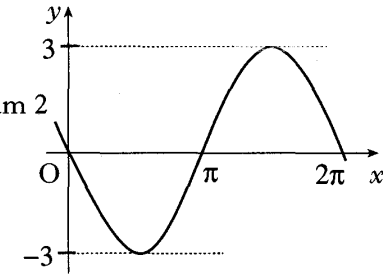


2

(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q.

Diagram 2



(b) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. 4

(c) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. 2

Ans

(a) $p = \sqrt{7}$, $q = -3$ (b) $4\cos(x + 0.848)$ (c) $-4\sin(x + 0.848)$

2007 P2

11. (a) Express $f(x) = \sqrt{3}\cos x + \sin x$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$. 4

(b) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$. 4

Ans

(a) $a = \frac{1}{2}$

(b) $b = \frac{3}{2}$

(c) $y = 3 \times 4^x$

$\log_{10} y = \log_{10} 3 + \log_{10}(4^x)$

$= \log_{10} 3 + x \log_{10}(4)$

So gradient of line $= \log_{10}(4)$

2006 P2

10. A curve has equation $y = 7\sin x - 24\cos x$.

(*a*) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \le a \le \frac{\pi}{2}$. 4

(*b*) Hence find, in the interval $0 \le x \le \pi$, the x-coordinate of the point on the curve where the gradient is 1. 3

Ans

(*a*) $k = 25$
$a = 1{\cdot}29$

(*b*) $x = 2{\cdot}82$

2005 P1

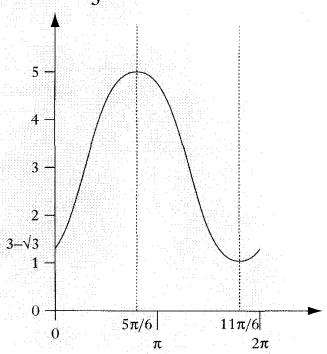
10. (*a*) Express $\sin x - \sqrt{3}\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \le a \le 2\pi$. 4

(*b*) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3}\cos x$ in the interval $0 \le x \le 2\pi$. 5

Ans

(*a*) $2\sin(x - \frac{\pi}{3})$

(*b*)



2004 P2

6. (*a*) Express $3\cos(x°) + 5\sin(x°)$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \le a \le 90$. 4

(*b*) Hence solve the equation $3\cos(x°) + 5\sin(x°) = 4$ for $0 \le x \le 90$. 3

Ans

(*a*) $\sqrt{34}\cos(x - 59)°$

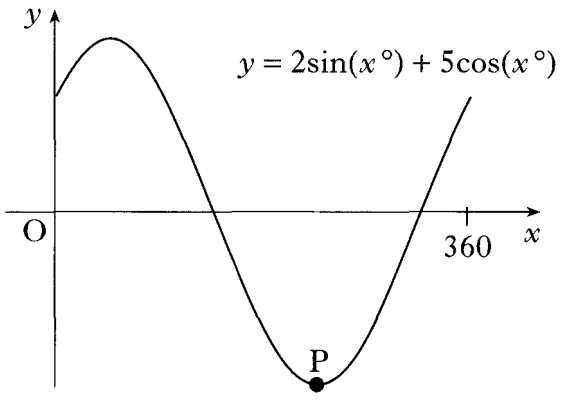
(*b*) $x = 12{\cdot}3$

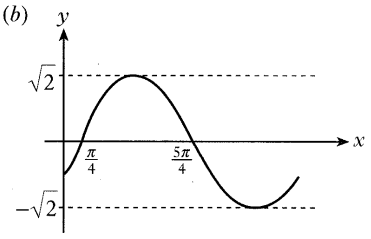
2003 P2

7. Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.

(*a*) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \le a < 360$. 4

(*b*) Find the coordinates of the minimum turning point P. 3

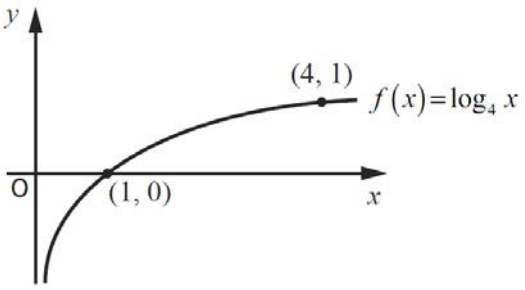


Ans	(*a*) $\sqrt{29}\sin(x + 68{\cdot}2)^\circ$ (*b*) $(201{\cdot}8^\circ, -\sqrt{29})$	
2002W P2	**4.** (*a*) Write $\sqrt{3}\sin x^\circ + \cos x^\circ$ in the form $k\sin(x+a)^\circ$ where $k > 0$ and $0 \le a < 360$. (*b*) Hence find the maximum value of $5 + \sqrt{3}\sin x^\circ + \cos x^\circ$ and determine the corresponding value of x in the interval $0 \le x \le 360$.	4 2
Ans	(*a*) $2\sin(x+30)^\circ$ (*b*) max = 7 when $x = 60$	
2002W P2	**4.** (*a*) Write $\sqrt{3}\sin x^\circ + \cos x^\circ$ in the form $k\sin(x+a)^\circ$ where $k > 0$ and $0 \le a < 360$. (*b*) Hence find the maximum value of $5 + \sqrt{3}\sin x^\circ + \cos x^\circ$ and determine the corresponding value of x in the interval $0 \le x \le 360$.	4 2
Ans	(*a*) $2\sin(x+30)^\circ$ (*b*) max = 7 when $x = 60$	
2002 P1	**9.** (*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \le a \le 2\pi$. (*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \le x \le 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis.	4 3
Ans	(*a*) $\sqrt{2}\sin(x - \frac{\pi}{4})$ (*b*) 	
2001 P2	**5.** Express $8\cos x^\circ - 6\sin x^\circ$ in the form $k\cos(x+a)^\circ$ where $k > 0$ and $0 < a < 360$.	4
Ans	$10\cos(x + 36{\cdot}9)^\circ$	
2000 P1	**10.** Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \le x \le 2\pi$.	6
Ans	max value $= \sqrt{2}$ when $x = \frac{7\pi}{4}$	

Specimen 2 P1	**6.** $f(x) = \sqrt{3}\sin x° - \cos x°$ (*a*) Express $f(x)$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \le a < 360$. (*b*) Hence solve the equation $f(x) = \sqrt{2}$ in the interval $0 \le a < 360$.	4 3
Ans	(*a*) compare $\sqrt{3}\sin x° - \cos x°$ with $k\sin x°\cos a° - k\cos x°\sin a°$ $k\cos a° = \sqrt{3}$, $k\sin a° = 1$ $k = 2$, $\tan a° = \frac{1}{\sqrt{3}} \Rightarrow a = 30$ (*b*) $2\sin(x - 30)° = \sqrt{2}$ $x - 30 = 45, 135$ $x = 75, 165$	
Specimen 1 P2	**6.** $f(x) = 2\cos x° + 3\sin x°$. (*a*) Express $f(x)$ in the form $k\cos(x - \alpha)°$ where $k > 0$ and $0 \le \alpha < 360$. (*b*) Hence solve $f(x) = 0{\cdot}5$ for $0 \le x < 360$. (*c*) Find the x-coordinate of the point nearest to the origin where the graph of $f(x) = 2\cos x° + 3\sin x°$ cuts the x-axis for $0 \le x < 360$.	4 3 2
Ans	(*a*) $\sqrt{13}\cos(x - 56{\cdot}3)°$ (*b*) 138·8, 334·3 (*c*) 146·3°	

2016 P1

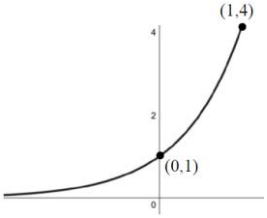
10. The diagram below shows the graph of the function $f(x)=\log_4 x$, where $x>0$.



The inverse function, f^{-1}, exists.

On the diagram in your answer booklet, sketch the graph of the inverse function. 2

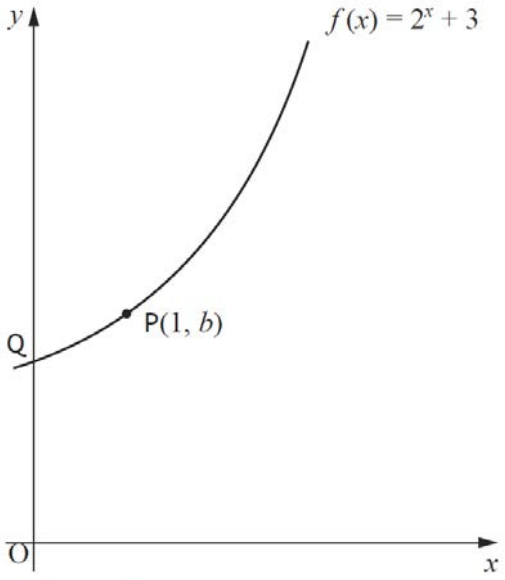
Ans



2015 P1

13. The function $f(x) = 2^x + 3$ is defined on $\mathbb{R}$, the set of real numbers.

The graph with equation $y = f(x)$ passes through the point P$(1, b)$ and cuts the y-axis at Q as shown in the diagram.



(a) What is the value of b? 1

(b) (i) Copy the above diagram.

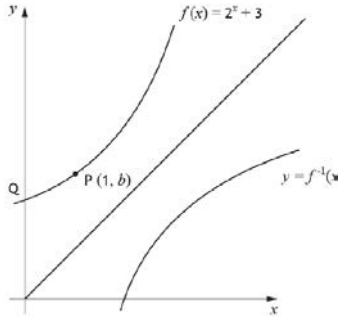
On the same diagram, sketch the graph with equation $y = f^{-1}(x)$. 1

(ii) Write down the coordinates of the images of P and Q. 3

(c) R $(3,11)$ also lies on the graph with equation $y = f(x)$.

Find the coordinates of the image of R on the graph with equation $y = 4 - f(x+1)$. 2

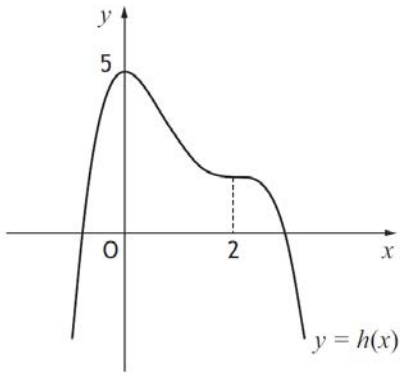
Ans



(a) b = 5 (b) (i) (ii) P(5, 1) Q(4, 0) (c) (2, -7)

NH EX P1

8. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.

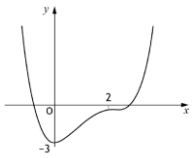


On separate diagrams sketch the graphs of:

(a) $y = 2 - h(x)$. 3

(b) $y = h'(x)$. 3

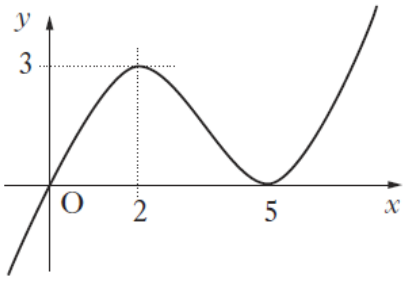
Ans



(a)

2014 P1

11. The diagram shows part of the graph of $y = f(x)$.

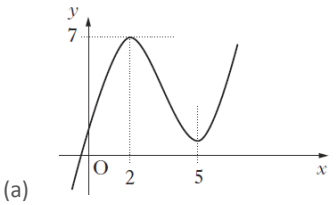


2

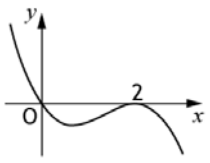
~~Which of the following diagrams could be~~ the graph of $y = 2f(x) + 1$?

(Sketch)

Ans



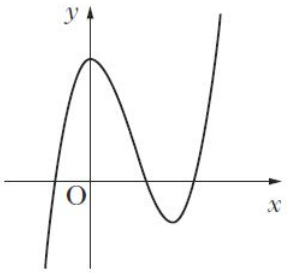
(a)



(b)

2013 P1

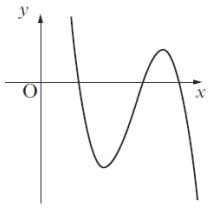
11. The diagram shows a cubic curve with equation $y = f(x)$.



Which of the following diagrams could show the curve with equation $y = -f(x - k)$, $k > 0$?

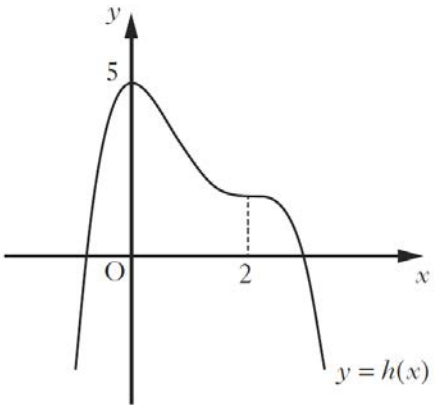
(Sketch)

Ans



2012 P2

4. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.

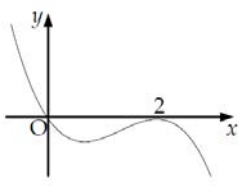


On separate diagrams sketch the graphs of:

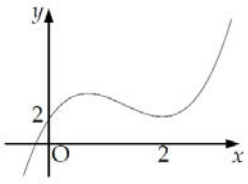
(*a*) $y = h'(x)$; 3

(*b*) $y = 2 - h'(x)$. 3

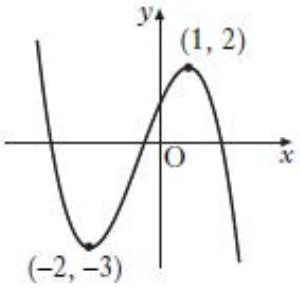
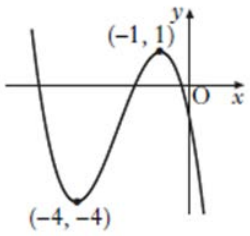
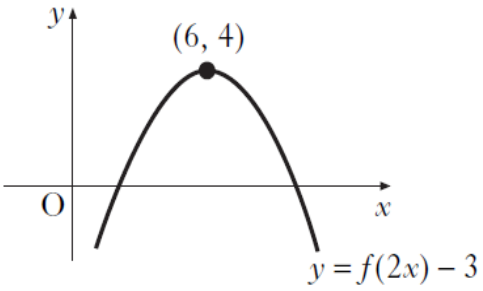
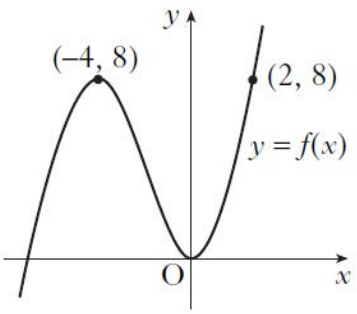
Ans

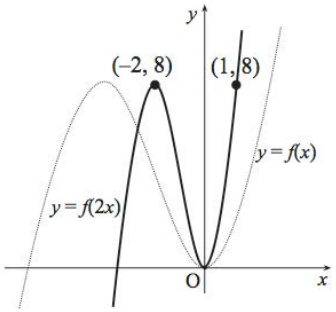
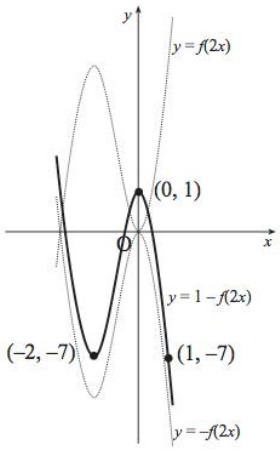
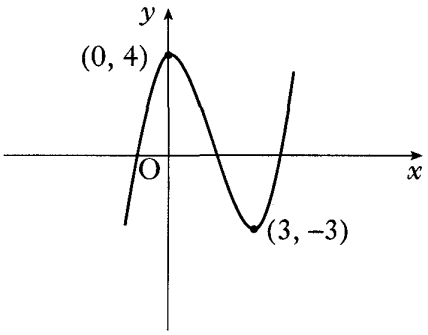
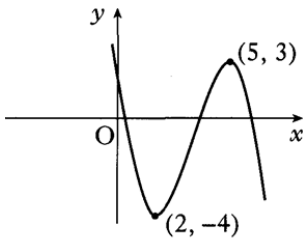
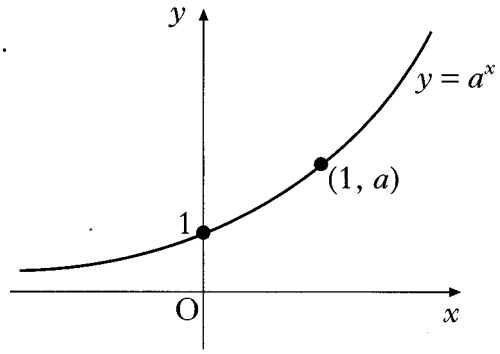


(a)

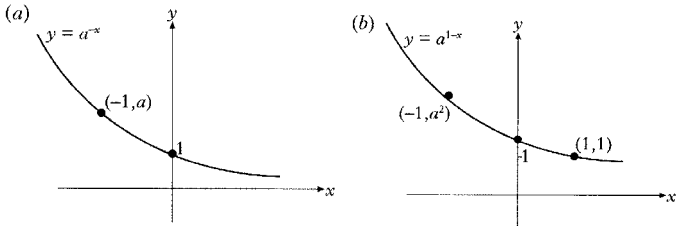


(b)

2011 P1	**3.** The diagram shows the graph of $y = f(x)$.  Which of the following shows the graph of $y = f(x + 2) - 1$? (Sketch the graph)	2
Ans	 	
2010 P1	**20.** The diagram shows the graph of $y = f(2x) - 3$.  What are the coordinates of the turning point on the graph of $y = f(x)$?	
Ans	(12, 7)	
2009 P1	**23.** The diagram shows a sketch of the function $y = f(x)$. (*a*) Copy the diagram and on it sketch the graph of $y = f(2x)$. (*b*) On a separate diagram sketch the graph of $y = 1 - f(2x)$.  	2 3

Ans	(a)  (b)  	
2008 P1	**8.** The diagram shows part of the graph of a function with equation $y = f(x)$.  ~~Which~~ of the following diagrams shows the graph with equation $y = -f(x - 2)$? (Sketch the graph)	2
Ans	 	
2007 P2	**9.** The diagram shows the graph of $y = a^x$, $a > 1$. On separate diagrams, sketch the graphs of: (a) $y = a^{-x}$; (b) $y = a^{1-x}$.  	2 2

Ans



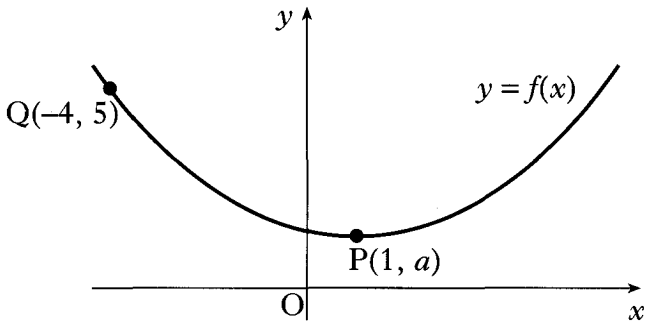
2006 P2

7. The diagram shows the graph of a function $y = f(x)$.

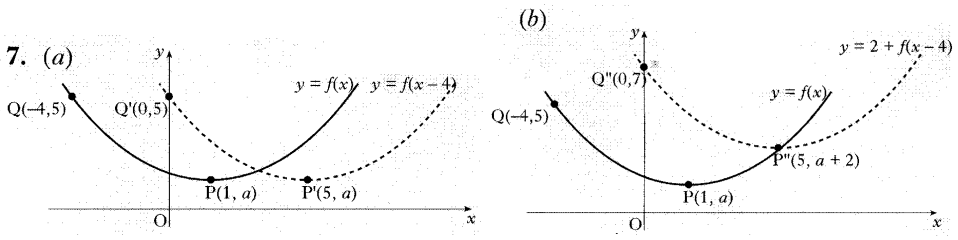
Copy the diagram and on it sketch the graphs of:

(a) $y = f(x - 4)$; 2

(b) $y = 2 + f(x - 4)$. 2



Ans

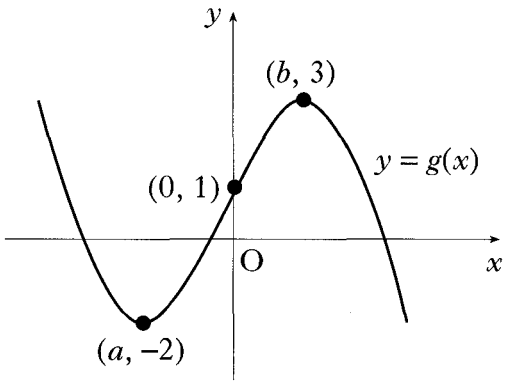


2004 P1

4. The diagram shows the graph of $y = g(x)$.

(a) Sketch the graph of $y = -g(x)$. 2

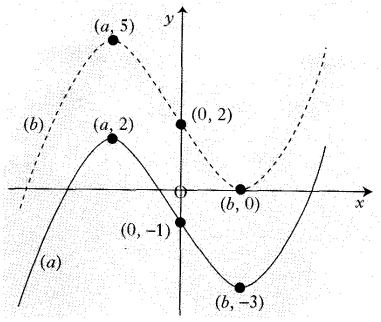
(b) On the same diagram, sketch the graph of $y = 3 - g(x)$. 2



Ans

(a) Reflect in x–axis.

(b) Translate +3 parallel to y–axis



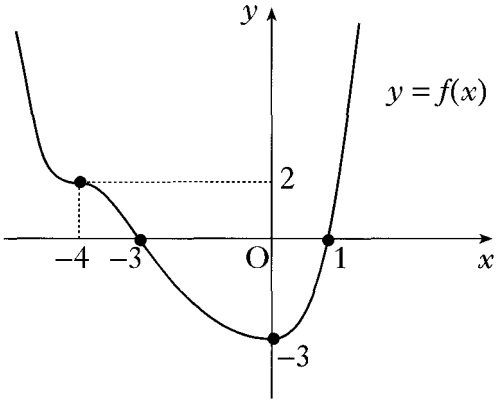
2003 P2

5. The diagram shows the graph of a function f.

f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

(a) Sketch the graph of $y = f(-x)$.

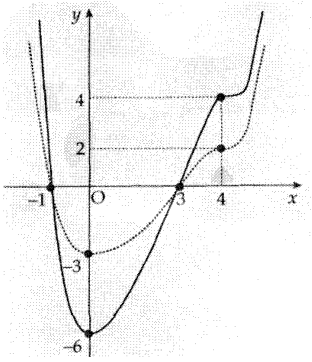
(b) On the same diagram, sketch the graph of $y = 2f(-x)$.



2

2

Ans

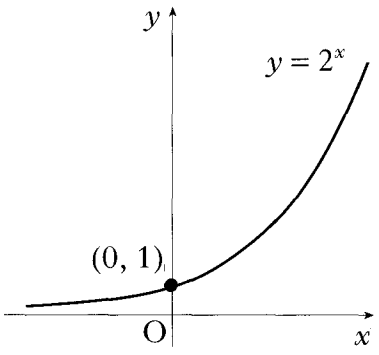


2002W P1

8. The diagram shows part of the graph of $y = 2^x$.

(a) Sketch the graph of $y = 2^{-x} - 8$.

(b) Find the coordinates of the points where it crosses the x and y axes.



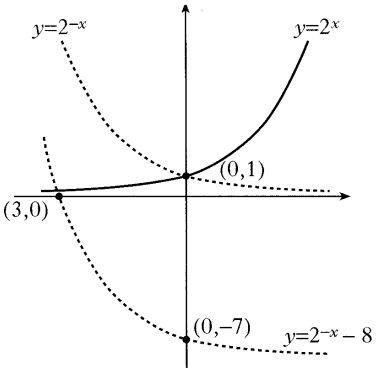
2

2

Ans

. (a) & (b)

Reflect graph in y-axis followed by a translation $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$



2002 P1

7. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. — 2

(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$; — 4

(ii) the graph of $y = 10 - f(x)$.

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. — 1

Ans

(*a*) $x = 7$

(*b*) $3x + 2y = 23$

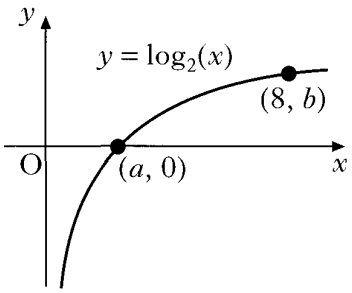
(*c*) (7, 1)

2001 P1

10. The diagram shows a sketch of part of the graph of $y = \log_2(x)$.

(*a*) State the values of a and b. — 1

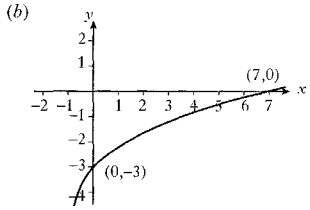
(*b*) Sketch the graph of $y = \log_2(x + 1) - 3$. — 3



Ans

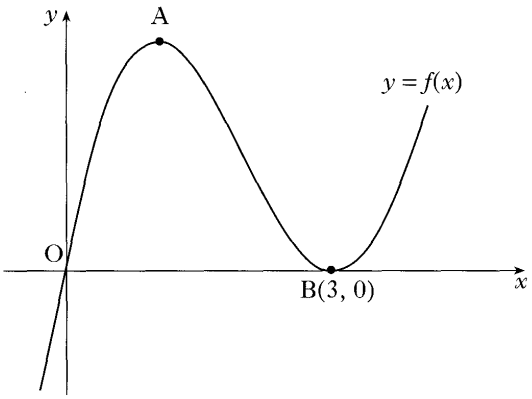
(*a*) $a = 1, \ b = 3$

(*b*)



2000 P1

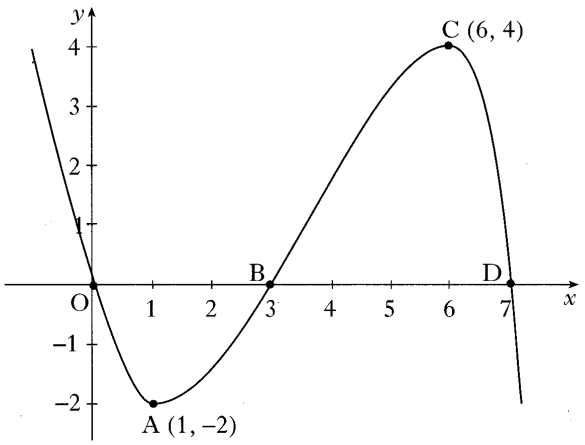
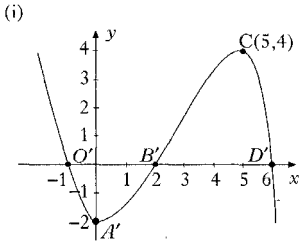
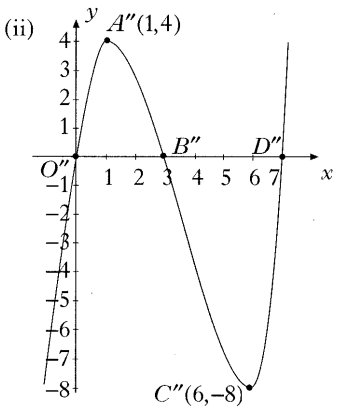
2. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).



(*a*) Find the coordinates of the turning point at A. — 4

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. — 2

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. — 1

Ans	(*a*) A = (1,4) (*b*) $f(x)$ needs to be translated 4 units up, 2 units left Sketch with A′ at (–1,8) and B′ at (1,4) (*c*) $4 < k < 8$	
Specimen 2 P2	**5.** Part of the graph of $y = f(x)$ is shown in the diagram. On separate diagrams, sketch the graphs of (i) $y = f(x + 1)$ (ii) $y = -2f(x)$. Indicate on each graph the images of O, A, B, C and D. 	5
Ans	(i)  (ii) 	

2017 P1	**1.** Functions f and g are defined on suitable domains by $f(x)=5x$ and $g(x)=2\cos x$. (a) Evaluate $f(g(0))$. (b) Find an expression for $g(f(x))$.	4
Ans	(b) $g(f(0))=10$ (c) $g(f(x))=2\cos 5x$	
2017 P1	**6.** A function, h, is defined by $h(x)=x^3+7$, where $x\in\mathbb{R}$. Determine an expression for $h^{-1}(x)$.	3
Ans	$h^{-1}(x)=(x-7)^{\frac{1}{3}}$	
2016 P1	**6.** Functions f and g are defined on $\mathbb{R}$, the set of real numbers. The inverse functions f^{-1} and g^{-1} both exist. (a) Given $f(x)=3x+5$, find $f^{-1}(x)$. (b) If $g(2)=7$, write down the value of $g^{-1}(7)$.	3 1
Ans	(a) $f^{-1}(x)=\frac{x-5}{3}$ (b) $g^{-1}(7)=2$	
2016 P1	**12.** The functions f and g are defined on $\mathbb{R}$, the set of real numbers by $f(x)=2x^2-4x+5$ and $g(x)=3-x$. (a) Given $h(x)=f(g(x))$, show that $h(x)=2x^2-8x+11$. (b) Express $h(x)$ in the form $p(x+q)^2+r$.	2 3
Ans	(a) *Proof* (b) $h(x)=2(x-2)^2+3$	
NH 2015 P1	**5.** A function g is defined on $\mathbb{R}$, the set of real numbers, by $g(x)=6-2x$. (a) Determine an expression for $g^{-1}(x)$. (b) Write down an expression for $g(g^{-1}(x))$.	2 1
Ans	(a) $g^{-1}(x)=\frac{6-x}{2}$ (b) x	

NH 2015 P2	**2.** Functions f and g are defined on suitable domains by $f(x) = 10 + x$ and $g(x) = (1 + x)(3 - x) + 2$. (a) Find an expression for $f(g(x))$. (b) Express $f(g(x))$ in the form $p(x + q)^2 + r$. (c) Another function h is given by $h(x) = \frac{1}{f(g(x))}$. What values of x cannot be in the domain of h?	2 3 2
Ans	(a) $12 + (1 + x)(3 - x)$ (b) $-(x - 1)^2 + 16$ (c) $x \neq 5$ *or* -3	
NH SP P1	**8.** $f(x)$ and $g(x)$ are functions, defined on the set of real numbers, such that $f(x) = 1 - \frac{1}{2}x$ and $g(x) = 8x^2 - 3$. (a) Given that $h(x) = g(f(x))$, show that $h(x) = 2x^2 - 8x + 5$.	3
Ans	Proof	
2014 P2	**3.** Functions f and g are defined on suitable domains by $f(x) = x(x - 1) + q$ and $g(x) = x + 3$. (a) Find an expression for $f(g(x))$. (b) Hence, find the value of q such that the equation $f(g(x)) = 0$ has equal roots.	2 4
Ans	*(a)* $(x + 3)(x + 2) + q$ *(b)* $q = \frac{1}{4}$	
2013 P1	**13.** A function f is defined on a suitable domain by $f(x) = \frac{x + 2}{x^2 - 7x + 12}$. What value(s) of x cannot be in this domain?	
Ans	3 and 4	
2013 P 1	**1.** The functions f and g are defined by $f(x) = x^2 + 1$ and $g(x) = 3x - 4$, on the set of real numbers. Find $g(f(x))$.	
Ans	$3x^2 - 1$	

2012 P2	**1.** Functions f and g are defined on the set of real numbers by • $f(x) = x^2 + 3$ • $g(x) = x + 4$. (a) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$. (b) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots.	3 3
Ans	(i) $(x + 4)^2 + 3$ (ii) $x^2 + 7$	
2010 P1	**11.** Functions f and g are defined on suitable domains by $f(x) = \cos x$ and $g(x) = x + \frac{\pi}{6}$. What is the value of $f\left(g\left(\frac{\pi}{6}\right)\right)$?	2
Ans	1/2	
2008 P1	**17.** A function f is given by $f(x) = \sqrt{9 - x^2}$. What is a suitable domain of f?	2
Ans	$-3 \leq x \leq 3$	
2008 P1	**23.** Functions f, g and h are defined on suitable domains by $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$. (a) Find expressions for $h(f(x))$ and $h(g(x))$.	3
Ans	(a) $h(f(x)) = \log_2(x^2 - x + 10)$ $h(g(x)) = \log_2(5 - x)$	
2007 P1	**3.** Functions f and g, defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$. Find: (a) $g(f(x))$; (b) $g(g(x))$.	2 2

Ans	(a) $g(f(x)) = -2x^2 - 1$ (b) $(g(g(x)) = 4x - 1$	
2006 P1	**3.** Two functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number. (a) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$. (b) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.	3 2
Ans	(a) $f(g(x)) = 2(2x - 3) + 3$ $g(f(x)) = 2(2x + 3) - 3$ (b) $16x^2 - 9$ minimum value $= -9$	
2005 P1	**4.** Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers. (a) Find $h(x)$ where $h(x) = g(f(x))$. (b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$. (ii) Hence state the range of the function h.	2 2 2
Ans	(a) $(3x - 1)^2 + 7$ (b) (i) $\left(\frac{1}{3}, 7\right)$ (ii) $y \geq 7$	
2003 P1	**9.** Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains. (a) Find an expression for $h(x)$ where $h(x) = f(g(x))$. (b) Write down any restriction on the domain of h.	2 1
Ans	(a) $\frac{1}{2x-1}$ (b) $x \neq \frac{1}{2}$	
2002W P1	**9.** The function f, defined on a suitable domain, is given by $f(x) = \frac{3}{x+1}$. (a) Find an expression for $h(x)$ where $h(x) = f(f(x))$, giving your answer as a fraction in its simplest form. (b) Describe any restriction on the domain of h.	3 1
Ans	(a) $\frac{3(x+1)}{x+4}$ (b) $x \neq -4$	

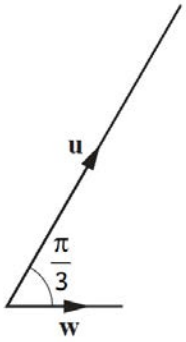
2002 P1	**3.** Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$. (a) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$.	2
Ans	(a) (i) $\sin(2x°)$ (ii) $2\sin(x°)$	
2001 P1	**7.** Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers. (a) Find expressions for: (i) $f(h(x))$; (ii) $g(h(x))$.	2
Ans	(a) (i) $\sin\left(x+\frac{\pi}{4}\right)$; (ii) $\cos\left(x+\frac{\pi}{4}\right)$	
2000 P2	**3.** $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$. (a) Find $p(x)$ where $p(x) = f(g(x))$. (b) If $q(x) = \frac{3}{3-x}$, $x \neq 3$, find $p(q(x))$ in its simplest form.	2 3
Ans	(a) $3 - \frac{3}{x}$ (b) x	
Specimen 2 P1	**8.** Functions f and g are defined on the set of real numbers by $f(x) = x - 1$ $g(x) = x^2$. (a) Find formulae for (i) $f(g(x))$ (ii) $g(f(x))$.	3
Ans	(a) $f(g(x)) = x^2 - 1$, $g(f(x)) = (x-1)^2$	

2017 P1

5. Vectors $\mathbf{u}$ and $\mathbf{v}$ are $\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$ respectively.

(a) Evaluate $\mathbf{u}.\mathbf{v}$. 1

(b)



Vector $\mathbf{w}$ makes an angle of $\frac{\pi}{3}$ with $\mathbf{u}$ and $|\mathbf{w}| = \sqrt{3}$.

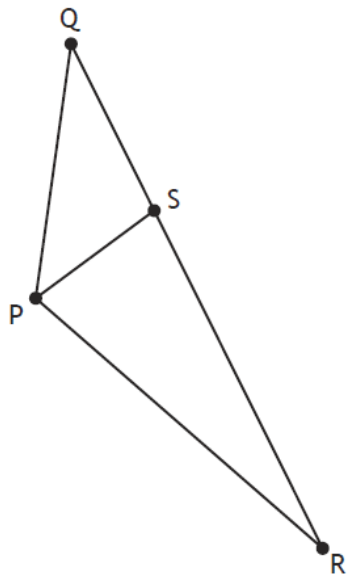
Calculate $\mathbf{u}.\mathbf{w}$. 3

Ans

(a) 1
(b) $|u| = 4.5$

2017 P2

5. In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.



(a) Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. 2

The point S divides QR in the ratio 1:2. 2

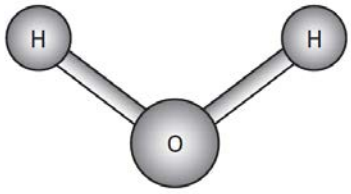
(b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$. 5

(c) Hence, find the size of angle QPS.

Ans	(a) $\overrightarrow{PQ} = -3\boldsymbol{i} - 4\boldsymbol{j} + 5\boldsymbol{k}$ (b) $Proof$ (c) 45.6° or 0.795 radians	
2016 p1	**7.** Three vectors can be expressed as follows: $\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$ $\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$ $\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ (a) Find $\overrightarrow{FH}$. (b) Hence, or otherwise, find $\overrightarrow{FE}$.	2 2
Ans	(a) $\overrightarrow{FH} = \boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ (b) $\overrightarrow{FE} = -\boldsymbol{i} - 5\boldsymbol{k}$	
2016 P1	**11.** (a) A and C are the points $(1, 3, -2)$ and $(4, -3, 4)$ respectively. Point B divides AC in the ratio $1 : 2$. Find the coordinates of B. (b) $k\overrightarrow{AC}$ is a vector of magnitude 1, where $k > 0$. Determine the value of k.	2 3
Ans	(a) B(2, 1, 0) (b) K = 1/9	

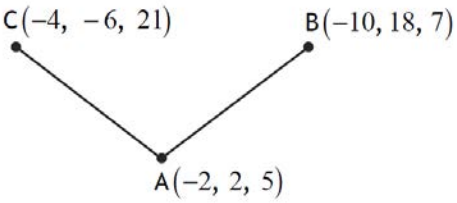
2016 P2

5. The picture shows a model of a water molecule.



Relative to suitable coordinate axes, the oxygen atom is positioned at point $A(-2, 2, 5)$.

The two hydrogen atoms are positioned at points $B(-10, 18, 7)$ and $C(-4, -6, 21)$ as shown in the diagram below.



(a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. — 2

(b) Hence, or otherwise, find the size of angle BAC. — 4

Ans

$\overrightarrow{AB} = \begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$

IVH 2015 P1

1. Vectors $\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ are perpendicular.

Determine the value of t. — 2

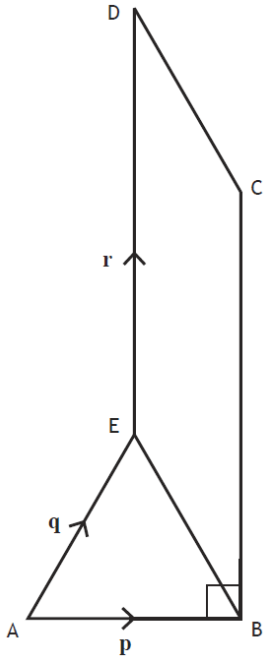
Ans

t = 9

2015 P2

6. Vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are represented on the diagram as shown.

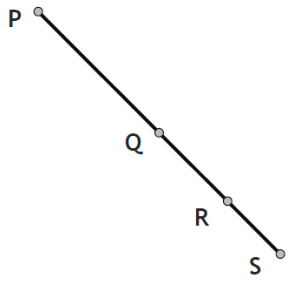
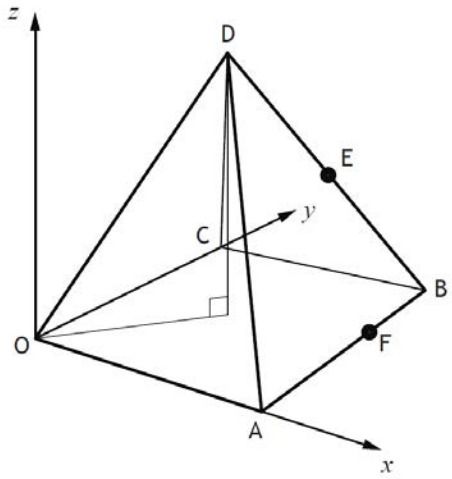
- BCDE is a parallelogram
- ABE is an equilateral triangle
- $|\mathbf{p}| = 3$
- Angle ABC = 90°

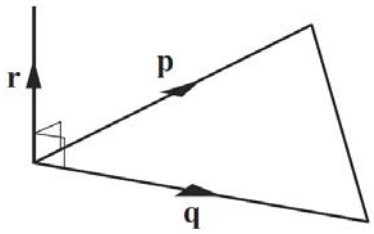
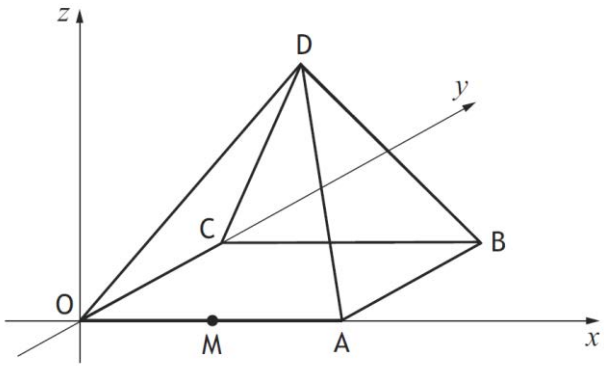


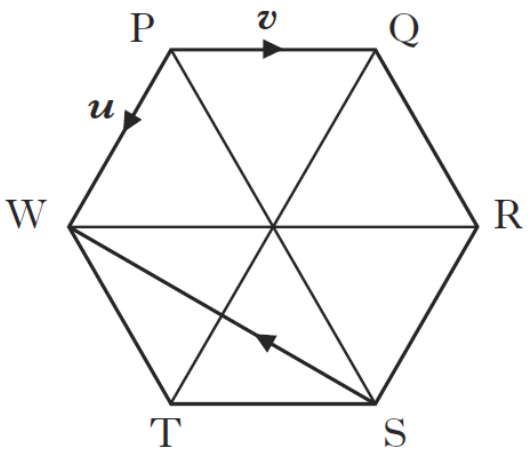
(a) Evaluate $\mathbf{p} \cdot (\mathbf{q}+\mathbf{r})$. — 3

(b) Express $\overrightarrow{EC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$. — 1

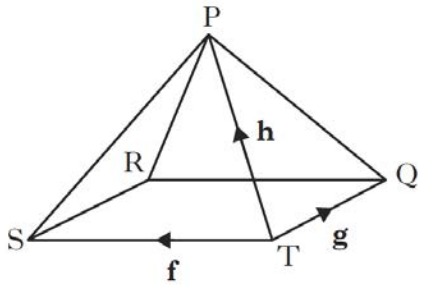
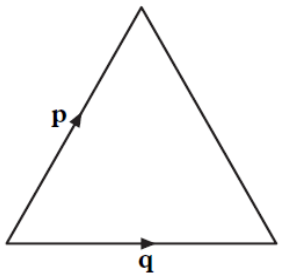
(c) Given that $\overrightarrow{AE} \cdot \overrightarrow{EC} = 9\sqrt{3} - \frac{9}{2}$, find $|\mathbf{r}|$. — 3

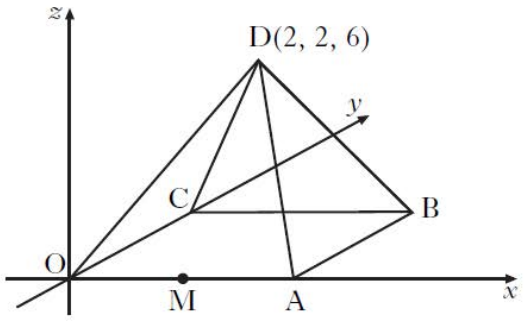
Ans	(a) $4\frac{1}{2}$ (b) $-\boldsymbol{q}+\boldsymbol{p}+\boldsymbol{r}$ (c) $\lvert\boldsymbol{r}\rvert = \frac{3\sqrt{3}}{\cos 30}$	
NH SP P1	**4.** Given that $2x^2 + px + p + 6 = 0$ has no real roots, find the range of values for p, where $p \in \mathbb{R}$.	4
Ans	$-4 < p < 12$	
NH SP P1	**3.** In the diagram, P has coordinates (–6, 3, 9), $\overrightarrow{PQ} = 6\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{PQ} = 2\overrightarrow{QR} = 3\overrightarrow{RS}$. Find the coordinates of S. 	5
Ans	S (5, 25, -2)	
NH SP P2	**6.** The points A(0, 9, 7), B(5, –1, 2), C(4, 1, 3) and D(x, –2, 2) are such that AB is perpendicular to CD. Determine the value of x.	5
Ans	x = -3	
NH SP P2	**1.**  A square based right pyramid is shown in the diagram. Square OABC has a side length of 60 units with edges OA and OC lying on the x-axis and y-axis respectively. The coordinates of D are (30, 30, 80). E is the midpoint of BD and F divides AB in the ratio 2:1. (a) Find the coordinates of E and F. (b) Calculate $\overrightarrow{ED}.\overrightarrow{EF}$. (c) Hence, or otherwise, calculate the size of angle DEF.	2 2 4

Ans	(a) E(45, 45, 40), F(60, 40, 0) (b) -1750 (c) 154°	
NH EX P2	**6.** An equilateral triangle with sides of length 3 units is shown.  Vector $\mathbf{r}$ is 2 units long and is perpendicular to both vectors $\mathbf{p}$ and $\mathbf{q}$. Calculate the value of the scalar product $\mathbf{p}.(\mathbf{p}+\mathbf{q}+\mathbf{r})$.	4
Ans	27/2	
NH EX P2	**5.** D,OABC is a square-based pyramid as shown.  O is the origin and OA = 4 units. M is the mid-point of OA. $\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ (a) Express $\overrightarrow{OB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$ and $\mathbf{k}$. (b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. (c) Find the size of angle BDM.	1 3 5
Ans	(a) $\overrightarrow{OB} = 4\boldsymbol{i} + 4\boldsymbol{j}$ (b) $\overrightarrow{DB} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}, \overrightarrow{DM} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$ (c) 40.3^o or 0.703 radians	

2014 P1	**19.** The diagram shows a regular hexagon PQRSTW. $\overrightarrow{PW}$ and $\overrightarrow{PQ}$ represent vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively.  What is $\overrightarrow{SW}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$?	
Ans	$-\boldsymbol{u} - 2\boldsymbol{v}$	
2014 P1	**16.** The unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a}.\boldsymbol{b} = \frac{2}{3}$. Determine the value of $\boldsymbol{a}.(\boldsymbol{a} + 2\boldsymbol{b})$.	
Ans	7/3	
2014 P1	**14.** The vectors $\boldsymbol{u} = \begin{pmatrix} 1 \\ k \\ k \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix}$ are perpendicular. What is the value of k?	2
Ans	6/7	
2014 P1	**6.** Given that $\boldsymbol{u} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, find $2\boldsymbol{u} - 3\boldsymbol{v}$ in component form.	
Ans	$\begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$	

2013 P1	**24.** (*a*) (i) Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear. (ii) Find the ratio in which T divides AB. (*b*) The point C lies on the *x*-axis. If TB and TC are perpendicular, find the coordinates of C.	4 5
Ans	(a) (ii) 2:3 (b) (7, 0, 0)	
2013 P1	**14.** Given that $\|\mathbf{a}\| = 3$, $\|\mathbf{b}\| = 2$ and $\mathbf{a}.\mathbf{b} = 5$, what is the value of $\mathbf{a}.(\mathbf{a} + \mathbf{b})$?	
Ans	(b) 14	
2013 P1	**12.** If $\mathbf{f} = 3\mathbf{i} + 2\mathbf{k}$ and $\mathbf{g} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, find $\|\mathbf{f} + \mathbf{g}\|$.	
Ans	$\sqrt{66}$ units	
2012 P2	**5.** A is the point (3, –3, 0), B is (2, –3, 1) and C is (4, *k*, 0). (*a*) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form. (ii) Show that $\cos \hat{ABC} = \frac{3}{\sqrt{2(k^2+6k+14)}}$. (*b*) If angle ABC = 30°, find the possible values of *k*.	7 5
Ans	(a) (i) $BA = \begin{pmatrix}1\\0\\-1\end{pmatrix}, BC = \begin{pmatrix}2\\k+3\\-1\end{pmatrix}$ K = -4, -2	
2012 P1	**17.** Given that $\mathbf{a} = \begin{pmatrix}3\\4\\0\end{pmatrix}$ and $\mathbf{a}.(\mathbf{a}+\mathbf{b}) = 7$, what is the value of $\mathbf{a}.\mathbf{b}$?	2
Ans	(b) -18	
2012 P1	**15.** If $\mathbf{u} = k\begin{pmatrix}3\\-1\\0\end{pmatrix}$, where $k>0$ and $\mathbf{u}$ is a unit vector, determine the value of *k*.	2
Ans	$\frac{1}{\sqrt{10}}$	

2012 P1	**10.** The diagram shows a square-based pyramid P,QRST. $\overrightarrow{TS}$, $\overrightarrow{TQ}$ and $\overrightarrow{TP}$ represent **f**, **g** and **h** respectively.  Express $\overrightarrow{RP}$ in terms of **f**, **g** and **h**.	2
Ans	- **f** - **g** + **h**	
2012 P1	**7.** If $\mathbf{u} = \begin{pmatrix} -3 \\ 1 \\ 2t \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix}$ are perpendicular, what is the value of t?	2
Ans	-3	
2011 P1	**14.** An equilateral triangle of side 3 units is shown. The vectors **p** and **q** are as represented in the diagram. What is the value of **p . q**?  	2
Ans	9/2	
2011 P1	**1.** Given that $\mathbf{p} = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$, express $2\mathbf{p} - \mathbf{q} - \frac{1}{2}\mathbf{r}$ in component form.	2
Ans	$\begin{pmatrix} 5 \\ 9 \\ -13 \end{pmatrix}$	
2011 P1	**15.** Given that the points S(–4, 5, 1), T(–16, –4, 16) and U(–24, –10, 26) are collinear, calculate the ratio in which T divides SU.	2
Ans	3:2	

2010 P1	**3.** Given that $\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$, find $3\mathbf{u} - 2\mathbf{v}$ in component form.	
Ans	$\begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$	
2011 P2	**1.** D,OABC is a square based pyramid as shown in the diagram below.  O is the origin, D is the point (2, 2, 6) and OA = 4 units. M is the mid-point of OA. (*a*) State the coordinates of B. **1** (*b*) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3** (*c*) Find the size of angle BDM. **5**	
Ans	(a) B (4, 4, 0) (b) $DB = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}, BC = \begin{pmatrix} 0 \\ -2 \\ 6 \end{pmatrix}$ (c) 40.3	
2010 P1	**10.** The vectors $x\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$ and $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ are perpendicular. What is the value of x?	**2**
Ans	1	

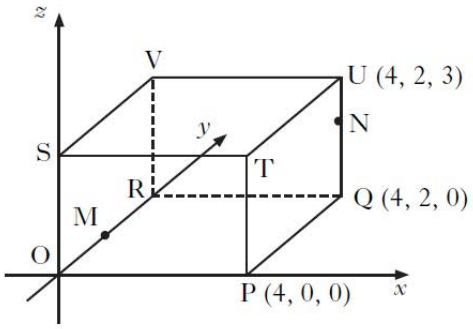
2010 P2

1. The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point (4, 0, 0),
Q is (4, 2, 0) and U is (4, 2, 3).

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



(*a*) State the coordinates of M and N. — 2

(*b*) Express $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. — 2

(*c*) Calculate the size of angle MVN. — 5

Ans

M(0, 1, 0) N(4, 2, 2) (b) $VM = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}, VN = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$ (c) θ = 76.7

2009 P1

17. The vector $\boldsymbol{u}$ has components $\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$. — 2

~~Which of~~ the following is a unit vector parallel to $\boldsymbol{u}$?

(Find)

Ans

$-\frac{3}{5}\boldsymbol{i} + \frac{4}{5}\boldsymbol{k}$

2009 P1

22. D, E and F have coordinates (10, –8, –15), (1, –2, –3) and (–2, 0, 1) respectively.

(*a*) (i) Show that D, E and F are collinear. — 4

(ii) Find the ratio in which E divides DF.

(*b*) G has coordinates (*k*, 1, 0). — 4

Given that DE is perpendicular to GE, find the value of *k*.

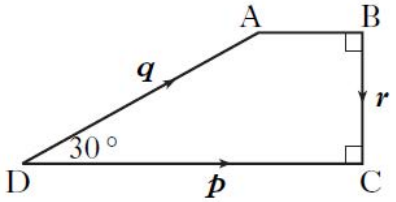
Ans

(ii) 3:1 (b) k = 7

2009 P2

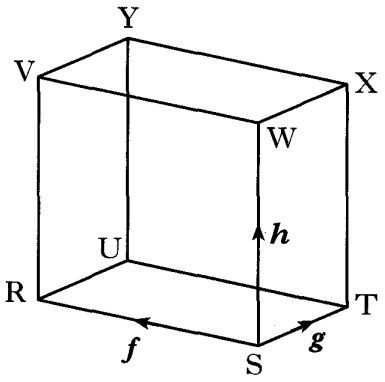
7. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented on the diagram shown where angle ADC = 30 °.

It is also given that $|\boldsymbol{p}| = 4$ and $|\boldsymbol{q}| = 3$.



(*a*) Evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$ and $\boldsymbol{r}.(\boldsymbol{p} - \boldsymbol{q})$. — 6

(*b*) Find $|\boldsymbol{q} + \boldsymbol{r}|$ and $|\boldsymbol{p} - \boldsymbol{q}|$. — 4

Ans	(a) $6\sqrt{3}$ and $\frac{9}{4}$	
2008 P1	**3.** The vectors $\boldsymbol{u} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$ are perpendicular. What is the value of k?	2
Ans	4	
2008 P1	**11.** E(–2, –1, 4), P(1, 5, 7) and F(7, 17, 13) are three collinear points. P lies between E and F. What is the ratio in which P divides EF?	2
Ans	1:2	
2008 P1	**12.** In the diagram RSTU, VWXY represents a cuboid. $\overrightarrow{SR}$ represents vector $\boldsymbol{f}$, $\overrightarrow{ST}$ represents vector $\boldsymbol{g}$ and $\overrightarrow{SW}$ represents vector $\boldsymbol{h}$. Express $\overrightarrow{VT}$ in terms of $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$.  	2
Ans	$\overrightarrow{VT} = -\boldsymbol{f} + \boldsymbol{g} - \boldsymbol{h}$	
2008 P1	**18.** Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $\lvert\boldsymbol{p}\rvert = 3$, $\lvert\boldsymbol{q}\rvert = 4$ and $\boldsymbol{p}.\boldsymbol{q} = 10$. Find the value of $\boldsymbol{q}.(\boldsymbol{p} + \boldsymbol{q})$.	2
Ans	26	

2008 P2

2. The diagram shows a cuboid OABC, DEFG.

F is the point (8, 4, 6).

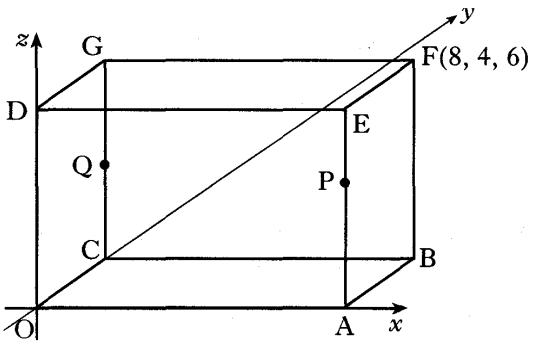
P divides AE in the ratio 2:1.

Q is the midpoint of CG.

(*a*) State the coordinates of P and Q. 2

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$. 2

(*c*) Find the size of angle QPA. 5



Ans

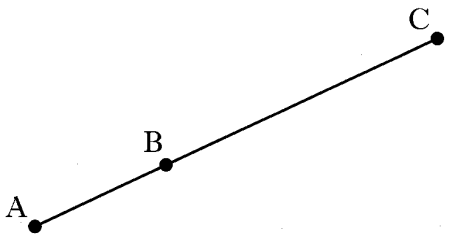
(a) P(8,0,4), Q(0,4,3) (b) $\overrightarrow{PQ} = \begin{pmatrix} -8 \\ 4 \\ -1 \end{pmatrix}$ $\overrightarrow{PA} = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$ (c) 83.6°

2007 P1

2. Relative to a suitable coordinate system A and B are the points (–2, 1, –1) and (1, 3, 2) respectively.

A, B and C are collinear points and C is positioned such that BC = 2AB.

Find the coordinates of C. 4



Ans

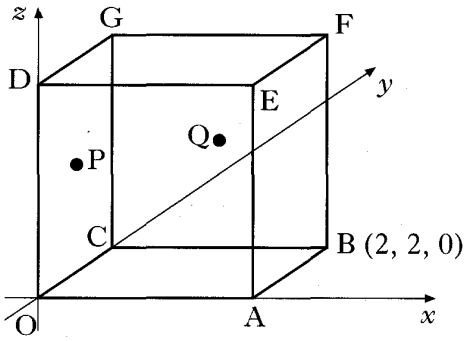
(7, 7, 8)

2007 P2

1. OABCDEFG is a cube with side 2 units, as shown in the diagram.

B has coordinates (2, 2, 0).

P is the centre of face OCGD and Q is the centre of face CBFG.



(*a*) Write down the coordinates of G. 1

(*b*) Find $\boldsymbol{p}$ and $\boldsymbol{q}$, the position vectors of points P and Q. 2

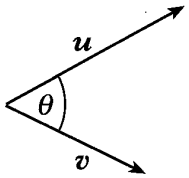
(*c*) Find the size of angle POQ. 5

Ans

(*a*) (0,2,2) (*b*) P = $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ Q = $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ (*c*) 30° / $\frac{\pi}{6}$ radians

2006 P1

9. $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.



(*a*) If $\boldsymbol{u}.\boldsymbol{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. — 2

(*b*) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. — 5

(*c*) Deduce the only possible value of k. — 1

(*d*) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$. — 3

Ans

(*a*) $k^3 + 3k^2 - k - 2 = 1$ *and complete*

(*b*) $(k + 3)(k + 1)(k - 1)$ stated explicitly

(*c*) $k = 1$

(*d*) $\cos\theta = \frac{1}{11}$

2006 P2

6. P is the point (–1, 2, –1) and Q is (3, 2, –4).

(*a*) Write down $\overrightarrow{PQ}$ in component form. — 1

(*b*) Calculate the length of $\overrightarrow{PQ}$. — 1

(*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$. — 1

Ans

(*a*) $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$

(*b*) $|\overrightarrow{PQ}| = 5$

(*c*) $\begin{pmatrix} \frac{4}{5} \\ 0 \\ -\frac{3}{5} \end{pmatrix}$ or $\begin{pmatrix} -\frac{4}{5} \\ 0 \\ \frac{3}{5} \end{pmatrix}$

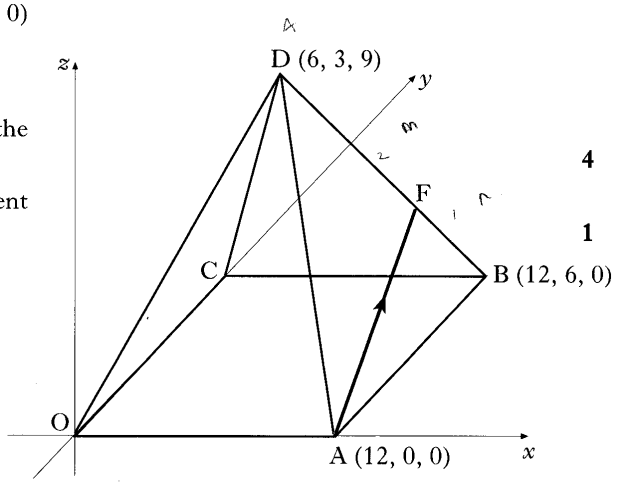
2005 P1

3. D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).

F divides DB in the ratio 2:1.

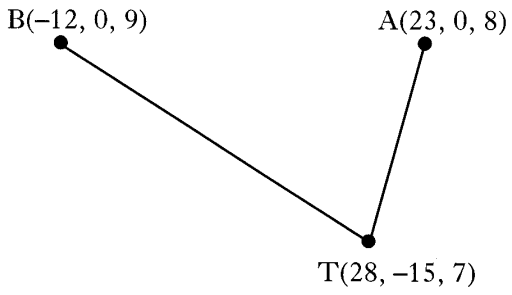
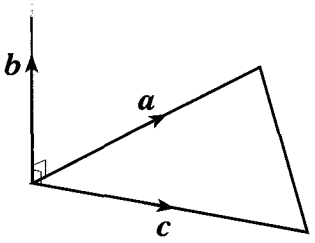
(*a*) Find the coordinates of the point F. — 4

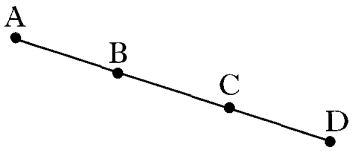
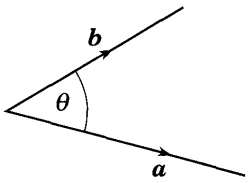
(*b*) Express $\overrightarrow{AF}$ in component form. — 1



4

1

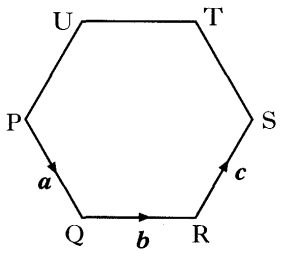
Ans	(*a*) F = (10,5,3) (*b*) $AF = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$	
2005 P2	**4.** The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops. Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(–12, 0, 9) and T(28, –15, 7). In the dark, Andrew and Bob locate Tracy using heat-seeking beams. (*a*) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form. (*b*) Calculate the angle between these two beams.  	2 5
Ans	(*a*) $\overrightarrow{TA} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$ $\overrightarrow{TB} = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$ (*b*) 50·9° **or** 0·889^c **or** 56·6 grads	
2005 P2	**10.** Vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram. Vector $\boldsymbol{b}$ is 2 units long and $\boldsymbol{b}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{c}$. Evaluate the scalar product $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$.  	4
Ans	$13\frac{1}{2}$	
2004 P1	**5.** A, B and C have coordinates (–3, 4, 7), (–1, 8, 3) and (0, 10, 1) respectively. (*a*) Show that A, B and C are collinear. (*b*) Find the coordinates of D such that $\overrightarrow{AD} = 4\overrightarrow{AB}$.	3 2
Ans	**5.** (*a*) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix} = \frac{3}{2} \times \overrightarrow{AB}$ $\overrightarrow{AB}$ and $\overrightarrow{AC}$ have common direction and common point, hence A, B and C are collinear. (*b*) D = (5, 20, –9)	

2004 P2	**2.** P, Q and R have coordinates (1, 3, –1), (2, 0, 1) and (–3, 1, 2) respectively. (*a*) Express the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in component form. (*b*) Hence or otherwise find the size of angle PQR.	2 5
Ans	(*a*) $\overrightarrow{QP} = \begin{pmatrix}-1\\3\\-2\end{pmatrix}$ $\overrightarrow{QR} = \begin{pmatrix}-5\\1\\1\end{pmatrix}$ (*b*) $P\hat{Q}R = 72{\cdot}0°$	
2003 P1	**3.** Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$. Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other.	2
Ans	Vectors are perpendicular.	
2003 P1	**6.** A and B are the points (–1, –3, 2) and (2, –1, 1) respectively. B and C are the points of trisection of AD, that is AB = BC = CD. Find the coordinates of D. 	3
Ans	(8, 3, –1)	
2003 P2	**9.** The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. If $\lvert\boldsymbol{a}\rvert = 5$, $\lvert\boldsymbol{b}\rvert = 4$ and $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. 	4
Ans	56·6°	
2002W P1	**2.** (*a*) If $\boldsymbol{u} = \begin{pmatrix}1\\7\\-2\end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix}1\\-2\\1\end{pmatrix}$, write down the components of $\boldsymbol{u} + 3\boldsymbol{v}$ and $\boldsymbol{u} - 3\boldsymbol{v}$. (*b*) Hence, or otherwise, show that $\boldsymbol{u} + 3\boldsymbol{v}$ and $\boldsymbol{u} - 3\boldsymbol{v}$ are perpendicular.	2 2
Ans	(*a*) • $u+3v=\begin{pmatrix}4\\1\\1\end{pmatrix}$ • $u-3v=\begin{pmatrix}-2\\13\\-5\end{pmatrix}$ (*b*) proof • $\begin{pmatrix}4\\1\\1\end{pmatrix}\bullet\begin{pmatrix}-2\\13\\5\end{pmatrix}$ $=-8+13-5=0$ **Alternative** • $(u+3v).(u-3v)$ $=\lvert u\rvert^2-9\lvert v\rvert^2$ $=54-9\times6=0$	

2002W P1

11. PQRSTU is a regular hexagon of side 2 units. $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{RS}$ represent vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively.

Find the value of $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$.



3

Ans

0

2002W P2

2. With reference to a suitable set of coordinate axes, A, B and C are the points (–8, 10, 20), (–2, 1, 8) and (0, –2, 4) respectively.

Show that A, B and C are collinear and find the ratio AB : BC.

4

Ans

$\vec{AB}=\begin{pmatrix}6\\-9\\-12\end{pmatrix}$ **or** $\vec{BC}=\begin{pmatrix}2\\-3\\-4\end{pmatrix}$ **or** $\vec{AC}=\begin{pmatrix}8\\-12\\-16\end{pmatrix}$

e.g. $\vec{AB}=3\vec{BC}\left(\text{or}\begin{pmatrix}6\\-9\\-12\end{pmatrix}=3\begin{pmatrix}2\\-3\\-4\end{pmatrix}\right)$

e.g. $\vec{AB}$, $3\vec{BC}$ have common direction,
B common pt. so A, B, C collinear
AB : BC = 3 : 1

2002 P1

2. The point Q divides the line joining P(–1, –1, 0) to R(5, 2, –3) in the ratio 2 : 1.

Find the coordinates of Q.

3

Ans

(3, 1, –2)

2002 P2

2. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

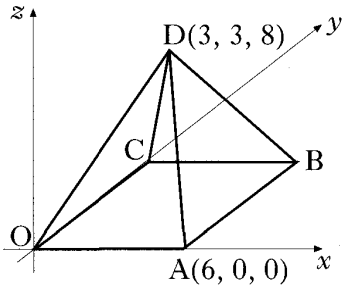
The coordinates of A and D are (6, 0, 0) and (3, 3, 8).

C lies on the y-axis.

(*a*) Write down the coordinates of B. 1

(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. 2

(*c*) Calculate the size of angle ADB. 4



Ans

(*a*) (6, 6, 0)

(*b*) $\overrightarrow{DA}=\begin{pmatrix}3\\-3\\-8\end{pmatrix}$ $\overrightarrow{DB}=\begin{pmatrix}3\\3\\-8\end{pmatrix}$

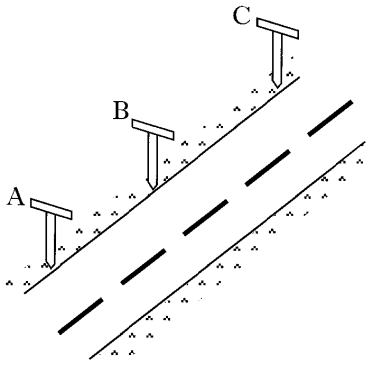
(*c*) $\cos A\hat{D}B=\dfrac{\overrightarrow{DA}.\overrightarrow{DB}}{|\overrightarrow{DA}||\overrightarrow{DB}|}$

38·7°

2001 P1

3. (*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points A(–8, –10, –2), B(–2, –1, 1) and C(6, 11, 5).

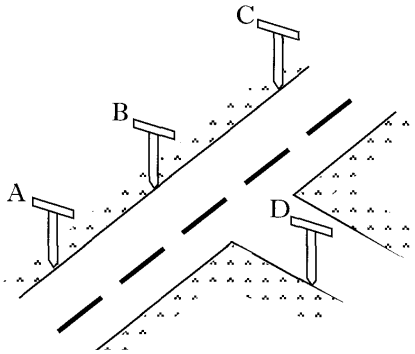
Determine whether or not the section of road ABC has been built in a straight line.



3

(*b*) A further T-rod is placed such that D has coordinates (1, –4, 4).

Show that DB is perpendicular to AB.



3

Ans

(*a*) the road ABC is straight

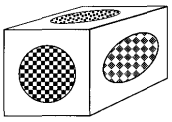
(*b*) $\overrightarrow{BD} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

$\overrightarrow{AB}.\overrightarrow{BD} = 0$

$\overrightarrow{AB}.\overrightarrow{BD} = 18 - 27 + 9 = 0$

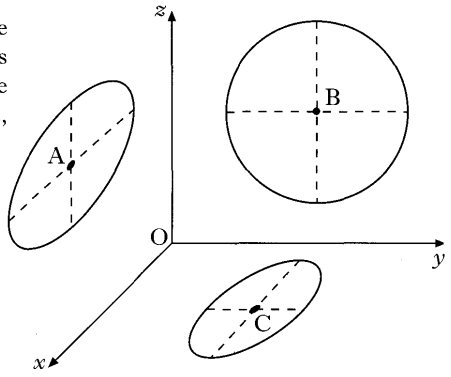
2001 P2

4. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.



The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A(6, 0, 7), B(0, 5, 6) and C(4, 5, 0).

Find the size of angle ABC.



7

Ans

71·5°

2000 P1

7. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

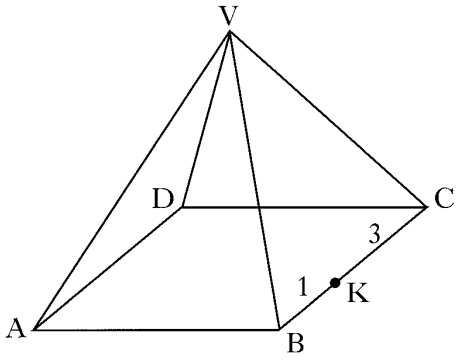
$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.

K divides BC in the ratio 1:3.

Find $\overrightarrow{VK}$ in component form.



3

Ans

$\begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$

2000 P2

7. For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular?

2

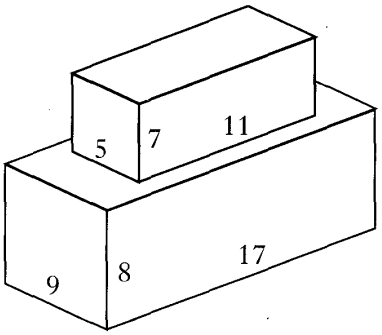
Ans

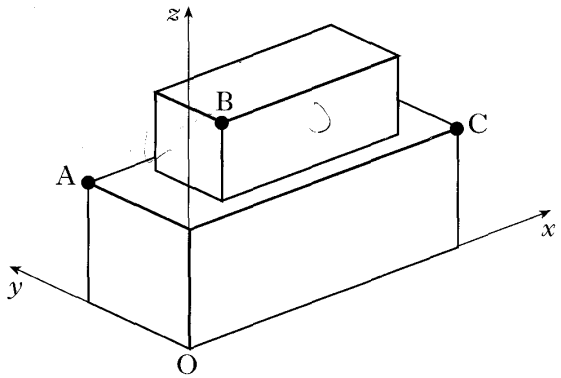
t = 4

2000 P2

9. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

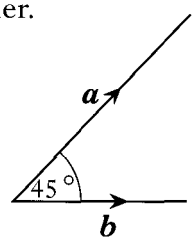
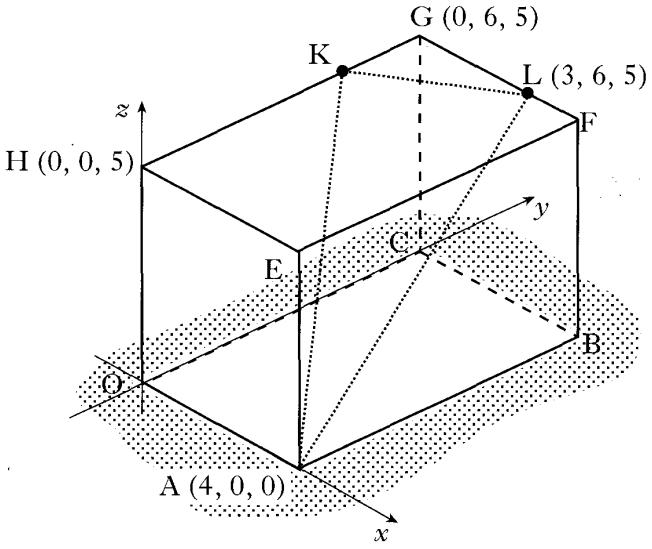
Coordinate axes are taken as shown.





(*a*) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8). Write down the coordinates of B. 1

(*b*) Calculate the size of angle ABC. 6

	(*a*) B = (3, 2, 15) (*b*) 92·5°	
Specimen 2 P1	**10.** The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, with $\lvert\boldsymbol{a}\rvert = 3$ and $\lvert\boldsymbol{b}\rvert = 2\sqrt{2}$. These vectors are inclined at an angle of 45 ° to each other. (*a*) Evaluate (i) $\boldsymbol{a}.\boldsymbol{a}$ (ii) $\boldsymbol{b}.\boldsymbol{b}$ (iii) $\boldsymbol{a}.\boldsymbol{b}$  (*b*) Another vector $\boldsymbol{p}$ is defined by $\boldsymbol{p} = 2\boldsymbol{a} + 3\boldsymbol{b}$. Evaluate $\boldsymbol{p}.\boldsymbol{p}$ and hence write down $\lvert\boldsymbol{p}\rvert$.	2 4
Ans	(*a*) $\boldsymbol{a}.\boldsymbol{a} = 9$, $\boldsymbol{b}.\boldsymbol{b} = 8$, $\boldsymbol{a}.\boldsymbol{b} = 6$ (*b*) $\boldsymbol{p}.\boldsymbol{p} = (2\boldsymbol{a} + 3\boldsymbol{b}).(2\boldsymbol{a} + 3\boldsymbol{b})$ $\lvert\boldsymbol{p}\rvert^2 = 4\boldsymbol{a}.\boldsymbol{a} + 9\boldsymbol{b}.\boldsymbol{b} + 12\boldsymbol{a}.\boldsymbol{b}$ $\lvert\boldsymbol{p}\rvert = \sqrt{180}$	
Specimen 2 P2	**2.** OABCEFGH is a cuboid. With axes as shown, O is the origin and the coordinates of A, H, G and L are (4, 0, 0), (0, 0, 5), (0, 6, 5) and (3, 6, 5) respectively. K lies two thirds of the way along HG, (ie HK:KG = 2:1).  (*a*) Determine the coordinates of K. (*b*) Write down the components of $\overrightarrow{AK}$ and $\overrightarrow{AL}$. (*c*) Calculate the size of angle KAL.	2 2 5

Ans

(*a*)

$\overrightarrow{HG} = \begin{pmatrix}0\\6\\0\end{pmatrix} \Rightarrow \overrightarrow{HK} = \begin{pmatrix}0\\4\\0\end{pmatrix} \Rightarrow K = (0,4,5)$

(*b*)

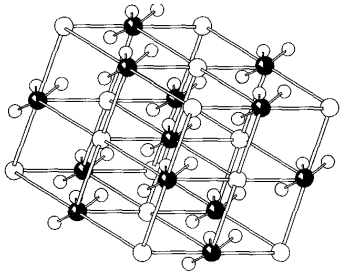
$\overrightarrow{AK} = \begin{pmatrix}-4\\4\\5\end{pmatrix}, \quad \overrightarrow{AL} = \begin{pmatrix}-1\\6\\5\end{pmatrix}$

(*c*) $\cos K\hat{A}L = \frac{4+24+25}{\sqrt{57}\sqrt{62}}$

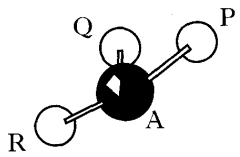
$K\hat{A}L = 26{\cdot}9°$

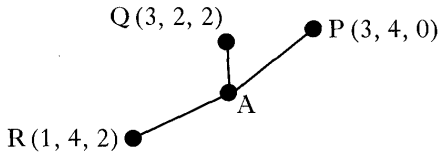
Specimen 1 P1

5. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown.





(*a*) Show that the cosine of angle PQR is $\frac{1}{2}$. — 5

(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2 : 1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. — 6

Ans

(*a*) $\cos PQR = \frac{\overrightarrow{PQ}.\overrightarrow{RQ}}{|\overrightarrow{PQ}||\overrightarrow{RQ}|}$

$= \frac{4}{\sqrt{8}\sqrt{8}}$

$= \frac{4}{8}$

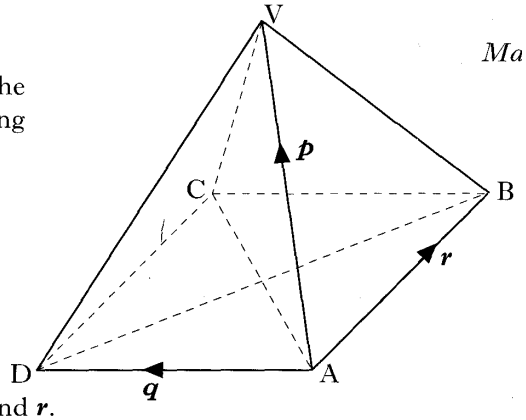
(*b*) (i) M(2,3,2) $T\left(\frac{7}{3}, \frac{10}{3}, \frac{4}{3}\right)$

(ii) substitution into $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

gives $PT = QT = RT = 2\sqrt{\frac{2}{3}}$

Specimen 1 P2

5. VABCD is a square-based pyramid. The length of AD is 3 units and each sloping face is an equilateral triangle.



$\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$ and $\overrightarrow{AB} = \boldsymbol{r}$.

(*a*) (i) Evaluate $\boldsymbol{p}.\boldsymbol{q}$.

(ii) Hence evaluate $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})$. — 3

(*b*) (i) Express $\overrightarrow{CV}$ in terms of $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$.

(ii) Hence show that angle CVA is 90°. — 4

Ans

(*a*) (i) 4·5

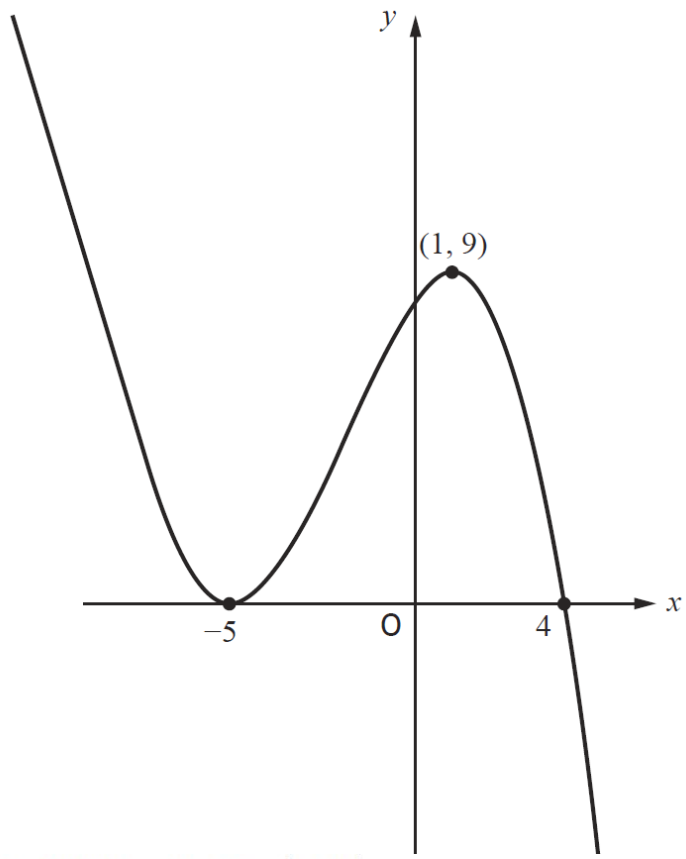
(ii) 9

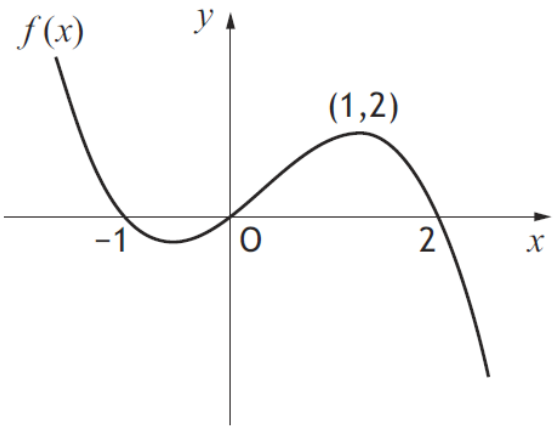
(*b*) (i) $\boldsymbol{p}-\boldsymbol{q}-\boldsymbol{r}$

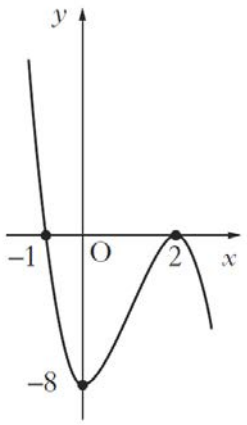
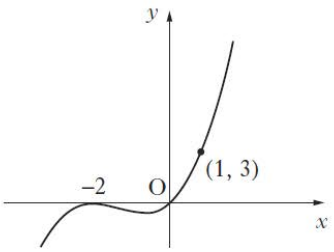
(ii) $\overrightarrow{CV}.\overrightarrow{AV} = \boldsymbol{p}.(\boldsymbol{p}-\boldsymbol{q}-\boldsymbol{r})$

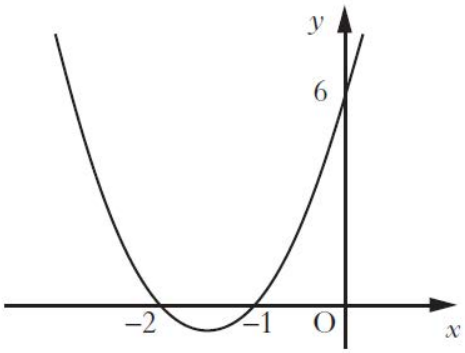
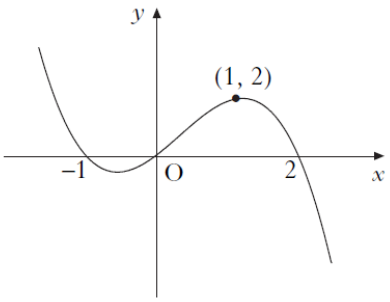
$= \boldsymbol{p}.\boldsymbol{p} - \boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})$

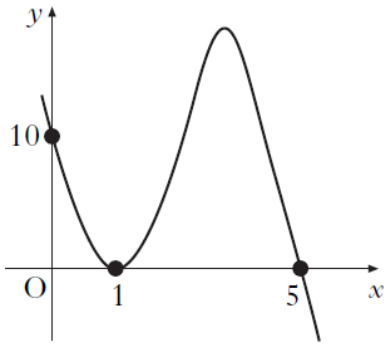
$= 0$

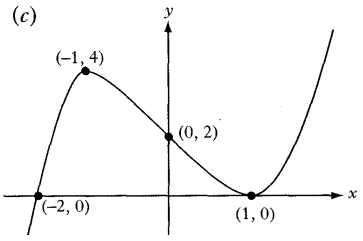
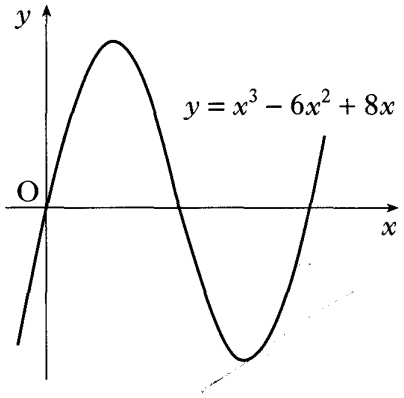
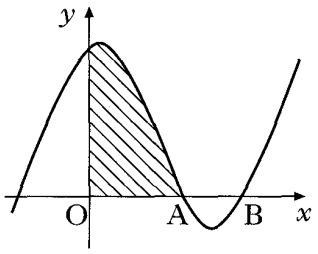
2017 P2	**2.** (a) Show that $(x-1)$ is a factor of $f(x)=2x^3-5x^2+x+2$. (b) Hence, or otherwise, solve $f(x)=0$.	2 3
Ans	(a) Proof (b) x = -1/2, x = 1, x = 2	
2016 P1	**15.** The diagram below shows the graph with equation $y=f(x)$, where $f(x)=k(x-a)(x-b)^2$.  (a) Find the values of a, b and k. (b) For the function $g(x)=f(x)-d$, where d is positive, determine the range of values of d for which $g(x)$ has exactly one real root.	3 1
Ans	(a) a = 4, b = -5, k = -1/12 (b) d > 9	
2016 P2	**3.** (a) (i) Show that $(x+1)$ is a factor of $2x^3-9x^2+3x+14$. (ii) Hence solve the equation $2x^3-9x^2+3x+14=0$.	2 3
Ans	(ii) x = -1, 2, 3.5	
NH 2015 P1	**3.** Show that $(x+3)$ is a factor of $x^3-3x^2-10x+24$ and hence factorise $x^3-3x^2-10x+24$ fully.	3

Ans	$(x+3)(x-4)(x-2)$	
NH SP P1	**7.** (a) Show that $(x+1)$ is a factor of $x^3-13x-12$.	3
	(b) Factorise $x^3-13x-12$ fully.	2
Ans	(b) $(x+1)(x+3)(x-4)$	
NH SP P1	**2.** Find the coordinates of the points of intersection of the curve $y=x^3-2x^2+x+4$ and the line $y=4x+4$.	5
Ans	$(-1,0),(0,4),(3,16)$	
NH Ex P1	**5.** For the polynomial, x^3-4x^2+ax+b • $x-1$ is a factor • -12 is the remainder when it is divided by $x-2$ (a) Determine the values of a and b. (b) Hence solve $x^3-4x^2+ax+b=0$.	
Ans	(a) a = -7, b = 10 (b) x = 1, x = 5, x = -2	
NH EX P1	**2.** The diagram shows the curve with equation $y=f(x)$, where $f(x)=kx(x+a)(x+b)$. The curve passes through (-1,0), (0,0), (1,2) and (2,0).  Find the values of a, b and k.	3
Ans	a = 1, b = -2, k = -1	

2014 P1	**22.** For the polynomial $6x^3 + 7x^2 + ax + b$, • $x + 1$ is a factor • 72 is the remainder when it is divided by $x - 2$. (*a*) Determine the values of a and b. (*b*) Hence factorise the polynomial completely.	4 3
Ans	(a) $a = -1$, $b = -2$ (b) $(x + 1)(3x + 2)(2x - 1)$	
2014 P1	**15.** The diagram shows a cubic curve passing through (–1, 0), (2, 0) and (0, –8).  What is the equation of the curve?	
Ans	$y = -2(x + 1)(x - 2)^2$	
2013 P 2	**3.** (*a*) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully.	4
Ans	$(x - 1)(x^2 + 4x + 5)$ with valid reason	
2013 P1	**17.** The diagram shows a curve with equation of the form $y = kx(x + a)^2$, which passes through the points (–2, 0), (0, 0) and (1, 3).  What are the values of a and k?	
Ans	$a = 2$, $k = 1/3$	

2013 P1	**6.** What is the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $(x - 2)$?	
Ans	4	
2012 P1	**13.** A parabola intersects the axes at $x = -2$, $x = -1$ and $y = 6$, as shown in the diagram.  What is the equation of the parabola? A $y = 6(x - 1)(x - 2)$ B $y = 6(x + 1)(x + 2)$ C $y = 3(x - 1)(x - 2)$ D $y = 3(x + 1)(x + 2)$	2
Ans	y = 3(x + 1)(x + 2)	
2011 P1	**7.** A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$. What is the remainder when $f(x)$ is divided by $(x - 1)$?	
Ans	4	
2011 P1	**17.** The diagram shows the graph of a cubic.  What is the equation of this cubic?	
Ans	y = -x(x + 1)(x - 2)	

2011P2	**2.** Functions f, g and h are defined on the set of real numbers by • $f(x) = x^3 - 1$ • $g(x) = 3x + 1$ • $h(x) = 4x - 5$. (a) Find $g(f(x))$. (b) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. (c) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$. (ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. (d) Hence solve $g(f(x)) + xh(x) = 0$.	2 1 5 1
Ans	(a) $g(f(x)) = 3x^3 - 2$ (b) proof (c) (ii) $(x - 1)(3x + 1)(x + 2)$ (d) $x = -2, -\frac{1}{3}, 1$	
2010 P1	**16.** The diagram shows the graph with equation $y = k(x - 1)^2(x + t)$.  What are the values of k and t?	
Ans	k = -2, t = -5	
2010 P1	**22.** (a) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$. (ii) Hence factorise $f(x)$ fully. (b) Solve $2x^3 + x^2 - 8x + 5 = 0$. (c) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G. Find the coordinates of G. (d) This tangent meets the curve again at the point H. Write down the coordinates of H.	5 1 5 1

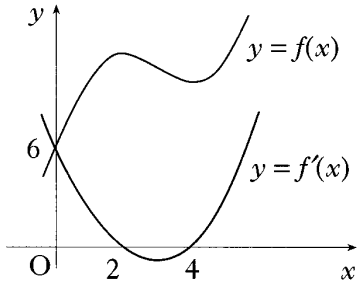
Ans	(ii) $(2x + 5)(x - 1)^2$ (b) $x = 1, -\frac{5}{2}$ (d) $H(-\frac{5}{2}, -8)$	
2008 P1	**21.** A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$. (*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. (*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$. (ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. (*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve.	6 5 4
Ans	(*a*) (–1, 4) maximum (1, 0) minimum (*b*) (i) $x = 1$, $f(x) = 0$ so $(x - 1)$ is a factor (ii) $(x - 1)(x - 1)(x + 2)$ (*c*) 	
2008 P1	**22.** The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$. (*a*) Find the coordinates of the points on the curve where the gradient of the tangent is –1. (*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. 	5 2
Ans	(a) (1,3), (3,-3) (b) (1,3)	
2007 P1	**8.** The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$. (*a*) Show that the graph cuts the x-axis at (3, 0). (*b*) Hence or otherwise find the coordinates of A. 	1 3
Ans	(*a*) To cut the x-axis, $y = 0$. So $0 = x^3 - 4x^2 + x + 6$ $= (x - 3)(x^2 - x - 2)$ $= (x - 3)(x - 2)(x + 1)$ So graph cuts x-axis at $x = -1, 3, 2$. (*b*) (2,0)	

2007 P2

10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).



(*a*) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:

(i) write down the values of a and b;

(ii) find the value of k. **3**

Ans

(*a*) (i) $a = 2, b = 4$

(ii) $k = \frac{3}{4}$

2005 P1

8. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **5**

(*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **2**

(*c*) Find the greatest and least values of f in the interval $-2 \le x \le 2$. **5**

Ans

(*a*) $(x - 3)(2x - 3)(x + 1)$

(*b*) $(-1,0), (\frac{3}{2}, 0), (3, 0)$

(*c*) greatest value = 9

least value = −35

2005 P2

11. (*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. **1**

(*b*) Hence find the range of values of p for which all the roots of the cubic equation are real. **7**

Ans

(*a*) $f(-1) = -1 + p - p + 1 = 0$

(*b*) $p \le -1, p \ge 3$

2004 P1

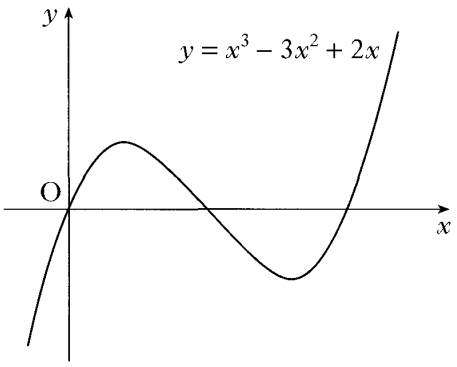
2. $f(x) = x^3 - x^2 - 5x - 3$.

(*a*) (i) Show that $(x + 1)$ is a factor of $f(x)$.

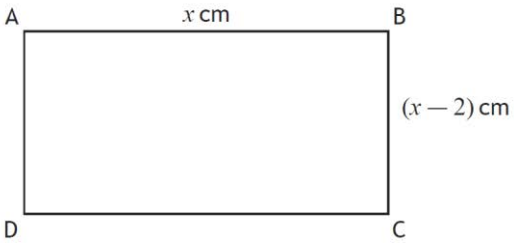
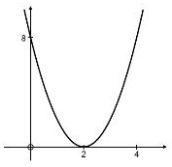
(ii) Hence or otherwise factorise $f(x)$ fully. **5**

(*b*) One of the turning points of the graph of $y = f(x)$ lies on the x-axis.

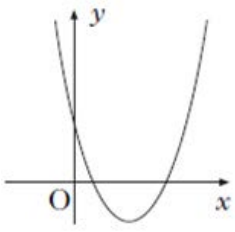
Write down the coordinates of this turning point. **1**

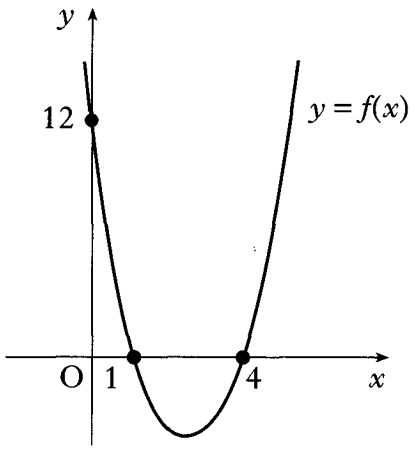
Ans	$(x + 1)(x + 1)(x - 3)$ $(-1, 0)$	
2003 P2	**1.** $f(x) = 6x^3 - 5x^2 - 17x + 6$. (*a*) Show that $(x - 2)$ is a factor of $f(x)$. (*b*) Express $f(x)$ in its fully factorised form.	4
Ans	(*b*) $(x - 2)(2x + 3)(3x - 1)$	
2002W P1	**5.** Given that $(x - 2)$ and $(x + 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d.	5
Ans	$c = -19$, $d = 6$	
2001 P2	**1.** (*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. (*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value.	3 2
Ans	(*a*) $k = -5$ (*b*) $x = -2, \frac{1}{2}, 1$	
2000 P2	**1.** The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$. (*a*) Find the equation of the tangent to this curve at the point where $x = 1$. (*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. 	5 5
Ans	(*a*) $x + y = 1$ (*b*) $(-1, -6)$	
Specimen 2 P1	**1.** Show that $x = 2$ is a root of the equation $y = 2x^3 + x^2 - 13x + 6 = 0$ and hence, or otherwise, find the other roots.	4

Ans	$2 \mid 2 \quad 1 \quad -13 \quad 6$ $\quad\quad\quad 4 \quad 10 \quad -6$ $\quad\quad 2 \quad 5 \quad -3 \quad 0$ remainder $= 0 \Rightarrow x = 2$ is a root $2x^2 + 5x - 3 = 0 \Rightarrow x = \frac{1}{2}, -3$	
Specimen 1 P1	**3.** (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. (*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes.	3 1
Ans	(*a*) $f(1) = 0$, $(x - 4)$, $(x - 1)$ (*b*) (1,0), (4,0), (0, −4)	

2017 P1	**4.** Find the value of k for which the equation $x^2+4x+(k-5)=0$ has equal roots.	3
Ans	k = 9	
2016 P2	**2.** Find the range of values for p such that $x^2-2x+3-p=0$ has no real roots.	3
Ans	p < 2	
NH 2015 P1	**8.** ABCD is a rectangle with sides of lengths x centimetres and $(x-2)$ centimetres, as shown.  If the area of ABCD is less than $15\,\text{cm}^2$, determine the range of possible values of x.	4
Ans	2 < x < 5	
NH SP P1	**8.** $f(x)$ and $g(x)$ are functions, defined on the set of real numbers, such that $f(x)=1-\frac{1}{2}x$ and $g(x)=8x^2-3$. (a) Given that $h(x)=g(f(x))$, show that $h(x)=2x^2-8x+5$. (b) Express $h(x)$ in the form $a(x+p)^2+q$. (c) Hence, or otherwise, state the coordinates of the turning point on the graph of $y=h(x)$. (d) Sketch the graph of $y=h(x)+3$, showing clearly the coordinates of the turning point and the y-axis intercept.	3 3 1 2
Ans	(a) Proof (b) $2(x-2)^2-3$ (c) (2, -3) (d) (d) 	
NH EX P2	**3.** Find the value of p such that the equation $x^2+(p+1)x+9=0$ has no real roots.	4
Ans	$-7<p<5$	
2014 P1	**17.** $3x^2+12x+17$ is expressed in the form $3(x+p)^2+q$. What is the value of q?	

Ans	5	
2013 P1	**21.** Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$.	3
Ans	$2(x + 3)^2 - 17$	
2013 P1	**19.** Solve $1 - 2x - 3x^2 > 0$, where x is a real number.	
Ans	$-1 < x < \frac{1}{3}$	
2013 P1	**3.** Calculate the discriminant of the quadratic equation $2x^2 + 4x + 5 = 0$.	
Ans	-24	
2012 P1	**3.** If $x^2 - 6x + 14$ is written in the form $(x - p)^2 + q$, what is the value of q?	2
Ans	5	
2012 P1	**19.** Solve $6 - x - x^2 < 0$.	
Ans	X < -3, x > 2	
2011 P1	**5.** If $x^2 - 8x + 7$ is written in the form $(x - p)^2 + q$, what is the value of q?	
Ans	-9	
2011 P1	**9.** The discriminant of a quadratic equation is 23. Here are two statements about this quadratic equation: (1) the roots are real; (2) the roots are rational. Which of the following is true? A Neither statement is correct. B Only statement (1) is correct. C Only statement (2) is correct. D Both statements are correct.	
Ans	B	

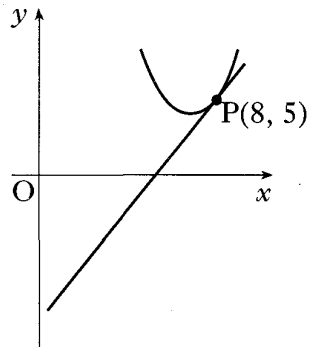
2010 P1	**6.** The roots of the equation $kx^2 - 3x + 2 = 0$ are equal. What is the value of k?	
Ans	9/8	
2010 P1	**18.** What is the solution of $x^2 + 4x > 0$, where x is a real number?	
Ans	$x < 4, x > 0$	
2009 P1	**12.** A function f is given by $f(x) = 2x^2 - x - 9$. Which of the following describes the nature of the roots of $f(x) = 0$? A No real roots B Equal roots C Real distinct roots D Rational distinct roots	
Ans	C	
2009 P1	**19.** For what values of x is $6 + x - x^2 < 0$? A $x > 3$ only B $x < -2$ only C $x < -2, x > 3$ D $-3 < x < 2$	
Ans	$x < -2, x > 3$	
2010 P1	**13.** ~~Which of the~~ following diagrams shows a parabola with equation $y = ax^2 + bx + c$, where • $a > 0$ • $b^2 - 4ac > 0$? (Sketch a graph showing the above info)	
Ans		

2008 P1	**10.** Here are two statements about the roots of the equation $x^2 + x + 1 = 0$: (1) the roots are equal; (2) the roots are real. Which of the following is true? A Neither statement is correct. B Only statement (1) is correct. C Only statement (2) is correct. D Both statements are correct.	2
Ans	A	
2008 P1	**16.** $2x^2 + 4x + 7$ is expressed in the form $2(x + p)^2 + q$. What is the value of q?	2
Ans	5	
2008 P1	**13.** The diagram shows part of the graph of a quadratic function $y = f(x)$. The graph has an equation of the form $y = k(x - a)(x - b)$.  What is the equation of the graph?	2
Ans	y = 3(x – 1)(x – 4)	
2007 P1	**4.** Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots.	4
Ans	$k < -\frac{1}{4}$	

2006 P2

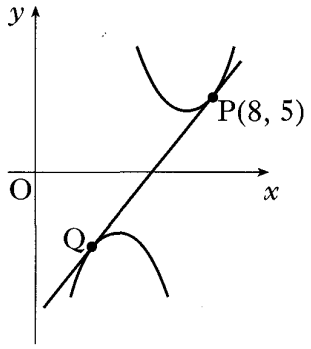
3. The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5).

(a) Find the equation of this tangent.



4

(b) Show that the tangent found in (a) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q.



5

Ans

(a) $y - 5 = 2(x - 8)$

(b) Q = (4, −3)

2004 P1

8. (a) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$.

(b) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing.

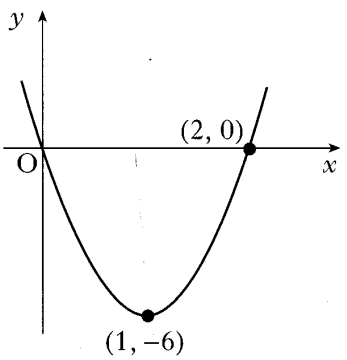
Ans

(a) $(x - 5)^2 + 2$

(b) $g'(x) = x^2 - 10x + 27$
$= (x - 5)^2 + 2$
$g'(x) > 0$ for all x and so $g(x)$ is always increasing.

2004 P1

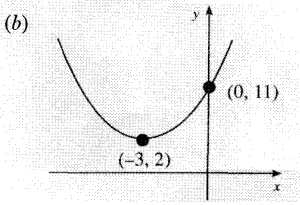
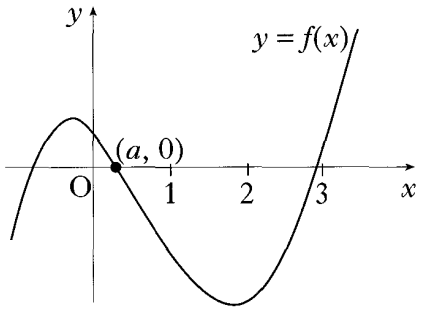
11. The diagram shows a parabola passing through the points (0, 0), (1, −6) and (2, 0).

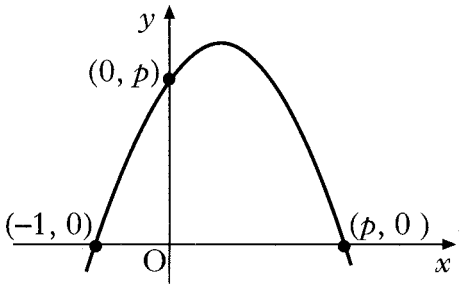


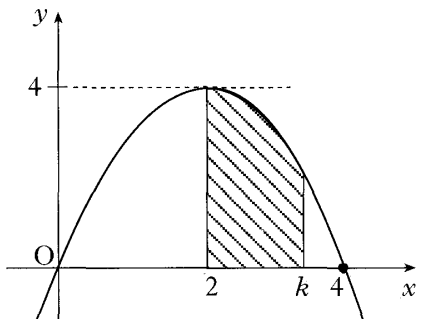
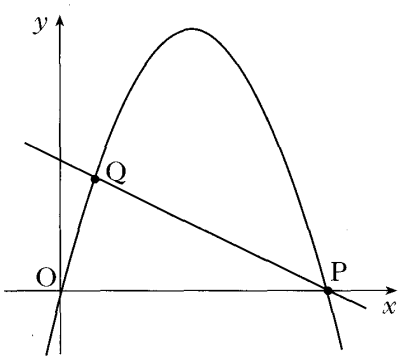
(a) The equation of the parabola is of the form $y = ax(x - b)$.

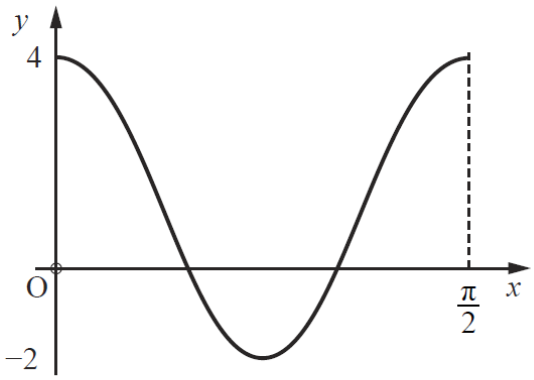
Find the values of a and b.

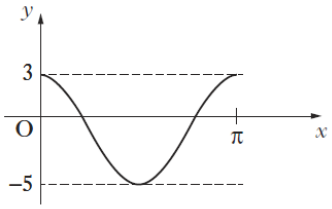
3

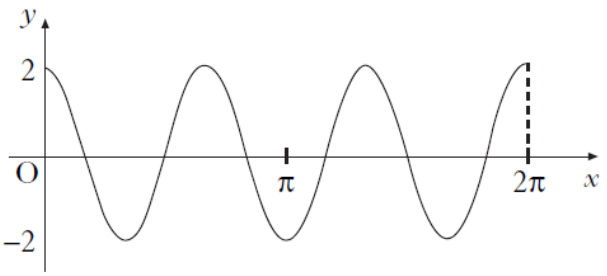
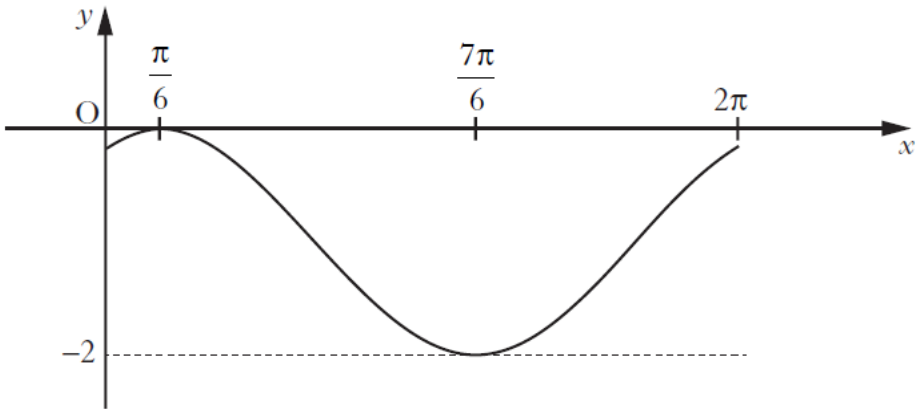
Ans	(*a*) $a = 6, b = 2$ (*b*) $f(x) = 2x^3 - 6x^2 + 8$	
2004 P2	**3.** Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p.	4
Ans	$b^2 - 4ac = p^2 - 4 \times 2 \times (-3)$ $= p^2 + 24$ p^2 is positive, so $b^2 - 4ac$ is positive too and roots are real.	
2003 P1	**2.** (*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. (*b*) Hence or otherwise sketch the graph of $y = f(x)$.	2 2
Ans	(*a*) $(x + 3)^2 + 2$ (*b*) 	
2003 P1	**7.** Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$.	5
Ans	$x^2 + 3x + 4 = 2x + 1$ $x^2 + x + 3 = 0$ $b^2 - 4ac = -11$ $b^2 - 4ac < 0$ therefore no intersection	
2002W P2	**6.** The graph of $f(x) = 2x^3 - 5x^2 - 3x + 1$ has been sketched in the diagram shown. Find the value of a correct to one decimal place. 	3
Ans	Evaluate $f(0{\cdot}1)$ and $f(0{\cdot}5)$, for example, to start with $a = 0{\cdot}2$	
2002 P1	**7.** (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$.	2
Ans	(*a*) $f(x) = (x-2) + 1$	

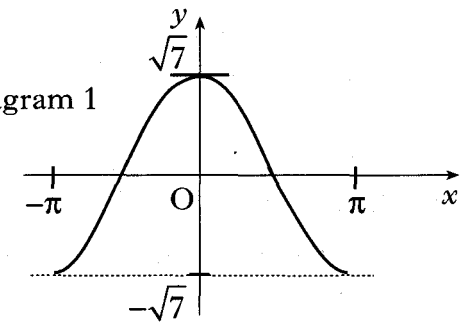
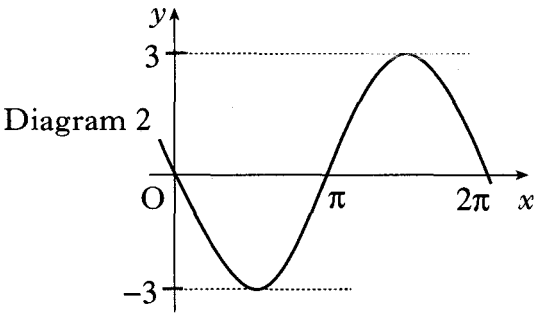
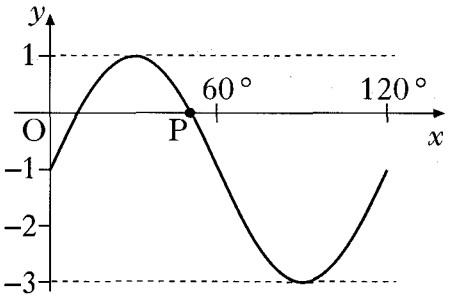
2002 P2	**9.** Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k.	5
Ans	(a) $f(x) = (x-2) + 1$	
2002 P2	**9.** Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k.	5
Ans	discriminant $= (-5k)^2 - 4(1-2k)(-2k)$ $= 9k^2 + 8k$ for real roots, discriminant ≥ 0 ie $9k^2 + 8k \geq 0$ $k(9k+8) \geq 0$ $k \geq 0$ or $k \leq \frac{-8}{9}$ no integers between 0 and $\frac{-8}{9}$ hence no integral values of k give non - real roots	
2001 P1	**2.** For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots?	3
Ans	$k = \frac{1}{4}$	
2001 P1	**4.** Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$.	2
Ans	$(x + 1)^2 - 9$	
2001 P2	**11.** The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$. (a) Show that the equation of the parabola is $y = p + (p - 1)x - x^2$. (b) For what value of p will the line $y = x + p$ be a tangent to this curve? 	3 3
Ans	(a) $y = k(x + 1)(x - p)$ $k = -1$ with justification ie substitute $(0,p)$ $y = -1(x + 1)(x - p)$ and complete (b) 2	

2000 P2	**4.** The parabola shown crosses the x-axis at (0, 0) and (4, 0), and has a maximum at (2, 4). The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$. (a) Find the equation of the parabola. 	2
Ans	(a) $y = 4x - x^2$	
Specimen 2 P2	**1.** The parabola shown in the diagram has equation $y = 4x - x^2$ and intersects the x-axis at the origin and P. The line PQ has equation $2y + x = 4$. Find the coordinates of P and Q. 	5
Ans	$4x - x^2 = 2 - \frac{1}{2}x \Rightarrow 2x^2 - 9x + 4 = 0$ $x = \frac{1}{2}, x = 4 \Rightarrow \text{P} = (4,0),\ \text{Q} = \left(\frac{1}{2}, \frac{7}{4}\right)$	
Specimen 1 P2	**3.** (a) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x + b)^2 + c$ where a, b and c are constants. (b) Hence, or otherwise, find the coordinates of the turning point of the function f.	3 1
Ans	(a) $2(x + 2)^2 - 11$ (b) $(-2, -11)$	
Specimen 1 P2	**8.** The roots of the equation $(x - 1)(x + k) = -4$ are equal. Find the values of k.	5
Ans	$-5, 3$	

NH 2015 P1	**4.** The diagram shows part of the graph of the function $y = p\cos qx + r$.  Write down the values of p, q and r.	3
Ans	p = 3, q = 4, r = 1	
NH SP P1	**12.** The voltage, $V(t)$, produced by a generator is described by the function $V(t) = 120\sin 100\pi t$, $t > 0$, where t is the time in seconds. (a) Determine the period of $V(t)$. (b) Find the first three times for which $V(t) = -60$.	2 6
Ans	(a) $\frac{1}{50}$ sec or 0·02 sec (b) $\frac{7}{600}$, $\frac{11}{600}$, and $\frac{19}{600}$ sec	
NH Ex P1	**4.** For the function $f(x) = 2 - 3\sin\left(x - \frac{\pi}{3}\right)$ in the interval $0 \le x < 2\pi$, determine which two of the following statements are true **and justify your answer**. Statement A The maximum value of $f(x)$ is −1. Statement B The maximum value of $f(x)$ is 5. Statement C The maximum value occurs when $x = \frac{5\pi}{6}$. Statement D The maximum value occurs when $x = \frac{11\pi}{6}$.	3
Ans	B & D with justification	
2014 P2	**6.** Solve the equation $\sin x - 2\cos 2x = 1$ for $0 \le x < 2\pi$.	5
Ans	0.848, 2.29 and $\frac{3\pi}{2}$	

2014 P1	**13.** What is the value of $\sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{5\pi}{4}\right)$?	
Ans	$\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}$	
2014 P1	**9.** $\sin x + \sqrt{3}\cos x$ can be written as $2\cos\left(x - \frac{\pi}{6}\right)$. The maximum value of $\sin x + \sqrt{3}\cos x$ is 2. What is the maximum value of $5\sin 2x + 5\sqrt{3}\cos 2x$?	
Ans	10	
2013 P1	**15.** Solve $\tan\left(\frac{x}{2}\right) = -1$ for $0 \le x < 2\pi$.	
Ans	$\frac{3\pi}{2}$	
2013 P1	**4.** ~~Which of the following shows~~ the graph of $y = 4\cos 2x - 1$, for $0 \le x \le \pi$? (Sketch)	
Ans		
2012 P1	**12.** Find the maximum value of $2 - 3\sin\left(x - \frac{\pi}{3}\right)$ and the value of x where this occurs in the interval $0 \le x \le 2\pi$.	
Ans	Max at 5 when x = $\frac{11\pi}{6}$	
2011 P1	**10.** Solve $2\cos x = \sqrt{3}$ for x, where $0 \le x < 2\pi$.	
Ans	$\frac{\pi}{6}$ and $\frac{11\pi}{6}$	

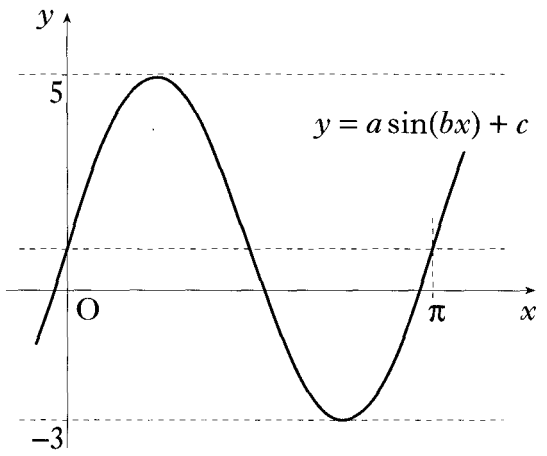
2010 P1	**4.** The diagram shows the graph with equation of the form $y = a\cos bx$ for $0 \le x \le 2\pi$.  What is the equation of this graph?	2
Ans	$y = 2\cos 3x$	
2009 P1	**11.** How many solutions does the equation $(4\sin x - \sqrt{5})(\sin x + 1) = 0$ have in the interval $0 \le x < 2\pi$?	2
Ans	1	
2009 P1	**14.** If $f(x) = 2\sin\left(3x - \frac{\pi}{2}\right) + 5$, what is the range of values of $f(x)$?	2
Ans	$3 \le f(x) \le 7$	
2012 P1	**9.** The diagram shows the curve with equation of the form $y = \cos(x + a) + b$ for $0 \le x \le 2\pi$.  What is the equation of this curve?	
Ans	$y = \cos\left(x - \frac{\pi}{6}\right) - 1$	
2008 P1	**6.** What is the solution of the equation $2\sin x - \sqrt{3} = 0$ where $\frac{\pi}{2} \le x \le \pi$?	2

Ans	$\frac{2\pi}{3}$	
2008 P2	**3.** (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$. Write down the value of p. Diagram 1  (ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$. Write down the value of q. Diagram 2  	2
Ans	$p = \sqrt{7}$, $q = -3$	
2007 P2	**4.** The diagram shows part of the graph of a function whose equation is of the form $y = a\sin(bx°) + c$. (*a*) Write down the values of a, b and c. (*b*) Determine the exact value of the x-coordinate of P, the point where the graph intersects the x-axis as shown in the diagram.  	3 3
Ans	(*a*) $a = 2$, $b = 3$, $c = -1$ (*b*) $x_P = 50°$	
2004 P1	**3.** Find all the values of x in the interval $0 \le x \le 2\pi$ for which $\tan^2(x) = 3$.	4
Ans	$x = \frac{\pi}{3}$ and $x = \frac{4\pi}{3}$ $x = \frac{2\pi}{3}$ and $x = \frac{5\pi}{3}$	

2003 P2

2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$.

Determine the values of a, b and c.



3

Ans

a = 4, b = 2, c = 1

2002W P1

4. (*a*) Write down the exact values of $\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$. 1

(*b*) If $\tan x = 4\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)$, find the exact values of x for $0 \le x \le 2\pi$. 2

Ans

(*a*) $\frac{\sqrt{3}}{2}$, $\frac{1}{2}$

(*b*) $\frac{\pi}{3}$, $\frac{4\pi}{3}$

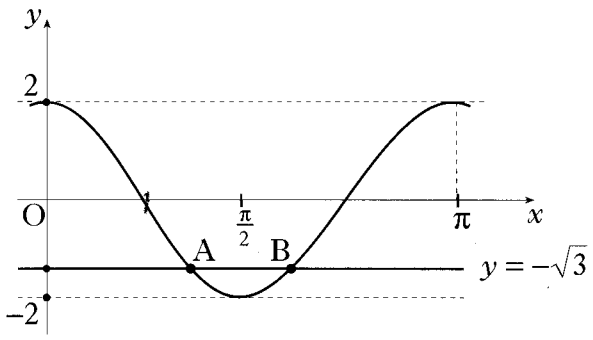
2002 P1

8. The diagram shows the graph of a cosine function from 0 to π.

(*a*) State the equation of the graph. 1

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at points A and B.

Find the coordinates of B. 3



Ans

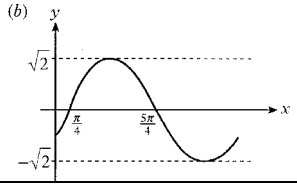
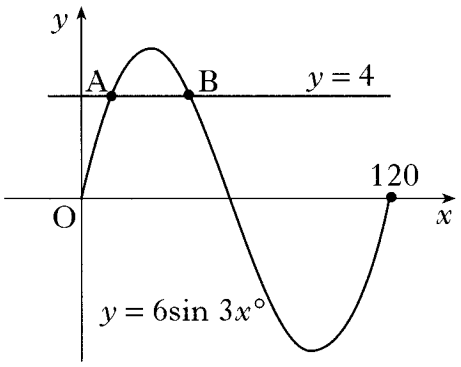
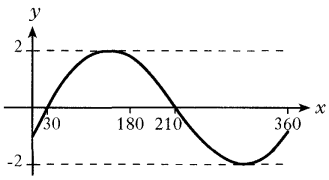
(*a*) $y = 2\cos(2x)$

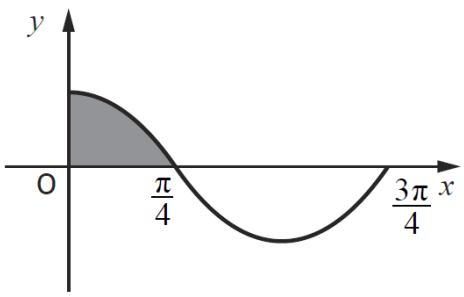
(*b*) B $\left(\frac{7\pi}{12}, -\sqrt{3}\right)$

2002 P1

9. (*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \le a \le 2\pi$. 4

(*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \le x \le 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis. 3

Ans	(a) $\sqrt{2}\sin(x-\frac{\pi}{4})$ (b) 	
Specimen 2 P2	**3.** The diagram shows part of the graph of $y = 6\sin 3x$ and the line with equation $y = 4$. Find the x-coordinates of A and B. 	3
Ans	$6\sin 3x = 4$ $3x = 41{\cdot}8,\ 138{\cdot}2,\ (401{\cdot}8)$ $x = 13{\cdot}9,\ 46{\cdot}1,\ (133{\cdot}6)$ $x_A = 13{\cdot}9,\ x_B = 46{\cdot}1$	
Specimen 1 P1	**8.** Sketch the graph of $y = 2\sin(x-30)°$ for $0 \le x < 360$.	4
Ans		

2017 P1	**13.** Find $\int \frac{1}{(5-4x)^{\frac{1}{2}}}dx, \quad x<\frac{5}{4}$.	4
Ans	$-\frac{1}{2}(5-4x)^{\frac{1}{2}}+c$	
2017 P2	**11.** (a) Show that $\frac{\sin 2x}{2\cos x}-\sin x\cos^2 x=\sin^3 x$, where $0<x<\frac{\pi}{2}$. (b) Hence, differentiate $\frac{\sin 2x}{2\cos x}-\sin x\cos^2 x$, where $0<x<\frac{\pi}{2}$.	3 3
Ans	(a) Proof (b) $3\sin^2 x\cos x$	
2016 P1	**5.** Find $\int 8\cos(4x+1)dx$.	2
Ans	$2\sin(4x+1)+c$	
2016 P2	**10.** (a) Given that $y=(x^2+7)^{\frac{1}{2}}$, find $\frac{dy}{dx}$. (b) Hence find $\int \frac{4x}{\sqrt{x^2+7}}dx$.	2 1
Ans	(a) $\frac{dy}{dx}=x(x^2+7)^{-\frac{1}{2}}$ (b) $4(x^2+7)^{\frac{1}{2}}(+c)$	
2016 P2	**11.** (a) Show that $\sin 2x\tan x=1-\cos 2x$, where $\frac{\pi}{2}<x<\frac{3\pi}{2}$. (b) Given that $f(x)=\sin 2x\tan x$, find $f'(x)$.	4 2
Ans	(a) Proof (b) $f'(x)=2\sin 2x$ or $f'(x)=4\sin x\cos x$	
NH 2015 P1	**12.** The diagram shows part of the graph of $y=a\cos bx$. The shaded area is $\frac{1}{2}$ unit^2.  What is the value of $\int_0^{\frac{3\pi}{4}}(a\cos bx)dx$?	2

Ans	-1/2	
NH 2015 P2	**7.** (a) Find $\int (3\cos 2x + 1)\,dx$. (b) Show that $3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x$. (c) Hence, or otherwise, find $\int (\sin^2 x - 2\cos^2 x)\,dx$.	2 2 2
Ans	(a) $\frac{3}{2}\sin 2x + c$ (b) proof (c) $-\frac{3}{4}\sin 2x - \frac{1}{2}x + c$	
NH SP P1	**1.** Find $\int \frac{3x^3+1}{2x^2}\,dx,\ x \neq 0$.	4
Ans	$\frac{3}{4}x^2 - \frac{1}{2}x^{-1} + C$	
NH SP P1	**10.** Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$.	3
Ans	$\frac{3\sqrt{3}}{2}$	
NH EX P2	**10.** Acceleration is defined as the rate of change of velocity. An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$. (a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. (b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. (c) Velocity is defined as the rate of change of displacement. Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$.	3 2 3
Ans	(a) $a(t) = -16\sin\left(2t - \frac{\pi}{2}\right)$ (b) $a(10) > 0$ therefore increasing (c) $s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$	
NH EX P2	**8.** Given that $\int_{\frac{\pi}{8}}^{a} 5\sin(4x - \frac{\pi}{2})dx = \frac{10}{4}$, $0 \leq a < \frac{\pi}{2}$, calculate the value of a.	6
Ans	$\frac{3\pi}{8}$	
NH EX P1	**10.** The gradient of a tangent to a curve is given by $\frac{dy}{dx} = 3\cos 2x$. The curve passes through the point $\left(\frac{7\pi}{6}, \sqrt{3}\right)$. Find y in terms of x.	4

Ans	$y = \frac{3}{2}\sin 2x + \frac{\sqrt{3}}{4}$	
2014 P2	**9.** Acceleration is defined as the rate of change of velocity. An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$. (a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. (b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. (c) Velocity is defined as the rate of change of displacement. Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$.	3 2 3
Ans	(a) $a(t) = -16\sin(2t - \frac{\pi}{2})$ (b) a(10) > 0, therefore increasing (c) $s(t) = 4\sin(2t - \frac{\pi}{2}) + 8$	
2014 P2	**5.** Given that $\int_4^t (3x+4)^{-\frac{1}{2}}\,dx = 2$, find the value of t.	5
Ans	t = 15	
2014 P2	**8.** What is the derivative of $\left(4 - 9x^4\right)^{\frac{1}{2}}$?	
Ans	$-18x^3\left(4 - 9x^4\right)^{-\frac{1}{2}}$	
2014 P1	**5.** Find $\int (2x+9)^5\,dx$.	
Ans	$\frac{1}{12}(2x+9)^6 + c$	
2013 P2	**6.** Given that $\int_0^a 5\sin 3x\,dx = \frac{10}{3}$, $0 \le a < \pi$, calculate the value of a.	
Ans	$a = \frac{\pi}{3}$	
2013 P1	**18.** Given that $y = \sin(x^2 - 3)$, find $\frac{dy}{dx}$.	

Ans	$2x\cos(x^2-3)$	
2013 P1	**16.** Find $\int(1-6x)^{-\frac{1}{2}}dx$ where $x<\frac{1}{6}$.	
Ans	$-\frac{1}{3}(1-6x)^{\frac{1}{2}}+c$	
2012 P1	**16.** If $y=3\cos^4 x$, find $\frac{dy}{dx}$.	
Ans	$-12\cos^3 x\sin x$	
2012 P1	**14.** Find $\int(2x-1)^{\frac{1}{2}}dx$ where $x>\frac{1}{2}$.	
Ans	$\frac{1}{3}(2x-1)^{\frac{3}{2}}+C$	
2011 P2	**6.** (*a*) The expression $3\sin x-5\cos x$ can be written in the form $R\sin(x+a)$ where $R>0$ and $0\le a<2\pi$. Calculate the values of R and a. (*b*) Hence find the value of t, where $0\le t\le 2$, for which $\int_0^t(3\cos x+5\sin x)\,dx=3$.	**4** **7**
Ans	(a) R = $\sqrt{34}$, a = 5.3 rads (b) t = 0.7	
2011 P1	**20.** On a suitable domain, D, a function g is defined by $g(x)=\sin^2\sqrt{x-2}$. Which of the following gives the real values of x in D and the corresponding values of $g(x)$?	**2**
Ans	$x\ge 2$ and $0\le g(x)\le 1$	
2011 P1	**13.** Given that $f(x)=4\sin 3x$, find $f'(0)$.	**2**
Ans	12	

2010 P2

6. (*a*) A curve has equation $y = (2x-9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$. **5**

(*b*) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x-axis at the point A. **1**

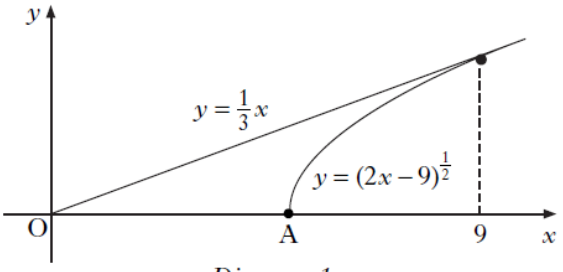


Diagram 1

Find the coordinates of point A.

(*c*) Calculate the shaded area shown in diagram 2. **7**

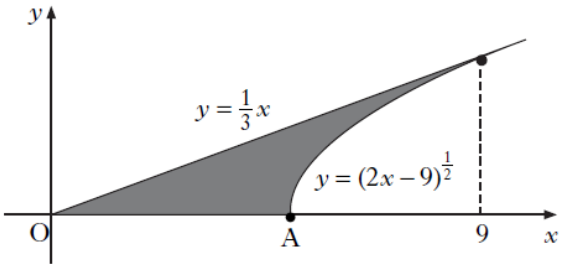


Diagram 2

Ans

(b) A$(\frac{9}{2},0)$ (c) $\frac{9}{2}$ units^2

2010 P1

9. Find $\int (2x^{-4} + \cos 5x)\,dx$. **2**

Ans

$-\frac{2}{3}x^{-3} + \frac{1}{5}\sin 5x + c$

2009 P2

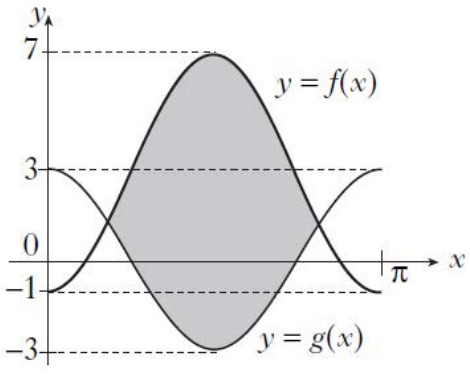
5. The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

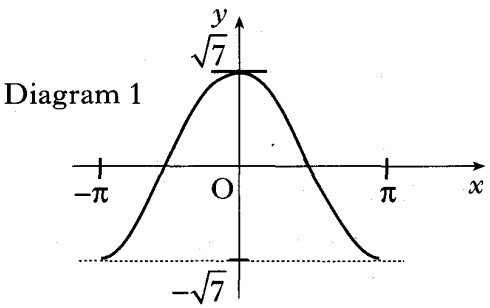
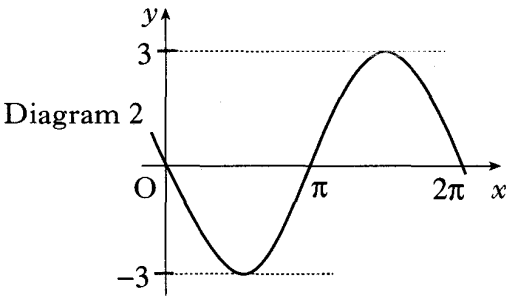
$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.

(*a*) Write down the values of m and n. **1**

(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \le x \le \pi$. **5**

(*c*) Calculate the shaded area. **6**

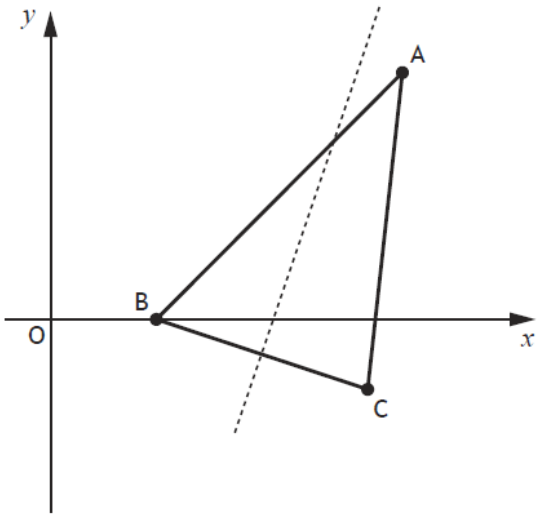


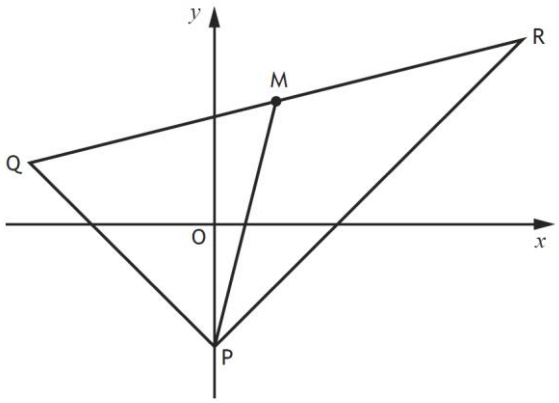
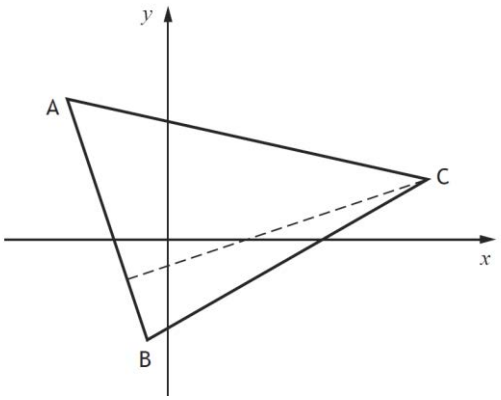
Ans	(a) m = 3, n = 2 (b) (0.6, 1.3) (2.6, 1.3) (c) 12.4 units^2	
2009 P1	**18.** Given that $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$ on a suitable domain, find $f'(x)$.	2
Ans	$3x(4 - 3x^2)^{-\frac{3}{2}}$	
2008 P1	**15.** What is the derivative of $(x^3 + 4)^2$?	2
Ans	$6x^2(x^3 + 4)$	
2008 P2	**3.** (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$. Write down the value of p.  (ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$. Write down the value of q.  (*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. (*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression.	2 4 2
Ans	*(a) p = √7 , q=-3 (b) 4Cos(x+0.848) (c) -4Sin(x+0.848)*	
2007 P1	**10.** Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$.	3
Ans	$\frac{1}{2}(3x^2 + 2)^{-\frac{1}{2}} \times 6x$ ($= 3x(3x^2 + 2)^{-\frac{1}{2}}$)	

2007 P2	**7.** Find the value of $\int_0^2 \sin(4x+1)\,dx$.	4
Ans	0.36	
2006 P1	**5.** A function f is defined by $f(x) = (2x-1)^5$. Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature.	7
Ans	point of inflexion at $(\frac{1}{2},0)$	
2006 P2	**9.** If $y = \frac{1}{x^3} - \cos 2x,\ x \neq 0$, find $\frac{dy}{dx}$.	4
Ans	$-3x^{-4} + 2\sin 2x$	
2005 P1	**5.** Differentiate $(1 + 2\sin x)^4$ with respect to x.	2
Ans	$4(1 + 2\sin(x))^3$ $\ldots\times 2\cos(x)$	
2004 P1	**6.** Given that $y = 3\sin(x) + \cos(2x)$, find $\frac{dy}{dx}$.	3
Ans	$3\cos(x) - 2\sin(2x)$	
2004 P1	**7.** Find $\int_0^2 \sqrt{4x+1}\ dx$.	5
Ans	$\frac{13}{3}$	
2003 P1	**8.** Find $\int_0 \frac{dx}{(3x+1)^{\frac{1}{2}}}$.	4
Ans	$\frac{2}{3}$	

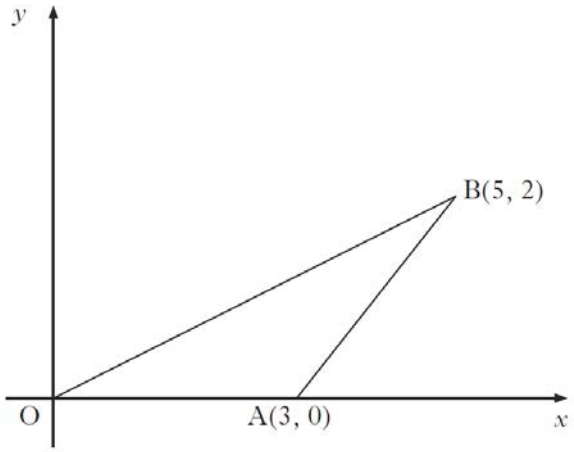
2003 P2	**6.** If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$.	4
Ans	$6 - \sqrt{3}$	
2002W P1	**10.** A function f is defined by $f(x) = 2x + 3 + \frac{18}{x-4}$, $x \neq 4$. Find the values of x for which the function is increasing.	5
Ans	$f'(x) = 2 - \frac{18}{(x-4)^2}$ $f'(x) = 0 \Rightarrow x = 1, 7$ $f'(x) > 0 \Rightarrow x < 1, x > 7$	
2002W P2	**8.** Find $\int_0^1 \left(\cos(3x) - \sin\left(\frac{1}{3}x + 1\right)\right) dx$ correct to 3 decimal places.	3
Ans	$a = -0.868$	
2002 P1	**10.** (a) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$.	2
	(b) Hence write down $\int \frac{x^2}{(8-x^3)^{\frac{1}{2}}} dx$.	1
Ans	(a) $f'(x) = -\frac{3}{2}x^2(8-x^3)^{-\frac{1}{2}}$ (b) $-\frac{2}{3}(8-x^3)^{\frac{1}{2}} + c$	
2002 P2	**6.** Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$.	4
Ans	$y = \sqrt{3}x + 1 - \frac{\pi}{\sqrt{3}}$	
2001 P2	**10.** A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$. Find y in terms of x.	4
Ans	$y = -\frac{3}{2}\cos(2x) + \frac{1}{4}\sqrt{3}$	

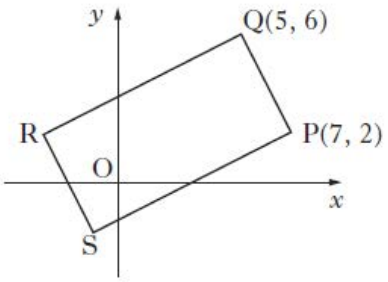
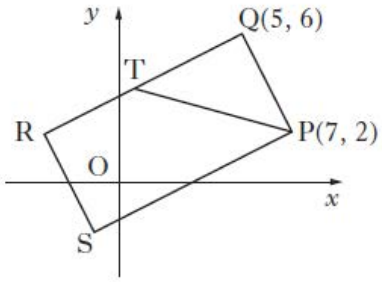
2000 P1	**8.** The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$. If $f'(x) = \sin(3x)$ express y in terms of x.	4
Ans	$y = -\frac{1}{3}\cos(3x) + \frac{7}{6}$	
2000 P2	**8.** Given that $f(x) = (5x-4)^{\frac{1}{2}}$, evaluate $f'(4)$.	3
Ans	$\frac{5}{8}$	
2000 P2	**10.** Find $\int \frac{1}{(7-3x)^2}\,dx$.	2
Ans	$\frac{1}{3(7-3x)} + c$	
Specimen 1 P1	**9.** Find $\frac{dy}{dx}$ given that $y = \sqrt{1+\cos x}$.	3
Ans	$-\frac{1}{2}\sin x\,(1+\cos x)^{-\frac{1}{2}}$	

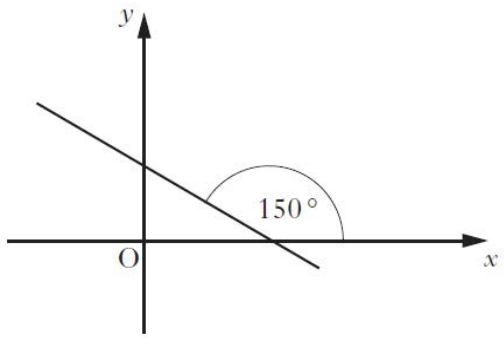
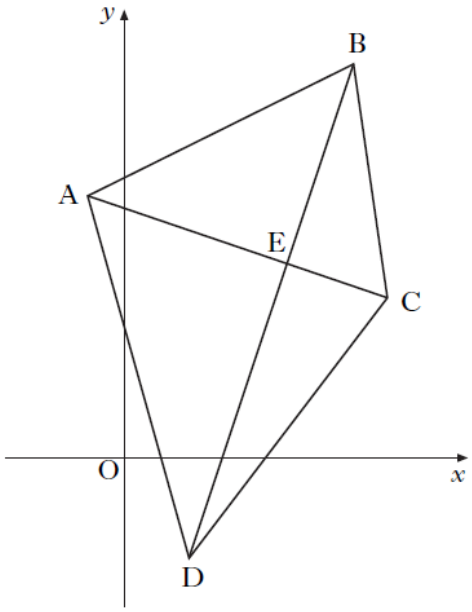
2017 P1	**7.** A (−3, 5), B (7, 9) and C (2, 11) are the vertices of a triangle. Find the equation of the median through C.	3
Ans	x = 2	
2017 P1	**11.** A and B are the points (−7, 2) and (5, a). AB is parallel to the line with equation $3y - 2x = 4$. Determine the value of a.	3
Ans	a = 10	
2017 P2	1. Triangle ABC is shown in the diagram below. The coordinates of B are (3,0) and the coordinates of C are (9,−2). The broken line is the perpendicular bisector of BC.  (a) Find the equation of the perpendicular bisector of BC. (b) The line AB makes an angle of 45° with the positive direction of the x-axis. Find the equation of AB. (c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC.	4 2 2
Ans	(a) $y = 3x - 19$ (b) $y = x - 3$ (c) (8, 5)	
2016 P1	**1.** Find the equation of the line passing through the point $(-2, 3)$ which is parallel to the line with equation $y + 4x = 7$.	2
Ans	$y + 4x = -5$	

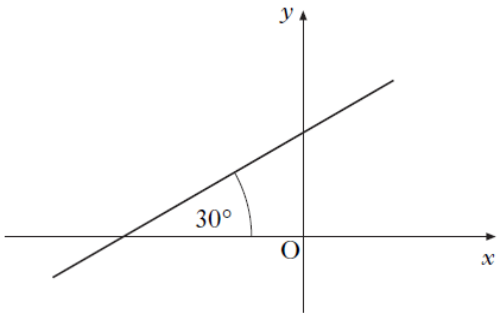
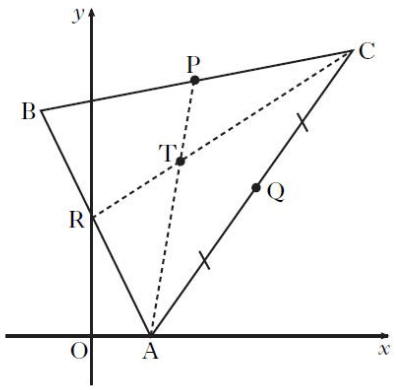
2016 P1	**1.** PQR is a triangle with vertices $P(0,-4)$, $Q(-6,2)$ and $R(10,6)$.  (a) (i) State the coordinates of M, the midpoint of QR. (ii) Hence find the equation of PM, the median through P. (b) Find the equation of the line, *L*, passing through M and perpendicular to PR. (c) Show that line *L* passes through the midpoint of PR.	1 2 3 3
Ans	(a) (i) M(2, 4) (ii) y = 4x – 4 (b) Y = -x + 6 (c) Proof	
NH 2015 P1	**9.** A, B and C are points such that AB is parallel to the line with equation $y + \sqrt{3}x = 0$ and BC makes an angle of $150°$ with the positive direction of the x-axis. Are the points A, B and C collinear?	3
Ans	No, as $120 \neq 150$	
NH 2015 P2	**1.** The vertices of triangle ABC are $A(-5, 7)$, $B(-1, -5)$ and $C(13, 3)$ as shown in the diagram. The broken line represents the altitude from C.  (a) Show that the equation of the altitude from C is $x - 3y = 4$. (b) Find the equation of the median from B. (c) Find the coordinates of the point of intersection of the altitude from C and the median from B.	4 3 2

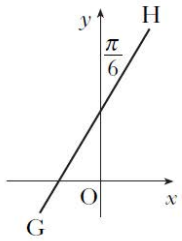
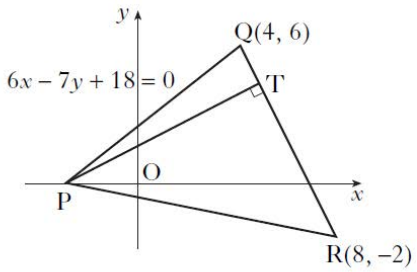
Ans	(a) proof (b) $y = 2x - 3$ (c) (1, -1)	
NH SP P1	**9.** (a) AB is a line parallel to the line with equation $y + 3x = 25$. A has coordinates $(-1, 10)$. Find the equation of AB. (b) $3y = x + 11$ is the perpendicular bisector of AB. Determine the coordinates of B.	1 5
Ans	(a) $y - 10 = -3(x + 1)$ (b) B(-3, 2)	
NH SP P1	**5.** Line l_1 has equation $\sqrt{3}y - x = 0$. (a) Line l_2 is perpendicular to l_1. Find the gradient of l_2. (b) Calculate the angle l_2 makes with the positive direction of the x-axis.	2 2
Ans	(a) $m_{l_2} = -\sqrt{3}$ (b) $\theta = \frac{2\pi}{3}$ or $120°$	
NH Ex P1	**6.** (a) Find the equation of l_1, the perpendicular bisector of the line joining P $(3,-3)$ and Q $(-1,9)$. (b) Find the equation of l_2 which is parallel to PQ and passes through R $(1,-2)$. (c) Find the point of intersection of l_1 and l_2. (d) Hence find the shortest distance between PQ and l_2.	4 2 3 2
Ans	*(a)* $y - 3 = \frac{1}{3}(x - 1)$ *(b)* $y - (-2) = -3(x - 1)$ *(c)* $X = -1/2, y = 5/2$ *(d)* $\sqrt{\frac{5}{2}}$	
NH Ex P1	**1.** The point P (5,12) lies on the curve with equation $y = x^2 - 4x + 7$. Find the equation of the tangent to this curve at P.	3
Ans	$u - 12 = 69x - 5)$	

2014 P2	**1.** A(3, 0), B(5, 2) and the origin are the vertices of a triangle as shown in the diagram.  (*a*) Obtain the equation of the perpendicular bisector of AB. (*b*) The median from A has equation $y + 2x = 6$. Find T, the point of intersection of this median and the perpendicular bisector of AB. (*c*) Calculate the angle that AT makes with the positive direction of the x-axis.	4 2 2
Ans	(a) $y - 1 = -1(x - 4)$ (b) x = 1, y = 4 (c) 116.6°	
2013 P1	**5.** The line L passes through the point (–2, –1) and is parallel to the line with equation $5x + 3y - 6 = 0$. What is the equation of L? *(d)*	
Ans	*(e)* $5x + 3y + 13 = 0$	

	2. The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6).  (*a*) Find the equation of QR.	3 3
	(*b*) The line from P with the equation $x + 3y = 13$ intersects QR at T.  Find the coordinates of T. (*c*) Given that T is the midpoint of QR, find the coordinates of R and S.	3 3
Ans	(a) $y - 6 = \frac{1}{2}(x - 5)$ (b) $y = 4, x = 1$ (c) R(-3, 2), S(-1, -2)	
2012 P1	**23.** (*a*) Find the equation of ℓ_1, the perpendicular bisector of the line joining P(3, –3) to Q(–1, 9). (*b*) Find the equation of ℓ_2 which is parallel to PQ and passes through R(1, –2). (*c*) Find the point of intersection of ℓ_1 and ℓ_2. (*d*) Hence find the shortest distance between PQ and ℓ_2.	4 2 3 2
Ans	(a) $x - 3y + 8 = 0$ (b) $3x + y - 1 = 0$ (c) $(-\frac{1}{2}, \frac{5}{2})$ (d) $\frac{\sqrt{10}}{2}$	

2012 P1	**4.** What is the gradient of the line shown in the diagram? 	
Ans	$-\frac{1}{\sqrt{3}}$	
2011 P1	**21.** A quadrilateral has vertices A(–1, 8), B(7, 12), C(8, 5) and D(2, –3) as shown in the diagram.  (*a*) Find the equation of diagonal BD. (*b*) The equation of diagonal AC is $x + 3y = 23$. Find the coordinates of E, the point of intersection of the diagonals. (*c*) (i) Find the equation of the perpendicular bisector of AB. (ii) Show that this line passes through E.	**2** **3** **5**
Ans	(a) $y - 3x = -9$ (b) E (5, 6) (c) (i) $y + 2x = 16$ (ii) proof	

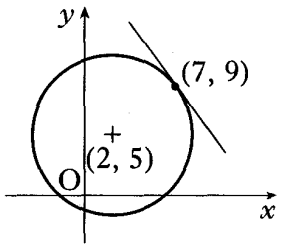
2011 P1	**8.** A line makes an angle of 30° with the positive direction of the x-axis as shown.  What is the gradient of the line?	
Ans	$\frac{1}{\sqrt{3}}$	
2011 P1	**2.** A line l has equation $3y + 2x = 6$. What is the gradient of any line parallel to l?	
Ans	$-\frac{2}{3}$	
2010 P1	**21.** Triangle ABC has vertices A(4, 0), B(–4, 16) and C(18, 20), as shown in the diagram opposite. Medians AP and CR intersect at the point T(6, 12).  (*a*) Find the equation of median BQ. (*b*) Verify that T lies on BQ. (*c*) Find the ratio in which T divides BQ.	3 1 2
Ans	(a) 2x + 55 – 72 = 0 (c) 2:1	
2010 P1	**1.** A line L is perpendicular to the line with equation $2x - 3y - 6 = 0$. What is the gradient of the line L?	
Ans	$-\frac{3}{2}$	
2009 P1	**3.** Triangle PQR has vertices at P(–3, –2), Q(–1, 4) and R(3, 6). PS is a median. What is the gradient of PS?	

Ans	$\frac{7}{4}$	
2009 P1	**5.** Here are two statements about the points S(2, 3) and T(5, –1): (1) The length of ST = 5 units; (2) The gradient of ST $= \frac{4}{3}$. Which of the following is true? A Neither statement is correct. B Only statement (1) is correct. C Only statement (2) is correct. D Both statements are correct.	
Ans	B	
2009 P1	**15.** The line GH makes an angle of $\frac{\pi}{6}$ radians with the y-axis, as shown in the diagram. What is the gradient of GH?  	
Ans	$\sqrt{3}$	
2009 P1	**21.** Triangle PQR has vertex P on the x-axis, as shown in the diagram. Q and R are the points (4, 6) and (8, –2) respectively. The equation of PQ is $6x - 7y + 18 = 0$. (*a*) State the coordinates of P. (*b*) Find the equation of the altitude of the triangle from P. (*c*) The altitude from P meets the line QR at T. Find the coordinates of T.  	1 3 4
Ans	(a) (-3, 0) (b) x – 2y + 3 = 0 (c) T(5, 4)	

2008 P1

5. The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?



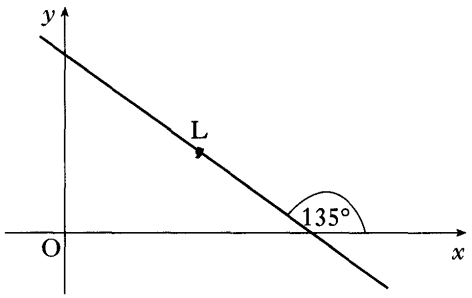
2

Ans

$y - 9 = -\frac{5}{4}(x - 7)$

2008 P1

7. The diagram shows a line L; the angle between L and the positive direction of the x-axis is 135°, as shown.



What is the gradient of line L?

2

Ans

-1

2008 P2

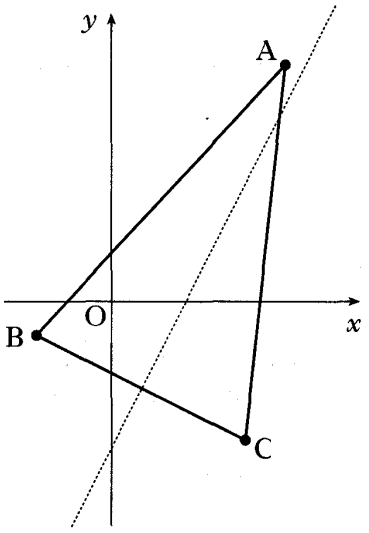
1. The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.

The broken line represents the perpendicular bisector of BC.

(*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$.

(*b*) Find the equation of the median from C.

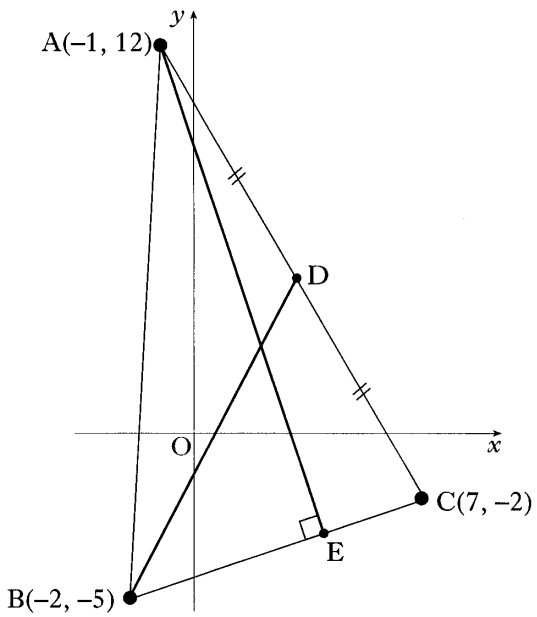
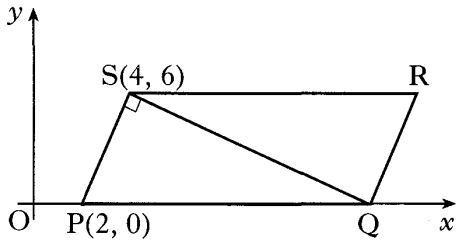
(*c*) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C.



4

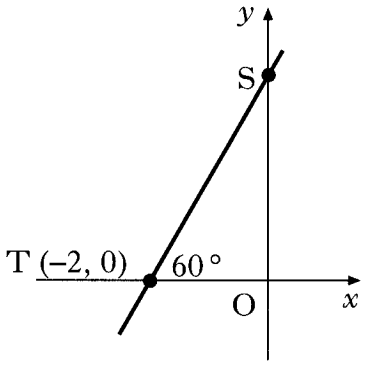
3

3

Ans	*(a) Proof (b) y = -3x+10 (c) (3,1)*	
2007 P1	**1.** Find the equation of the line through the point (–1, 4) which is parallel to the line with equation $3x - y + 2 = 0$.	3
Ans	y = 3x + 7	
2006 P1	**1.** Triangle ABC has vertices A(–1, 12), B(–2, –5) and C(7, –2). (*a*) Find the equation of the median BD. (*b*) Find the equation of the altitude AE. (*c*) Find the coordinates of the point of intersection of BD and AE. 	3 3 3
Ans	(*a*) $y - 5 = 2(x - 3)$ or $y + 5 = 2(x - (-2))$ etc (*b*) $y - 12 = -3(x - (-1))$ (*c*) (2, 3)	
2006 P2	**1.** PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the *x*-axis, as shown. The diagonal QS is perpendicular to the side PS.  (*a*) Show that the equation of QS is $x + 3y = 22$. (*b*) Hence find the coordinates of Q and R.	4 2
Ans	(*a*) proof $m_{PS} = 3$ $m_{QS} = -\frac{1}{3}$ $y - 6 = -\frac{1}{3}(x - 4)$ (*b*) Q = (22, 0) R = (24, 6)	

2005 P1

1. Find the equation of the line ST, where T is the point (–2, 0) and angle STO is 60 °.



3

Ans

$y - 0 = \sqrt{3}(x - (-2))$

2004 P1

1. The point A has coordinates (7, 4). The straight lines with equations $x + 3y + 1 = 0$ and $2x + 5y = 0$ intersect at B.

(*a*) Find the gradient of AB. 3

(*b*) Hence show that AB is perpendicular to only one of these two lines. 5

Ans

(*a*) $m_{AB} = 3$

(*b*) $m_{AB} = 3 \Rightarrow m_{perp} = -\frac{1}{3}$

$y = -\frac{1}{3}x - \frac{1}{3}$

$m_{l_1} = -\frac{1}{3}$

$m_{l_2} = -\frac{2}{5}$

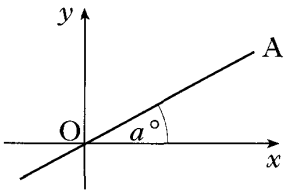
so only the 1st line is perpendicular to AB.

2004 P2

1. (*a*) The diagram shows line OA with equation $x - 2y = 0$.

The angle between OA and the x-axis is a °.

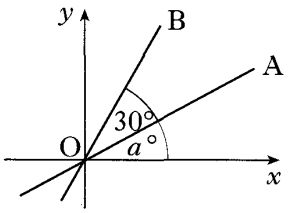
Find the value of a.



3

(*b*) The second diagram shows lines OA and OB. The angle between these two lines is 30 °.

Calculate the gradient of line OB correct to 1 decimal place.

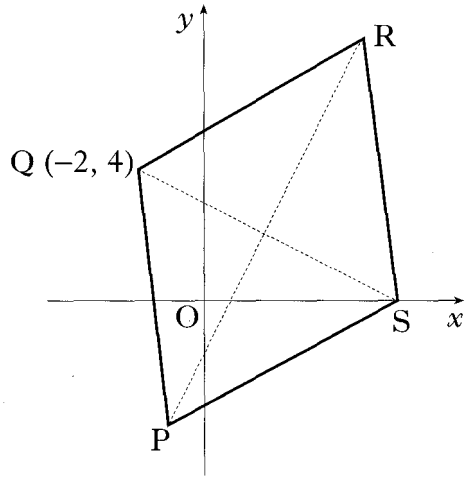
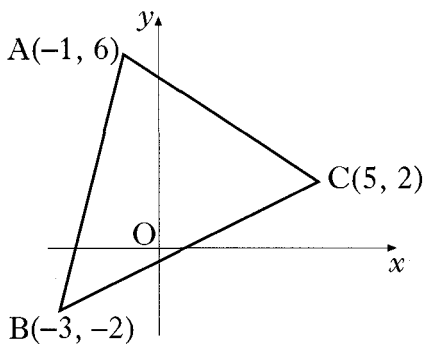


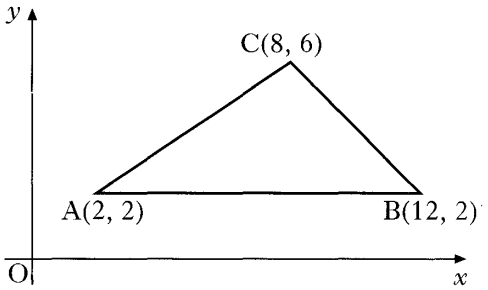
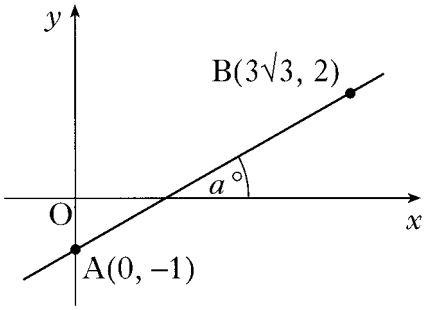
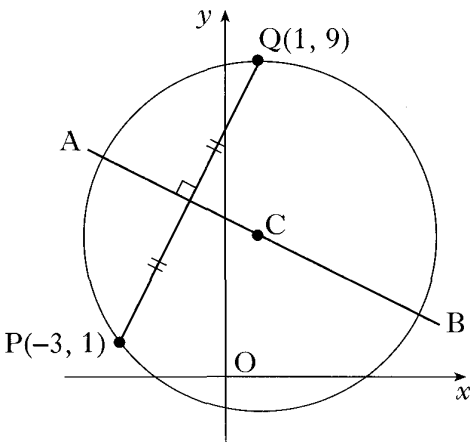
1

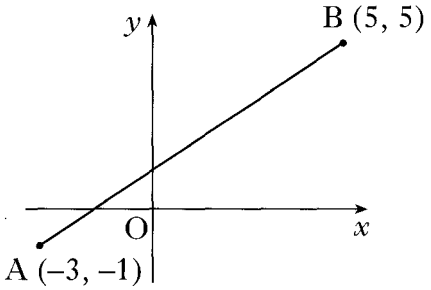
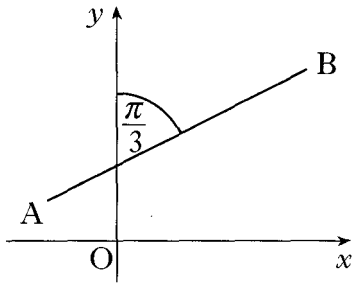
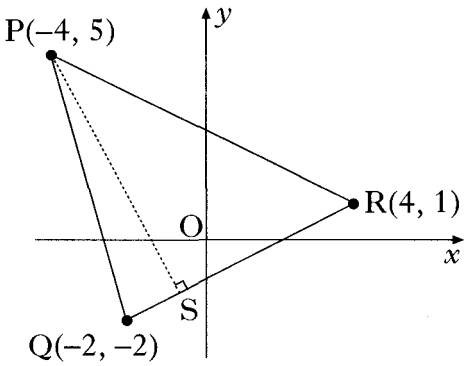
Ans

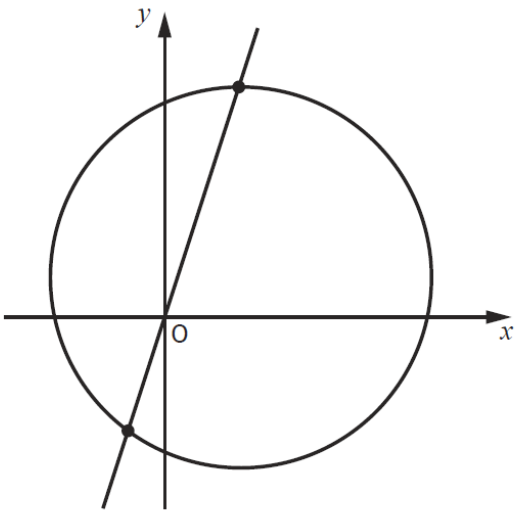
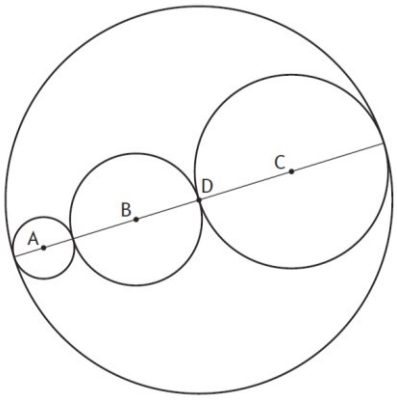
(*a*) $a = 26{\cdot}6°$

(*b*) 1·5

2003 P1	**1.** Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x + y - 1 = 0$.	3
Ans	$x - 4y + 13 = 0$	
2002W P1	**1.** (*a*) Find the equation of the straight line through the points A(–1, 5) and B(3, 1). (*b*) Find the size of the angle which AB makes with the positive direction of the x-axis.	2 2
Ans	(*a*) $y + x = 4$ (*b*) 135°	
2002W P2	**1.** The diagram shows a rhombus PQRS with its diagonals PR and QS. PR has equation $y = 2x - 2$. Q has coordinates (–2, 4). (*a*) (i) Find the equation of the diagonal QS. (ii) Find the coordinates of T, the point of intersection of PR and QS. (*b*) R is the point (5, 8). Write down the coordinates of P. 	6 2
Ans	(*a*) $2y + x = 6$, T(2, 2) (*b*) P(–1, –4)	
2002 P2	**1.** Triangle ABC has vertices A(–1, 6), B(–3, –2) and C(5, 2). Find (*a*) the equation of the line p, the median from C of triangle ABC. (*b*) the equation of the line q, the perpendicular bisector of BC. (*c*) the coordinates of the point of intersection of the lines p and q. 	3 4 1
Ans	(*a*) $y = 2$ (*b*) $y = -2x + 2$ (*c*) (0, 2)	

2001 P1	**1.** Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point $(2, -1)$.	3
Ans	$2x + 3y = 1$	
2001 P2	**7.** Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6). (*a*) Write down the equation of l_1, the perpendicular bisector of AB. (*b*) Find the equation of l_2, the perpendicular bisector of AC. (*c*) Find the point of intersection of lines l_1 and l_2. (*d*) Hence find the equation of the circle passing through A, B and C. 	1 4 1 2
Ans	(*a*) $x = 7$ (*b*) $3x + 2y = 23$ (*c*) $(7, 1)$ (*d*) $(x-7)^2 + (y-1)^2 = 26$	
2000 P1	**3.** Find the size of the angle $a°$ that the line joining the points A(0, –1) and B($3\sqrt{3}$, 2) makes with the positive direction of the x-axis. 	3
Ans	$30°$	
2000 P2	**2.** (*a*) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9). 	4

Ans	(*a*) $x + 2y = 9$	
Specimen 2 P1	**2.** A and B are the points (–3, –1) and (5, 5). Find the equation of the perpendicular bisector of AB. 	4
Ans	$m_{AB} = \frac{3}{4} \Rightarrow m_{perp} = -\frac{4}{3}$ midpoint $= (1,2)$ $y - 2 = -\frac{4}{3}(x-1)$	
Specimen 2 P1	**4.** The line AB makes an angle of $\frac{\pi}{3}$ radians with the y-axis, as shown in the diagram. Find the exact value of the gradient of AB. 	2
Ans	angle between line and x-axis $= \frac{\pi}{2} - \frac{\pi}{3}$ gradient $= \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$	
Specimen 1 P1	**1.** P(–4, 5), Q(–2, –2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. 	3
Ans	$y = -2x - 3$	
Specimen 1 P2	**1.** ABCD is a parallelogram. A, B and C have coordinates (2, 3), (4, 7) and (8, 11). Find the equation of DC.	3
Ans	$y = 2x - 5$	

2017 P1	**2.** The point P (–2, 1) lies on the circle $x^2+y^2-8x-6y-15=0$. Find the equation of the tangent to the circle at P.	4
Ans	$y=-3x-5$	
2017 P2	**3.** The line $y=3x$ intersects the circle with equation $(x-2)^2+(y-1)^2=25$.  Find the coordinates of the points of intersection.	5
Ans	(-1, -3), (2, 6)	
2017 P2	**10.** (a) Show that the points A(–7, –2), B(2, 1) and C(17, 6) are collinear.	3
	Three circles with centres A, B and C are drawn inside a circle with centre D as shown.  The circles with centres A, B and C have radii r_A, r_B and r_C respectively. • $r_A=\sqrt{10}$ • $r_B=2r_A$ • $r_C=r_A+r_B$ (b) Determine the equation of the circle with centre D.	4

Ans

(a)

Method 1

$m_{AB} = \frac{3}{9} = \frac{1}{3}$ see Note 1

$m_{BC} = \frac{5}{15} = \frac{1}{3}$

$\ldots \Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.

Method 2

$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ see Note 1

$\overrightarrow{BC} = \begin{pmatrix} 15 \\ 5 \end{pmatrix} \therefore \overrightarrow{AB} = \frac{3}{5}\overrightarrow{BC}$

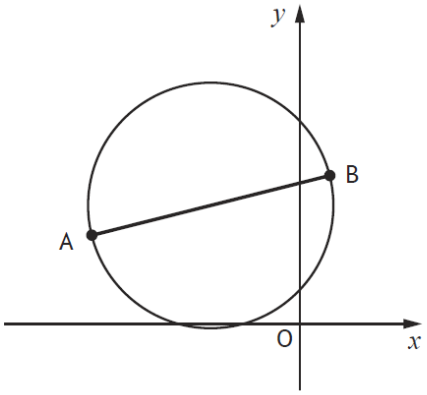
$\ldots \Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.

(b) $(x-8)^2 + (y-3)^2 = 360$

2016 P1

4. A and B are the points $(-7, 3)$ and $(1, 5)$.

AB is a diameter of a circle.



Find the equation of this circle. **3**

Ans $(x+3)^2 + (y-4)^2 = 17$

2016 P1

8. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact. **5**

Ans (2, 1)

2016 P2

4. Circles C_1 and C_2 have equations $(x+5)^2 + (y-6)^2 = 9$ and $x^2 + y^2 - 6x - 16 = 0$ respectively.

(a) Write down the centres and radii of C_1 and C_2. **4**

(b) Show that C_1 and C_2 do not intersect. **3**

Ans

(a) $C_1 = 3$, $C_2 = 5$

NH 2015 P2

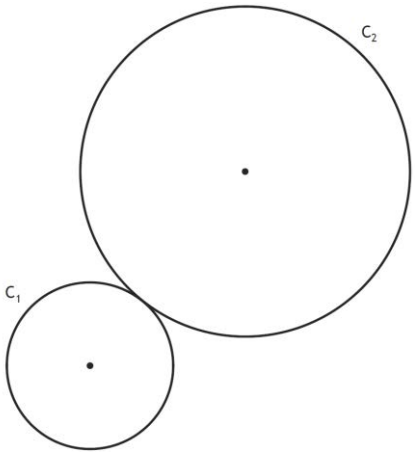
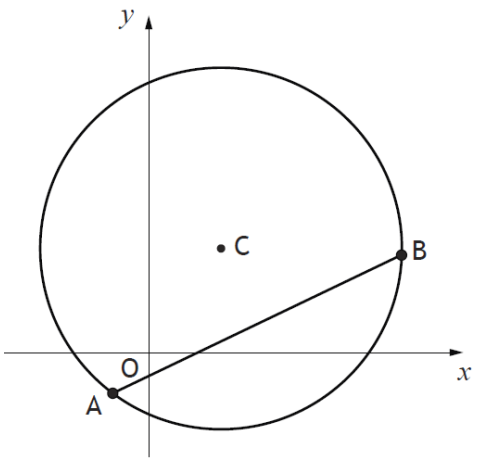
11. T$(-2, -5)$ lies on the circumference of the circle with equation

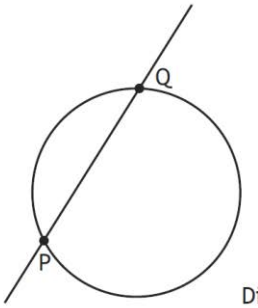
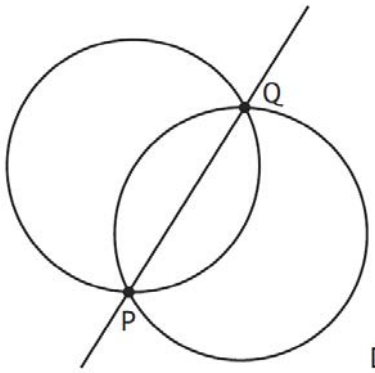
$$(x+8)^2 + (y+2)^2 = 45.$$

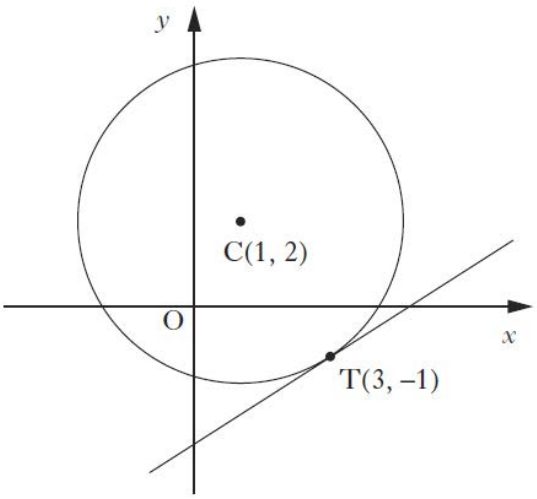
(a) Find the equation of the tangent to the circle passing through T. **4**

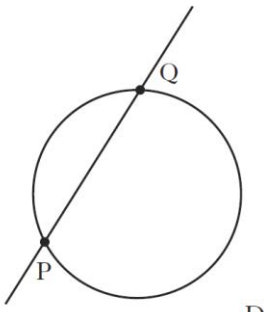
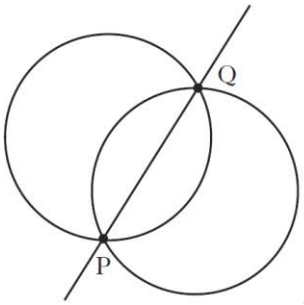
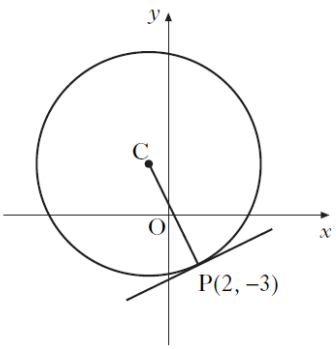
(b) This tangent is also a tangent to a parabola with equation $y = -2x^2 + px + 1 - p$, where $p > 3$.

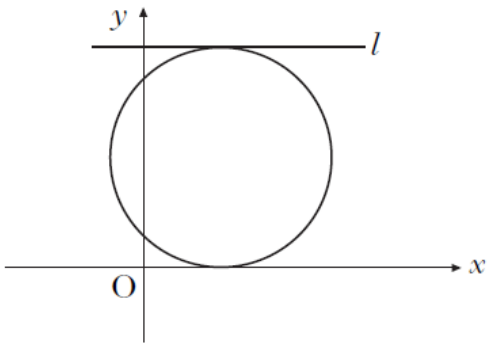
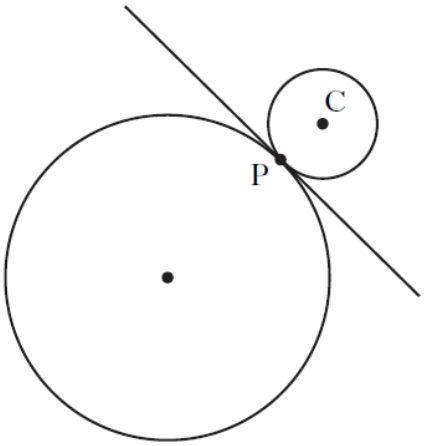
Determine the value of p. **6**

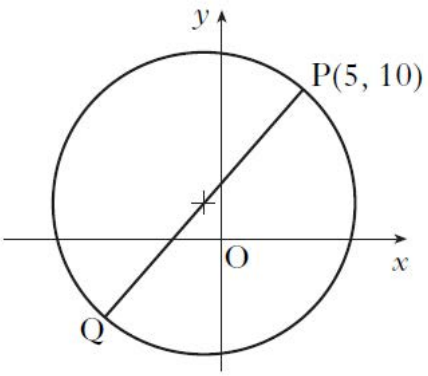
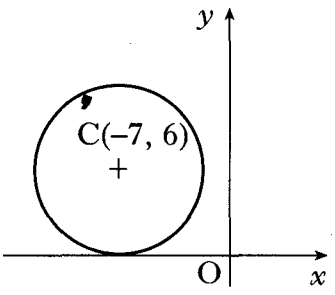
Ans	(a) $y = 2x - 1$ (b) $p = 10$	
NH 2015 P1	**14.** The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points. What is the value of k?	
Ans	If r = 6, k = 25. If r = $\sqrt{61}$, k = 0	
NH 2015 P1	**5.** Circle C_1 has equation $x^2 + y^2 + 6x + 10y + 9 = 0$. The centre of circle C_2 is $(9, 11)$. Circles C_1 and C_2 touch externally. (a) Determine the radius of C_2. A third circle, C_3, is drawn such that: • both C_1 and C_2 touch C_3 internally • the centres of C_1, C_2 and C_3 are collinear. (b) Determine the equation of C_3. 	4 4
Ans	(a) 15 (b) $(x-6)^2 + (y-7)^2 = 400$	
NH SP P2	**5.**  Points A(–1, –1) and B(7, 3) lie on the circumference of a circle with centre C, as shown in the diagram. (a) Find the equation of the perpendicular bisector of AB. CB is parallel to the x-axis. (b) Find the equation of the circle, passing through A and B, with centre C.	4 4
Ans	(a) $y - 1 = -2(x - 3)$ (b) $(x-2)^2 + (y-3)^2 = 25$	

NH EX P2	**2.** (a) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x-y+5=0$ intersecting the circle $x^2+y^2-6x-2y-30=0$ at the points P and Q.  Diagram 1 Find the coordinates of P and Q. (b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.  Diagram 2 Determine the equation of this second circle.	6 6
Ans	(a) P(-3, -1), Q(1, 7) (b) $(x+5)^2+(y-5)^2=40$	
2014 P2	**8.** Given that the equation $$x^2+y^2-2px-4py+3p+2=0$$ represents a circle, determine the range of values of p.	5
Ans	$p<-\frac{2}{5}$, p > 1	
2014 P1	**23.** (*a*) Find P and Q, the points of intersection of the line $y=3x-5$ and the circle C_1 with equation $x^2+y^2+2x-4y-15=0$. (*b*) T is the centre of C_1. Show that PT and QT are perpendicular. (*c*) A second circle C_2 passes through P, Q and T. Find the equation of C_2.	4 3 3

Ans	(a) (3, 4), (1, -2) (b) Proof (c) $(x-2)^2+(y-1)^2=10$	
2014 P1	**2.** The diagram shows a circle with centre C(1, 2) and the tangent at T(3, –1).  What is the gradient of this tangent?	
Ans	2/3	
2013 P1	**22.** A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$. (*a*) Write down the centre and calculate the radius of C_1. 2 (*b*) The point P(3, 2) lies on the circle C_1. 3 Find the equation of the tangent at P. (*c*) A second circle C_2 has centre (10, –1). The radius of C_2 is half of the radius of C_1. 3 Show that the equation of C_2 is $x^2 + y^2 - 20x + 2y + 93 = 0$. (*d*) Show that the tangent found in part (*b*) is also a tangent to circle C_2. 4	
Ans	(a) $\sqrt{32}$ (b) $y-2=-1(x-3)$ (c) Proof (d) Proof	

2012 P2	**2.** (*a*) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.  Diagram 1 Find the coordinates of P and Q. (*b*) Diagram 2 shows the circle from (*a*) and a second congruent circle, which also passes through P and Q.  Diagram 2 Determine the equation of this second circle.	6 6
Ans	(a) P(-3, -1), Q(1, 7) (b) $(x + 5)^2 + (y - 5)^2 = 40$	
2011 P1	**6.** The point P(2, –3) lies on the circle with centre C as shown. The gradient of CP is –2. What is the equation of the tangent at P?  	
Ans	$y + 3 = \frac{1}{2}(x - 2)$	
2011 P2	**7.** Circle C_1 has equation $(x + 1)^2 + (y - 1)^2 = 121$. A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1. The circles have no points of contact. What is the range of values of p?	**9**

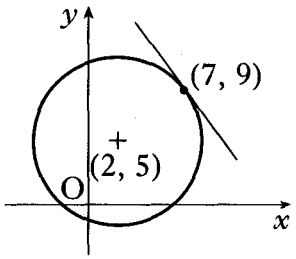
Ans	-23 < p < 13	
2010 P1	**8.** The equation of the circle shown in the diagram is $x^2 + y^2 - 6x - 10y + 9 = 0$. The x-axis and the line l are parallel tangents to the circle.  What is the equation of line l?	
Ans	y = 10	
2010 P2	**3.** (*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$. (ii) Find the coordinates of the point of contact, P.	**5**
	(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.  The line $y = 3 - x$ is a common tangent at the point P. The radius of the larger circle is three times the radius of the smaller circle. Find the equation of the smaller circle.	**6**
Ans	(a) (ii) P(-1, 4) (b) $(x - 1)^2 + (y - 6)^2 = 8$	

2009 P1	**2.** A circle has equation $x^2 + y^2 + 8x + 6y - 75 = 0$. What is the radius of this circle?	
Ans	10	
2009 P1	**9.** The line with equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 5$ at the points J and K. What are the x-coordinates of J and K?	
Ans	$x_J = 1$, $x_K = -1$	
2009 P2	**4.** (*a*) Show that the point P(5, 10) lies on circle C_1 with equation $(x + 1)^2 + (y - 2)^2 = 100$.	**1**
	(*b*) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. 	**5**
	(*c*) Two circles, C_2 and C_3, touch circle C_1 at Q. The radius of each of these circles is twice the radius of circle C_1. Find the equations of circles C_2 and C_3.	**4**
Ans	(b) $3x + 4y + 45 = 0$ (c) C_2: $(x - 5)^2 + (y - 10)^2 = 400$ C_3: $(x + 19)^2 + (y + 22)^2 = 400$	
2008 P1	**2.** The x-axis is a tangent to a circle with centre (–7, 6) as shown in the diagram.  What is the equation of the circle?	**2**
Ans	$(x + 7)^2 + (y - 6)^2 = 36$	

2008 P1

5. The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?



2

Ans

$y - 9 = -\frac{5}{4}(x - 7)$

2008 P2

4. (*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. 2

(*b*) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect. 4

(*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.

Find the coordinates of the points of intersection of the two circles. 5

Ans

(a) (-4,-2), $\sqrt{58}$ (b) d = $\sqrt{128}$ (c) (3,1), (-1,5)

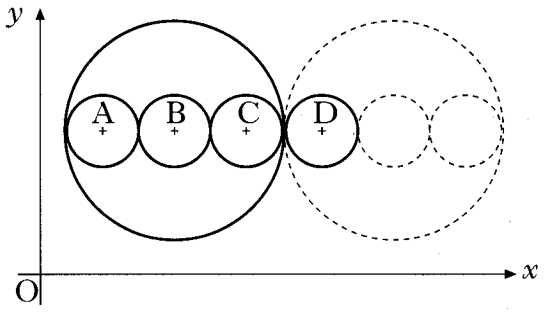
2007 P1

5. The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.



5

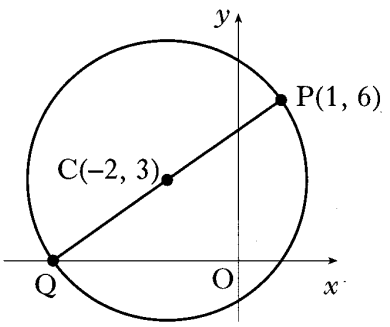
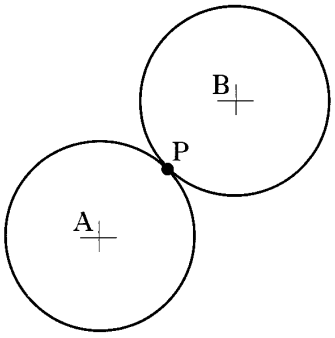
Ans

$(x - 15)^2 + (y - 8)^2 = 2^2$

2007 P2

3. Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle.

6

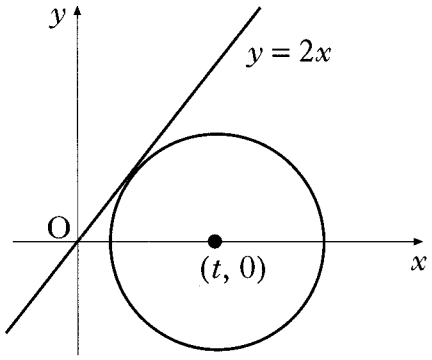
Ans	$x^2 + (6 - 2x)^2 + 6x - 4(6 - 2x) - 7 = 0$ $x^2 + 36 - 24x + 4x^2 + 6x - 24 + 8x - 7 = 0$ $5x^2 - 10x + 5 = 0$ $5(x^2 - 1) = 0$ Only one root so line is tangential to circle. Point of contact is (1,4).	
2006 P1	**2.** A circle has centre C(–2, 3) and passes through P(1, 6). (*a*) Find the equation of the circle. (*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q. 	2 4
Ans	(*a*) $r^2 = 18$ (*b*) $y - 0 = -(x - (-5))$	
2006 P2	**4.** The circles with equations $(x - 3)^2 + (y - 4)^2 = 25$ and $x^2 + y^2 - kx - 8y - 2k = 0$ have the same centre. Determine the radius of the larger circle.	5
Ans	$\sqrt{37}$, 5 and "2nd" circle	
2005 P1	**2.** Two congruent circles, with centres A and B, touch at P. Relative to suitable axes, their equations are $x^2 + y^2 + 6x + 4y - 12 = 0$ and $x^2 + y^2 - 6x - 12y + 20 = 0$. (*a*) Find the coordinates of P. (*b*) Find the length of AB. 	3 2
Ans	(*a*) P = (0,2) (*b*) AB = 10	

2005 P1

11. (*a*) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units.

Write down the equation of the circle. 1

(*b*) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle. 5



Ans

(*a*) $(x - t)^2 + (y - 0)^2 = 2^2$

(*b*) $t = \sqrt{5}$

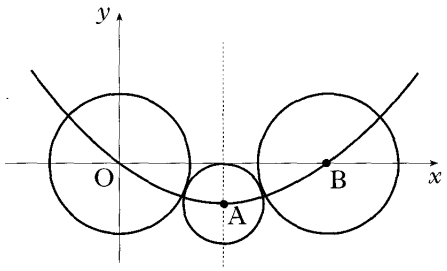
2003 P1

11. • O, A and B are the centres of the three circles shown in the diagram below.

• The two outer circles are congruent and each touches the smallest circle.

• Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.

• The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(*a*) (i) State the coordinates of A and find the length of the line OA. 2

(ii) Hence find the equation of the circle with centre B. 3

(*b*) The equation of the parabola can be written in the form $y = px(x + q)$.

Find the values of p and q. 2

Ans

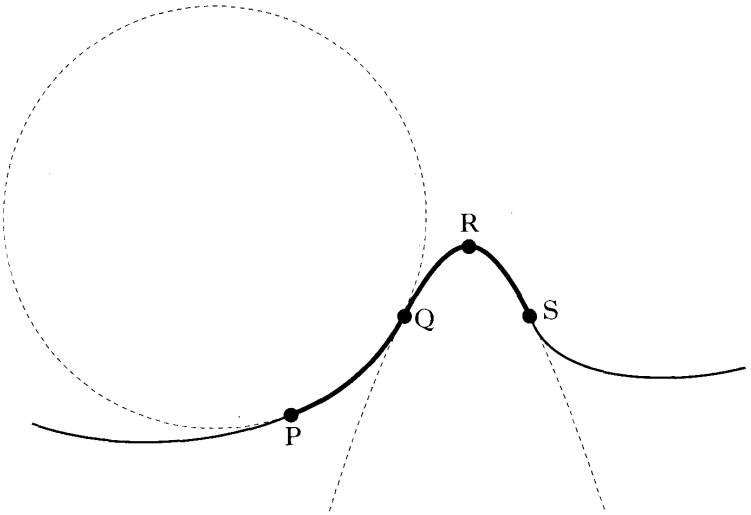
(*a*) (i) A(12, –5), OA = 13

(ii) $(x - 24)^2 + y^2 = 64$

(*b*) $p = \frac{5}{144}$, $q = -24$

2002W P1

6. The side view of part of a roller coaster ride is shown by the path PQRS.
The curve PQ is an arc of the circle with equation $x^2 + y^2 + 4x - 10y + 9 = 0$.
The curve QRS is part of the parabola with equation $y = -x^2 + 6x - 5$.
The point Q has coordinates (2, 3).



(*a*) Find the equation of the tangent to the circle at Q. — 4

(*b*) Show that this tangent to the circle at Q is also the tangent to the parabola at Q. — 2

Ans

(*a*) $y - 2x = -1$

(*b*) proof

For parabola

- $\frac{dy}{dx} = -2x + 6$
- $m = 2$

(same as gradient of tangent to circle)

2002W P2

10. The line $y + 2x = k$, $k > 0$, is a tangent to the circle $x^2 + y^2 - 2x - 4 = 0$.

(*a*) Find the value of k. — 7

(*b*) Deduce the coordinates of the point of contact. — 2

Ans

(*a*) $x^2 + (k - 2x)^2 - 2x - 4 = 0$ and apply discriminant
$k=7$

(*b*) (3, 1)

2002 P1

1. The point P(2, 3) lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P. — 4

Ans

$2y + 3x = 12$

2001 P1

11. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

(*a*) (i) Show that the radius of circle P is $4\sqrt{2}$. — 4

(ii) Hence show that circles P and Q touch.

(*b*) Find the equation of the tangent to circle Q at the point $(-4, 1)$. — 3

(*c*) The tangent in (*b*) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$. — 3

Ans

(*a*) $r_P = 4\sqrt{2}$

$r_P + r_Q = 6\sqrt{2}$

$C_P = (4,5)$

$C_P C_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ and "so touch"

(*b*) $y = x + 5$

(*c*) $x = 2 \pm 2\sqrt{3}$

2000 P1

6. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? — 5

Ans

for all k

2000 P2

2. (*a*) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9). — 4

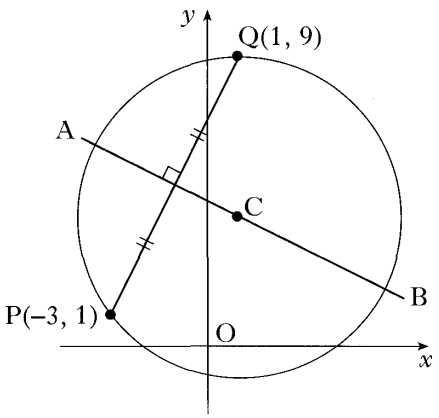
(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. — 3

(*c*) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T. — 2



Ans

(*a*) $x + 2y = 9$

(*b*) $(x-1)^2 + (y-4)^2 = 25$

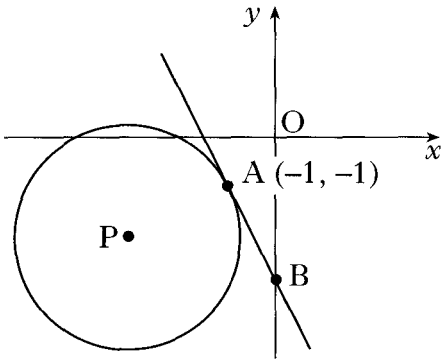
(*c*) (i) $y = 9$

(ii) T = (–9, 9)

Specimen 2 P1

5. (*a*) The diagram shows a circle, centre P, with equation $x^2 + y^2 + 6x + 4y + 8 = 0$.

Find the equation of the tangent at the point A (–1, –1) on the circle.



4

(*b*) The tangent crosses the *y*-axis at B. Find the equation of the circle with AB as diameter.

3

Ans

(*a*) P = (–3,–2)

$m_{PA} = \frac{1}{2} \Rightarrow m_{tgt} = -2$

$y - 1 = -2(x - (-1))$

(*b*) B = (0,3)

centre C = $\text{mid}_{AB} = (-\frac{1}{2}, -2)$

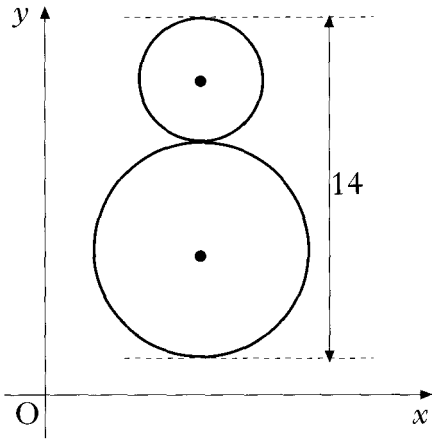
radius^2 = $CB^2 = \frac{5}{4}$

$(x + \frac{1}{2})^2 + (y + 2)^2 = \frac{5}{4}$

Specimen 1 P1

6. A bakery firm makes ginger-bread men each 14 cm high with a circular "head" and "body". The equation of the "body" is $x^2 + y^2 - 10x - 12y + 45 = 0$ and the line of centres is parallel to the *y*-axis.

Find the equation of the "head".



5

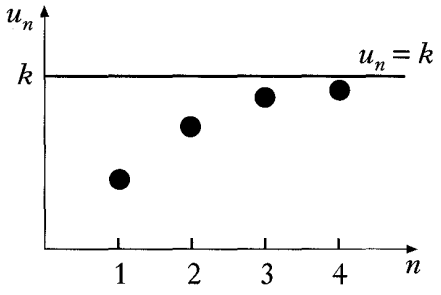
Ans

$(x - 5)^2 + (y - 13)^2 = 9$

2017 P1	**9.** A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant. (a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. (b) (i) Explain why this sequence approaches a limit as $n \to \infty$. (ii) Calculate this limit.	2 1 2
Ans	(a) m = ¼ (b) (i) a limit exists as the recurrence relation is linear and $-1 < \frac{1}{4} < 1$ (ii) 8	
2017 P2	**8.** Sequences may be generated by recurrence relations of the form $u_{n+1} = ku_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$. (a) Show that $u_2 = 5k^2 - 20k - 20$. (b) Determine the range of values of k for which $u_2 < u_0$.	2 4
Ans	(a) Proof (b) $-1 < k < 5$ with eg sketch or table of signs	
2016 P1	**3.** A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_3 = 6$. (a) Find the value of u_4. (b) Explain why this sequence approaches a limit as $n \to \infty$. (c) Calculate this limit.	1 1 2
Ans	(a) $u_4 = 12$ (b) A limit exists as the recurrence relation is linear and $-1 < \frac{1}{3} < 1$ (c) $l = 15$	
NH 2015 P2	**3.** A version of the following problem first appeared in print in the 16th Century. A frog and a toad fall to the bottom of a well that is 50 feet deep. Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well. The toad climbs 13 feet each day before resting. Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well. Their progress can be modelled by the recurrence relations: • $f_{n+1} = \frac{1}{3}f_n + 32, \quad f_1 = 32$ • $t_{n+1} = \frac{3}{4}t_n + 13, \quad t_1 = 13$ where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in. (a) Calculate t_2, the height of the toad at the end of the second day. (b) Determine whether or not either of them will eventually escape from the well.	1 5

Ans	(a) 22.75 (b) $52 > 50$ $\therefore$ toad will escape	
NH SP P2	**2.** A wildlife reserve has introduced conservation measures to build up the population of an endangered mammal. Initially the reserve population of the mammal was 2000. By the end of the first year there were 2500 and by the end of the second year there were 2980. It is believed that the population can be modelled by the recurrence relation: $u_{n+1} = au_n + b$, where a and b are constants and n is the number of years since the reserve was set up. (a) Use the information above to find the values of a and b. (b) Conservation measures will end if the population stabilises at over 13 000. Will this happen? Justify your answer.	4 3
Ans	(a) a = 0.96, b = 580 (b) stabilizes at 14500	
NH EX P2	**1.** A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$. (a) Determine the values of u_1 and u_2. (b) A second sequence is given by 4, 5, 7, 11, ... It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$. Find the values of p and q. (c) Either the sequence in (a) or the sequence in (b) has a limit. (i) Calculate this limit. (ii) Why does this other sequence not have a limit?	1 3 3
Ans	(a) $u_1 = 8, u_2 = -4$ (b) p = 2 or q = -3 (c) (i) $l = 0, \quad -1 < p < 1$ (ii) outside interval $-1 < p < 1$	
2014 P1	**10.** A sequence is defined by the recurrence relation $u_{n+1} = (k-2)u_n + 5$ with $u_0 = 3$. For what values of k does this sequence have a limit as $n \to \infty$?	
Ans	$1 < k < 3$	
2014 P1	**1.** A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 1$, with $u_2 = 15$. What is the value of u_4?	

Ans	3	
2013 P2	**1.** The first three terms of a sequence are 4, 7 and 16. The sequence is generated by the recurrence relation $u_{n+1} = mu_n + c$, with $u_1 = 4$. Find the values of m and c.	4
Ans	$m = 3$, $c = -5$	
2012 P1	**1.** A sequence is defined by the recurrence relation $u_{n+1} = 3u_n + 4$, with $u_0 = 1$. Find the value of u_2.	
Ans	25	
2011 P2	**3.** (*a*) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$. Write down the values of u_1 and u_2. **1** (*b*) A second sequence is given by 4, 5, 7, 11, ... It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$. Find the values of p and q. **3** (*c*) Either the sequence in (*a*) or the sequence in (*b*) has a limit. (i) Calculate this limit. (ii) Why does the other sequence not have a limit? **3**	
Ans	(a) $u_1 = 8$, $u_2 = -4$ (b) $p = 2$, $q = 3$ (c) (i) $L = 0$ (ii) $a = 2$, $2 > 1$	
2010 P1	**2.** A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 3$ and $u_0 = 1$. What is the value of u_2?	
Ans	13	
2010 P1	**7.** A sequence is generated by the recurrence relation $u_{n+1} = \frac{1}{4}u_n + 7$, with $u_0 = -2$. What is the limit of this sequence as $n \to \infty$?	
Ans	28/3	

2009 P1	**1.** A sequence is defined by $u_{n+1} = 3u_n + 4$ with $u_1 = 2$. What is the value of u_3?	
Ans	34	
2009 P1	**6.** A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}7u_n + 10$. What is the limit of this sequence as $n \to \infty$?	
Ans	100/3	
2008 P1	**4.** A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n - 240$. What is the limit of this sequence as $n \to \infty$?	2
Ans	-400	
2008 P1	**1.** A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}3u_n + 6$ with $u_{10} = 10$. What is the value of u_{12}?	2
Ans	8.7	
2007 P1	**7.** A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{4}u_n + 16,\ u_0 = 0$. (*a*) Calculate the values of u_1, u_2 and u_3. Four terms of this sequence, u_1, u_2, u_3 and u_4 are plotted as shown in the graph. As $n \to \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence. (*b*) (i) Give a reason why this sequence has a limit. (ii) Find the exact value of k. 	3 3
Ans	(*a*) $u_1 = 16$ $u_2 = 20$ $u_3 = 21$ (*b*) (i) $-1 < \frac{1}{4} < 1$ (ii) $\frac{64}{3}$	

2006 P1	**4.** A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}8u_n + 12$, $u_0 = 4$. (*a*) State why this sequence has a limit. (*b*) Find this limit.	 1 2
Ans	(*a*) sequence has limit since $-1 < 0{\cdot}8 < 1$ (*b*) limit = 60	
2005 P1	**6.** (*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. (*b*) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$. (i) Express u_1 and u_2 in terms of m. (ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit.	2 5
Ans	(*a*) $k = -\frac{1}{4}$ (*b*) (i) $u_1 = 3m + 5$, $u_2 = m(3m + 5) + 5$ (ii) $m = -2$	
2004 P2	**4.** A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$. (*a*) Write down the condition on k for this sequence to have a limit. (*b*) The sequence tends to a limit of 5 as $n \to \infty$. Determine the value of k.	 1 3
Ans	(*a*) $-1 < k < 1$ (*b*) $k = \frac{2}{5}$	
2003 P1	**4.** A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$. (*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. (*b*) Find the limit of this recurrence relation as $n \to \infty$.	 2 2
Ans	(a) p = 1/3, q = 11 (*b*) 16.5	
2002W P2	**3.** (*a*) Calculate the limit as n $\to \infty$ of the sequence defined by $u_{n+1} = 0{\cdot}9u_n + 10$, $u_0 = 1$. (*b*) Determine the least value of n for which u_n is greater than half of this limit and the corresponding value of u_n.	3 2
Ans	(*a*) 100 (*b*) $n = 7$, $u_7 = 52{\cdot}65$	

2002 P2

4. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0·5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.

(*a*) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run? 3

(*b*) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition? 3

Ans

(*a*) $-1 < 0{\cdot}8 < 1$ and
limit = 2·5 metres

(*b*) trim 25%

2001 P2

3. On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1·5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

(*a*) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

Let u_n and u_{n+1} represent the amounts that he owes at the starts of two successive months. Write down a recurrence relation involving u_{n+1} and u_n. 2

(*b*) Find the date and the amount of the final payment. 4

Ans

(*a*) $u_{n+1} = 1{\cdot}015u_n - 300$, $u_0 = 2500$

(*b*) Dec 1st, £290·68

Specimen 2 P2

4. Two sequences are defined by the recurrence relations

$$u_{n+1} = 0{\cdot}2u_n + p, \quad u_0 = 1 \quad \text{and}$$
$$v_{n+1} = 0{\cdot}6v_n + q, \quad v_0 = 1.$$

(*a*) Explain why each of these sequences has a limit.

(*b*) If both sequences have the same limit, express p in terms of q. 3

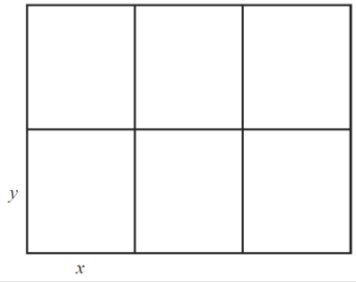
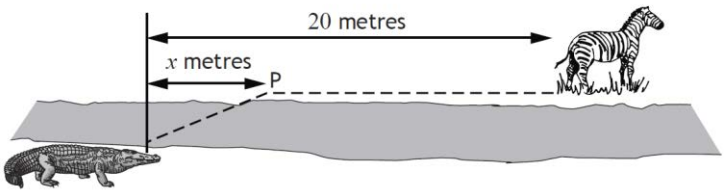
Ans

(*a*) $-1 < 0{\cdot}2 < 1$ and $-1 < 0{\cdot}6 < 1$

(*b*)
Limit $= \frac{p}{0{\cdot}8}$ and Limit $= \frac{q}{0{\cdot}4} \Rightarrow p = 2q$

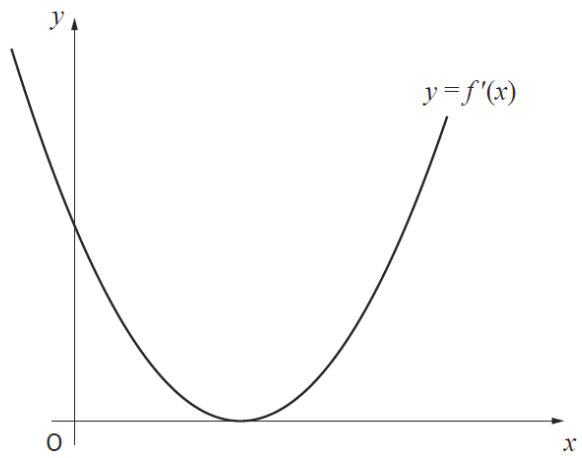
Specimen 1 P1	**2.** A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 3u_n + 5$ with first term u_1. (*a*) Explain why this sequence has a limit as n tends to infinity. (*b*) Find the **exact** value of this limit.	1 2
Ans	(*a*) $-1 < 0 \cdot 3 < 1$ (*b*) $\frac{50}{7}$	

2017 P1	**3.** Given $y=(4x-1)^{12}$, find $\frac{dy}{dx}$.	2
Ans	$\frac{dy}{dx}=48(4x-1)^{11}$	
2017 P1	**8.** Calculate the rate of change of $d(t)=\frac{1}{2t}$, $t\neq 0$, when $t=5$.	3
Ans	-1/50	
2017 P2	**4.** (a) Express $3x^2+24x+50$ in the form $a(x+b)^2+c$. (b) Given that $f(x)=x^3+12x^2+50x-11$, find $f'(x)$. (c) Hence, or otherwise, explain why the curve with equation $y=f(x)$ is strictly increasing for all values of x.	3 2 2
Ans	(a) $3(x+4)^2+2$ (b) $f'(x)=3x^2+24x+50$ (c) $(x+4)^2\geq 0\ \forall x$, $\therefore 3(x+4)^2+2>0 \Rightarrow$ always, strictly increasing	
2017 P2	**7.** (a) Find the x-coordinate of the stationary point on the curve with equation $y=6x-2\sqrt{x^3}$. (b) Hence, determine the greatest and least values of y in the interval $1\leq x\leq 9$.	4 3
Ans	(a) x = 2.24 (b) greatest y = 8, least y = 0	
2016 P1	**2.** Given that $y=12x^3+8\sqrt{x}$, where $x>0$, find $\frac{dy}{dx}$.	3
Ans	$\frac{dy}{dx}=4x^{-\frac{1}{2}}$	
2016 P1	**9.** (a) Find the x-coordinates of the stationary points on the graph with equation $y=f(x)$, where $f(x)=x^3+3x^2-24x$. (b) Hence determine the range of values of x for which the function f is strictly increasing.	4 2
Ans	(a) x = -4, x = 2 (b) x < -4, x > 2	
NH 2015 P1	**2.** Find the equation of the tangent to the curve $y=2x^3+3$ at the point where $x=-2$.	4
Ans	y = 24x + 35	
NH 2015 P1	**7.** A function f is defined on a suitable domain by $f(x)=\sqrt{x}\left(3x-\frac{2}{x\sqrt{x}}\right)$. Find $f'(4)$.	4

Ans	$9\frac{1}{8}$	
2016 P2	**7.** A council is setting aside an area of land to create six fenced plots where local residents can grow their own food. Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.  (a) The area of land being set aside is 108 m^2. Show that the total length of fencing, L metres, is given by $$L(x) = 9x + \frac{144}{x}.$$ (b) Find the value of x that minimises the length of fencing required.	3 6
Ans	Min at x = 4	
NH 2015 P2	**8.** A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river. Crocodiles travel at different speeds on land and in water. The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.  The time taken, T, measured in tenths of a second, is given by $$T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$$ (a) (i) Calculate the time taken if the crocodile does not travel on land. (ii) Calculate the time taken if the crocodile swims the shortest distance possible. (b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time.	1 1 8
Ans	(a) (i) 10.4 (ii) 11 (b) 9.8 seconds	

NJ SP P1

11. The diagram shows the graph of $y = f'(x)$. The x-axis is a tangent to this graph.

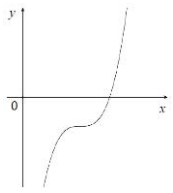


(a) Explain why the function $f(x)$ is never decreasing. 1

(b) On a graph of $y = f(x)$, the y-coordinate of the stationary point is negative. Sketch a possible graph for $y = f(x)$. 2

Ans

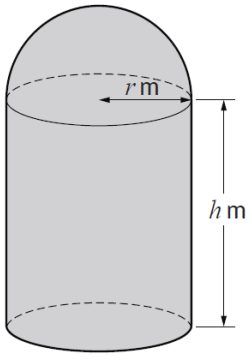
(a) $m = f'(x) \geq 0$ stated explicitly (b)



NH SP P2

8. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

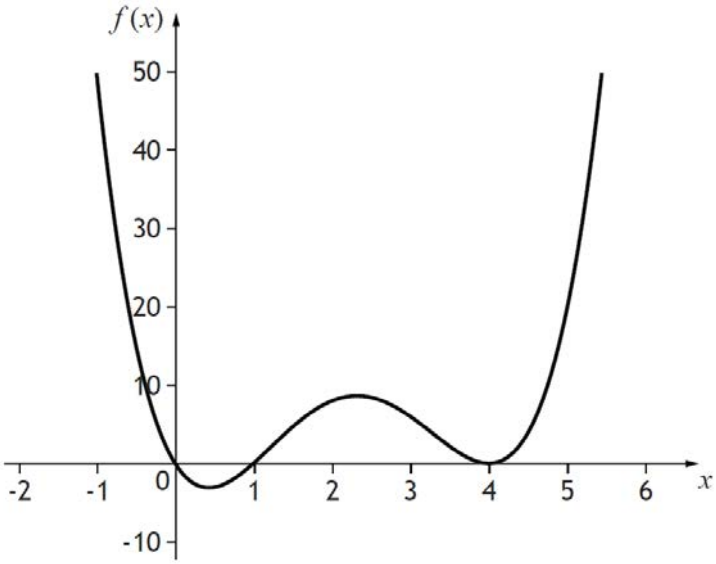
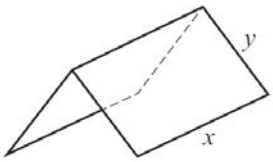
The volume of the **cylindrical** part of the container needs to be 100 cubic metres.

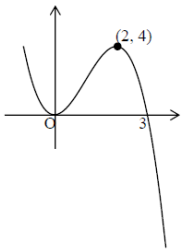
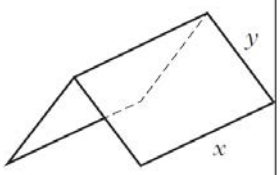


(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by: 3

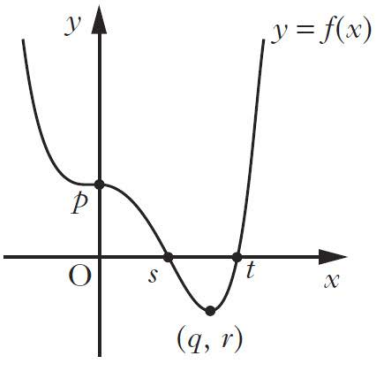
$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$

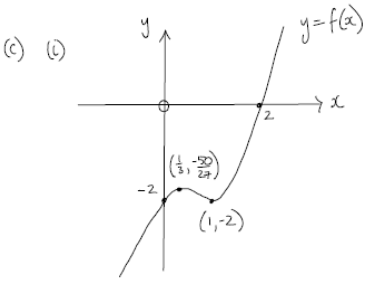
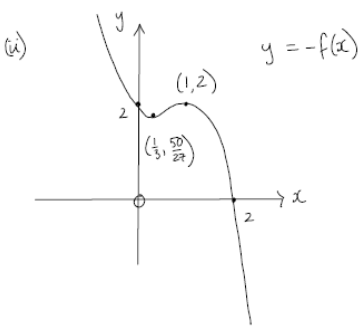
(b) Determine the value of r which minimises the amount of metal needed to build the container. 6

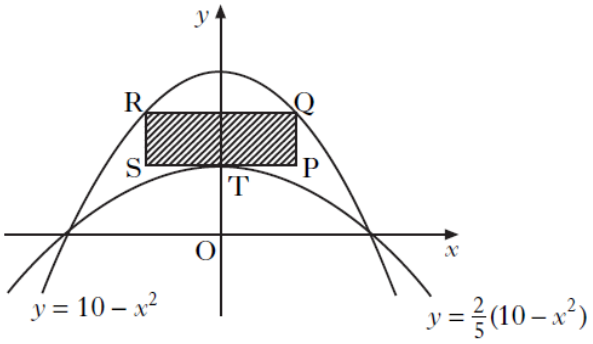
Ans	(b) r = 2.20m	
NH SP P2	**3.** The diagram shows the graph of $f(x) = x(x-p)(x-q)^2$.  (a) Determine the values of p and q. (b) Find the equation of the tangent to the curve when $x = 1$.	1 4
Ans	(a) p = 1, q = 4 (b) y = 9(x – 1)	
NH EX P2	**9.** A manufacturer is asked to design an open-ended shelter, as shown:  The frame of the shelter is to be made of rods of two different lengths: • x metres for top and bottom edges; • y metres for each sloping edge. The total length, L metres, of the rods used in a shelter is given by: $$L = 3x + \frac{48}{x}$$ To minimise production costs, the total length of rods used for a frame should be as small as possible. (a) Find the value of x for which L is a minimum. The rods used for the frame cost £8·25 per metre. The manufacturer claims that the minimum cost of a frame is less than £195. (b) Is this claim correct? Justify your answer.	5 2

Ans	(a) 4cm (b) No, (£198 > £195)	
2014 P2	**2.** A curve has equation $y = x^4 - 2x^3 + 5$. Find the equation of the tangent to this curve at the point where $x = 2$.	4
Ans	$y - 5 = 8(x - 2)$	
2014 P1	**21.** A curve has equation $y = 3x^2 - x^3$. (*a*) Find the coordinates of the stationary points on this curve and determine their nature. (*b*) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve.	6 2
Ans	(a) Min at (0, 0), Max at (2, 4) (b)  	
2013 P2	**7.** A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions. Condition 1 The frame of a shelter is to be made of rods of two different lengths: • x metres for top and bottom edges; • y metres for each sloping edge.  Condition 2 The frame is to be covered by a rectangular sheet of material. The total area of the sheet is 24 m^2. (*a*) Show that the total length, L metres, of the rods used in a shelter is given by $$L = 3x + \frac{48}{x}.$$ (*b*) These rods cost £8·25 per metre. To minimise production costs, the total length of rods used for a frame should be as small as possible. (i) Find the value of x for which L is a minimum. (ii) Calculate the minimum cost of a frame.	3 7
Ans	(b) (i) $x = 4$ (ii) £198	

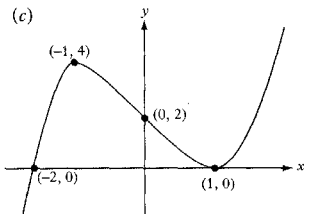
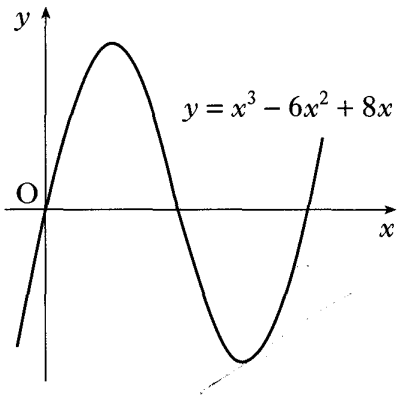
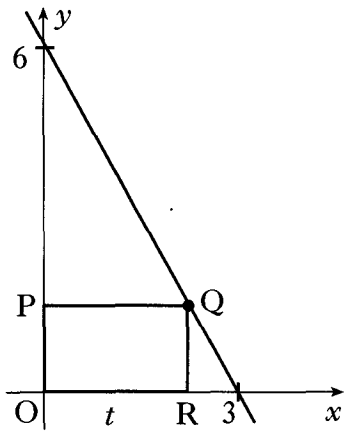
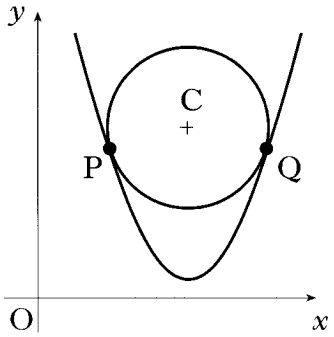
2013 P2	**3.** (*a*) Given that $(x-1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully. (*b*) Show that the curve with equation $y = x^4 + 4x^3 + 2x^2 - 20x + 3$ has only one stationary point. Find the x-coordinate and determine the nature of this point.	5
Ans	Min T.P at $x = 1$	
2013 P1	**2.** The point P (5, 12) lies on the curve with equation $y = x^2 - 4x + 7$. What is the gradient of the tangent to this curve at P?	
Ans	6	
2012 P1	**8.** The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$. What is the rate of change of V with respect to r, at $r = 2$?	
Ans	16π	
2012 P1	**2.** What is the gradient of the tangent to the curve with equation $y = x^3 - 6x + 1$ at the point where $x = -2$?	
Ans	6	
2012 P1	**6.** If $y = 3x^{-2} + 2x^{\frac{3}{2}}$, $x > 0$, determine $\frac{dy}{dx}$.	
Ans	$-6x^{-3} + 3x^{\frac{1}{2}}$	
2012 P1	**18.** The graph of $y = f(x)$ shown has stationary points at $(0, p)$ and (q, r). Here are two statements about $f(x)$: (1) $f(x) < 0$ for $s < x < t$; (2) $f'(x) < 0$ for $x < q$. Which of the following is true? A Neither statement is correct. B Only statement (1) is correct. C Only statement (2) is correct. D Both statements are correct.	

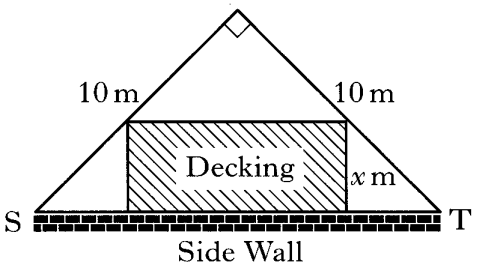
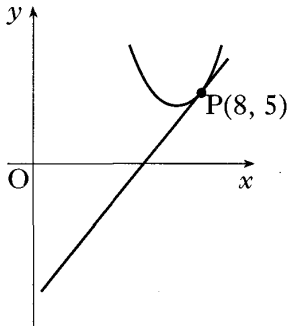


Ans	B	
2012 P2	**3.** A function f is defined on the domain $0 \le x \le 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$. Determine the maximum and minimum values of f.	
Ans	Max = 6 Min = -2	
2011 P1	**4.** A tangent to the curve with equation $y = x^3 - 2x$ is drawn at the point $(2, 4)$. What is the gradient of this tangent?	**2**
Ans	10	
2011 P1	**22.** A function f is defined on the set of real numbers by $f(x) = (x - 2)(x^2 + 1)$. (*a*) Find where the graph of $y = f(x)$ cuts: (i) the x-axis; (ii) the y-axis. (*b*) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. (*c*) On separate diagrams sketch the graphs of: (i) $y = f(x)$; (ii) $y = -f(x)$.	**2** **8** **3**
Ans	(a) (i) (2,0) (ii) (0,-2) (b) Max tp at (1/3, -50/27) Min tp at (1, -2) (c) (i)  (ii) 	

2010 P1	**15.** The derivative of a function f is given by $f'(x) = x^2 - 9$. Here are two statements about f: (1) f is increasing at $x = 1$; (2) f is stationary at $x = -3$. Which of the following is true? A Neither statement is correct. B Only statement (1) is correct. C Only statement (2) is correct. D Both statements are correct.	**2**
Ans	C	
2010 P1	**17.** If $s(t) = t^2 - 5t + 8$, what is the rate of change of s with respect to t when $t = 3$?	**2**
Ans	1	
2010 P2	**5.** The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.  A rectangle PQRS is placed between the two parabolas as shown, so that: • Q and R lie on the upper parabola; • RQ and SP are parallel to the x-axis; • T, the turning point of the lower parabola, lies on SP. (*a*) (i) If TP = x units, find an expression for the length of PQ. (ii) Hence show that the area, A, of rectangle PQRS is given by $$A(x) = 12x - 2x^3.$$ (*b*) Find the maximum area of this rectangle.	3 6
Ans	(a) (i) $6 - x^2$ (b) Area = 11.3 units^2	

2009 P1	**4.** A curve has equation $y = 5x^3 - 12x$. What is the gradient of the tangent at the point $(1, -7)$?	**2**
Ans	3	
2009 P1	**8.** What is the derivative of $\frac{1}{4x^3}$, $x \neq 0$?	**2**
Ans	$-\frac{3}{4x^4}$	
2009 P1	**20.** $A = 2\pi r^2 + 6\pi r$. What is the rate of change of A with respect to r when $r = 2$?	**2**
Ans	14π	
2009 P2	**1.** Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature.	**8**
Ans	Max TP (-1, 17) Min TP (3, -15)	
2009 P2	**2.** Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$. (a) (i) Find $p(x)$ where $p(x) = f(g(x))$. (ii) Find $q(x)$ where $q(x) = g(f(x))$. (b) Solve $p'(x) = q'(x)$.	**3** **3**
Ans	(a) (i) $3x^2 - 5$ (ii) $9x^2 + 6x - 1$ (b) $x = -\frac{1}{2}$	
2008 P1	**21.** A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$. (a) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. (b) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$. (ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. (c) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve.	6 5 4

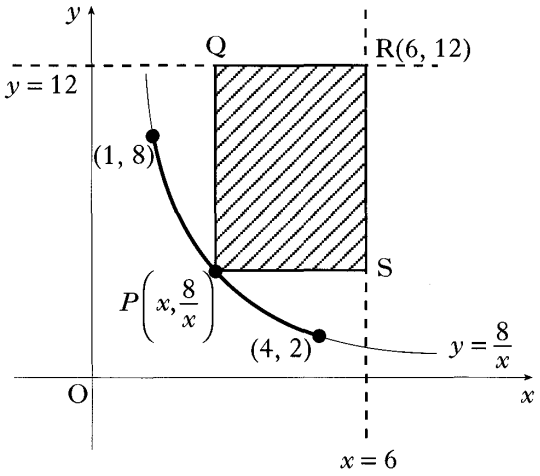
Ans	(*a*) (–1, 4) maximum (1, 0) minimum (*b*) (i) $x = 1$, $f(x) = 0$ so $(x - 1)$ is a factor (ii) $(x - 1)(x - 1)(x + 2)$ (*c*) 	
2008 P1	**22.** The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$. (*a*) Find the coordinates of the points on the curve where the gradient of the tangent is –1. (*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. 	5 2
Ans	(a) (1,3), (3,-3) (b) (1,3)	
2008 P2	**6.** In the diagram, Q lies on the line joining (0, 6) and (3, 0). OPQR is a rectangle, where P and R lie on the axes and OR = t. (*a*) Show that QR = $6 - 2t$. (*b*) Find the coordinates of Q for which the rectangle has a maximum area. 	3 6
Ans	(a) proof (b) (1.5,3)	
2007 P2	**5.** A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q. (*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. (*b*) Find the coordinates of P. (*c*) Find the coordinates of C, the centre of the circle. 	5 2 2

Ans	(*a*) Q = (12,10) (*b*) P = (4,10) (*c*) C = (8,11)	
2007 P2	**6.** A householder has a garden in the shape of a right-angled isosceles triangle. It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.  (*a*) (i) Find the exact value of ST. (ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by $A = \left(10\sqrt{2}\right)x - 2x^2$. (*b*) Find the dimensions of the decking which maximises its area.	3 5
Ans	(*a*) (i) ST = $\sqrt{200}$ (ii) Length of decking = $\sqrt{200} - 2x$ So A = $x(\sqrt{200} - 2x)$ = $(10\sqrt{2}) - 2x^2$ (*b*) $x = \frac{10\sqrt{2}}{4}$ length = $5\sqrt{2}$	
2006 P2	**3.** The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5). (*a*) Find the equation of this tangent. 	4
Ans	(*a*) $y - 5 = 2(x - 8)$	

2006 P2

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by $A = 80 - 12x - \frac{48}{x}$. 3

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. 8

Ans

(*a*) (i) PS = $6 - x$

RS = $12 - \frac{8}{x}$

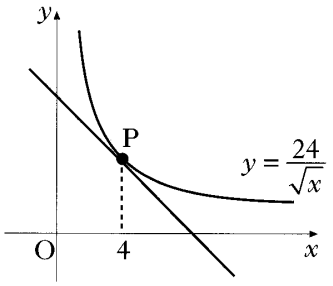
(ii) Area = $(6 - x)\left(12 - \frac{8}{x}\right)$ and complete

(*b*) max.A = 32 at $x = 2$ **and** min.A = 20 at $x = 1$ **or** $x = 4$

2005 P2

6. The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$.



6

Ans

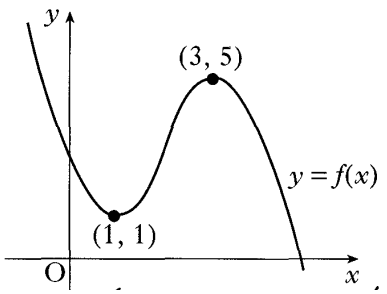
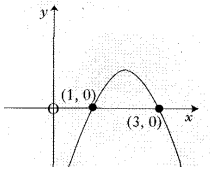
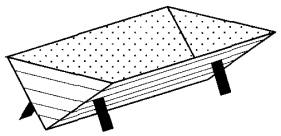
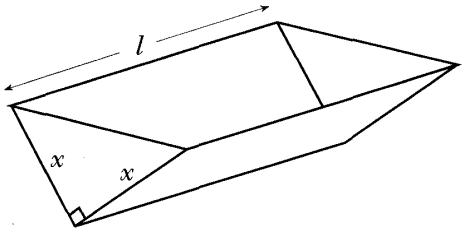
$y - 12 = -\frac{3}{2}(x - 4)$

2004 P2

5. The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

(*a*) Find the value of x for which the gradient of the tangent at P is 12. 5

(*b*) Hence find the equation of the tangent at P. 2

Ans	(a) $x = 2$ (b) $y = 12x - 8$	
2004 P2	**7.** The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1, 1) and (3, 5). Sketch the graph of $y = f'(x)$. 	3
Ans		
2003 P1	**5.** Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$.	5
Ans	$\frac{3}{16}$	
2003 P2	**4.** (a) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$.	5
Ans	(a) $y = 4x - 2$	
2003 P2	**8.** An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.  The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.  (a) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. (b) Find the value of x which minimises this surface area.	3 5

Ans

(*a*) length = $\frac{108000}{\frac{1}{2}x^2}$

$SA = 2 \times \frac{1}{2}x^2 + 2x \times \text{length}$

$SA = x^2 + \frac{432000}{x}$

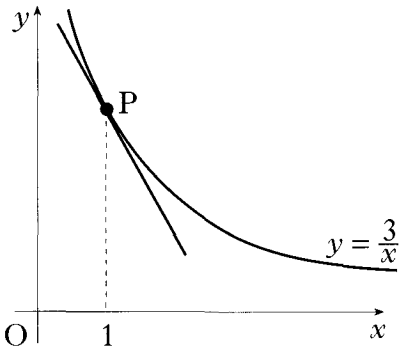
$\frac{dA}{dx} = 2x - \frac{432000}{x^2}$

$\frac{dA}{dx} = 0$ Justify minimum using, e.g. nature table

$x = 60$ (*b*) 60

2002W P1

3. Find the equation of the tangent to the curve with equation $y = \frac{3}{x}$ at the point P where $x = 1$.



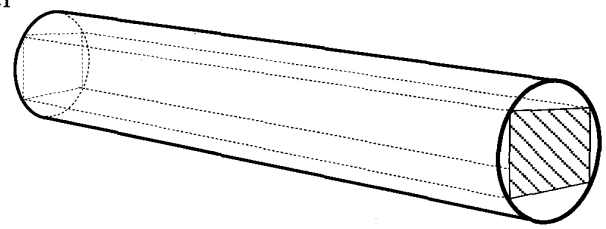
5

Ans

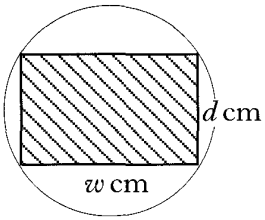
$y + 3x = 6$

2002W P2

7. A rectangular beam is to be cut from a cylindrical log of diameter 20 cm.



The diagram shows a cross-section of the log and beam where the beam has a breadth of w cm and a depth of d cm.



The strength S of the beam is given by

$S = 1{\cdot}7\ w\ (400 - w^2)$.

Find the dimensions of the beam for maximum strength.

5

Ans

Solve $\frac{dS}{dw} = 0$ and test for max/min

$d = 20\sqrt{\frac{2}{3}}$

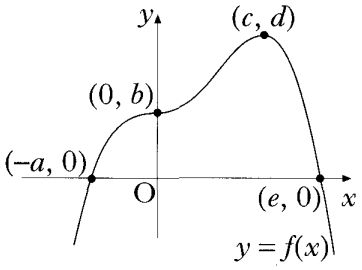
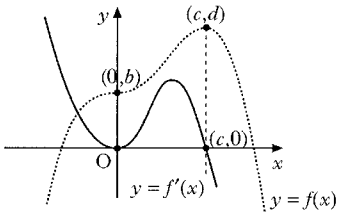
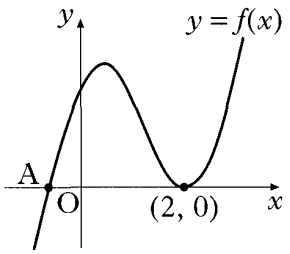
2002 P1

4. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis.

4

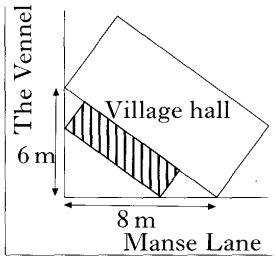
Ans

(2, 4)

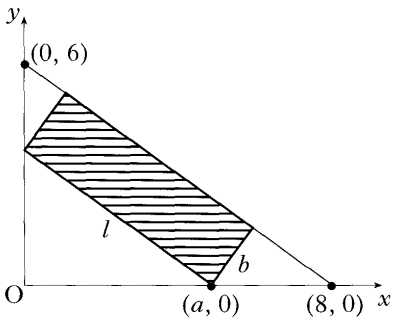
2002 P1	**6.** The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown. There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d). Sketch the graph of the derived function f'. 	3
Ans		
2002 P2	**3.** The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$. (*a*) Find the x-coordinate of the maximum turning point. (*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. (*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. 	5 3 2
Ans	(*a*) $f'(x) = 6x^2 - 14x + 4 = 0$ $x = \frac{1}{3}$ (*b*) $(x-2)(2x+1)(x-2)$ (*c*) $A(-\frac{1}{2}, 0)$, $x < -\frac{1}{2}$	

2002 P2

10. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



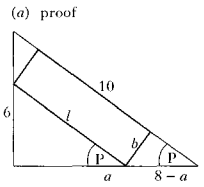
(a) (i) Show that $l = \frac{5}{4}a$. 3

(ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8-a)$. 4

(b) Find the value of a which produces the largest area of the extension.

Ans

(a) proof



$\cos P = \frac{8}{10} = \frac{a}{l} \Rightarrow l = \frac{10}{8}a = \frac{5}{4}a$

$\sin P = \frac{6}{10} = \frac{b}{8-a} \Rightarrow b = \frac{6}{10}(8-a)$

Area $= lb = \frac{5}{4}a \times \frac{6}{10}(8-a) = \frac{3}{4}a(8-a)$

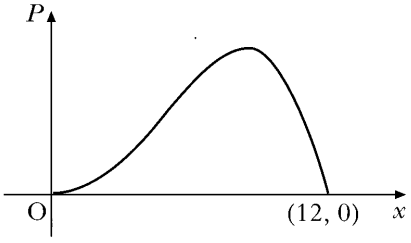
(b) Solve $\frac{dA}{da} = 0$ and test for max/min

$a = 4$

2001 P1

6. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \le x \le 12$.

Find the value of x which gives the maximum profit.



5

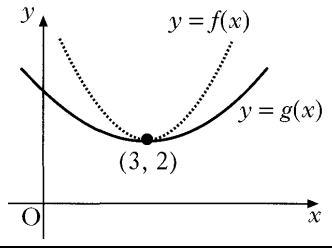
Ans

$x = 9$

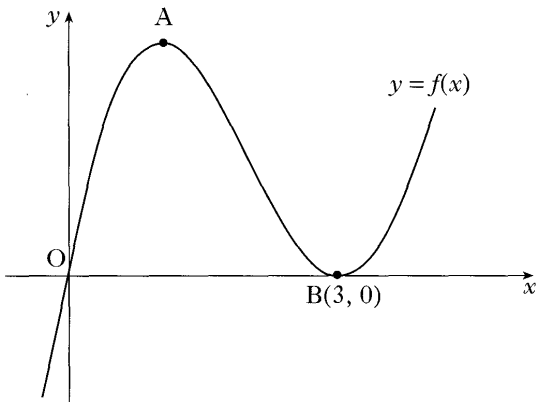
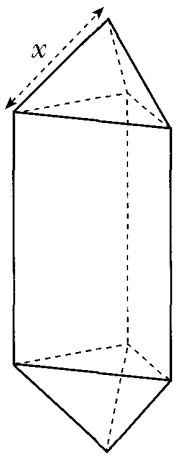
2001 P1

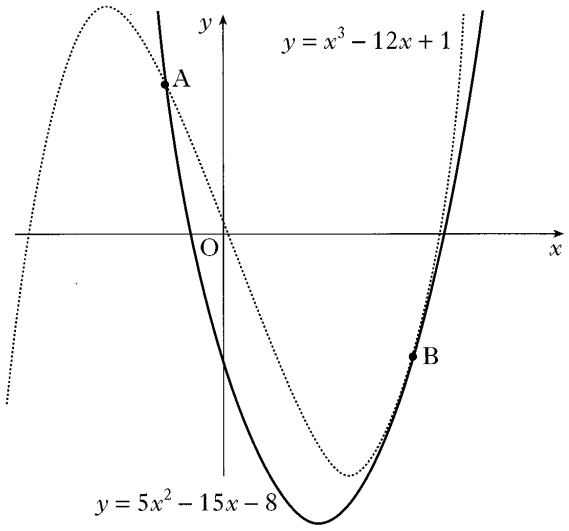
9. The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at (3, 2).

Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$.



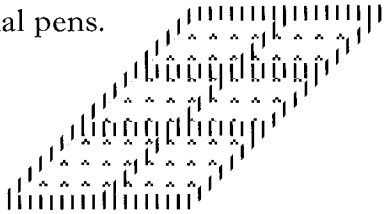
2

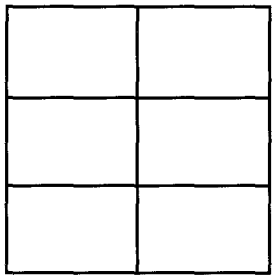
Ans	straight line for f' through $(3,0)$, $m_{f'} > 0$ straight line for g' through $(3,0)$, $m_{f'} > m_{g'} > 0$	
2001 P2	**2.** A curve has equation $y = x - \frac{16}{\sqrt{x}}, \quad x > 0.$ Find the equation of the tangent at the point where $x = 4$.	6
Ans	$y = 2x - 12$	
2000 P1	**2.** A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).  (*a*) Find the coordinates of the turning point at A.	4
Ans	(*a*) A = (1,4)	
2000 P2	**6.** A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end. The surface area, A, of the solid is given by $$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$ where x is the length of each edge of the tetrahedron. Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid.  	6
Ans	$x = 2$	

2000 P1	**4.** The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$. The two curves intersect at A and touch at B, ie at B the curves have a common tangent.  (*a*) (i) Find the x-coordinates of the points on the curves where the gradients are equal. (ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i).	4 1
Ans	(*a*) (i) $x = \frac{1}{3}$ and $x = 3$ (ii) parallel and coincident	
Specimen 2 P1	**3.** The point P(–1, 7) lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P.	3
Ans	$y - 7 = -10(x - (-1))$	
Specimen 2 P2	**9.** A curve has equation $y = 2x^3 + 3x^2 + 4x - 5$. Prove that this curve has no stationary points.	5
Ans	$\frac{dy}{dx} = 6x^2 + 6x + 4$ $b^2 - 4ac = -60$ $6x^2 + 6x + 4$ has no roots $\frac{dy}{dx} = 0$ has no solutions so curve has no stationary points	

Specimen 2 P1

10. A zookeeper wants to fence off six individual animal pens.





Each pen is a rectangle measuring x metres by y metres, as shown in the diagram.

(*a*) (i) Express the total length of fencing in terms of x and y. 4

(ii) Given that the total length of fencing is 360 m, show that the total area, A m^2, of the six pens is given by $A(x) = 240x - \frac{16}{3}x^2$. 6

(*b*) Find the values of x and y which give the maximum area and write down this maximum area.

Ans

10. (*a*) length $= 9y + 8x = 360$

$A = 3y \times 2x = 2x.3.\frac{1}{9}(360 - 8x)240x - \frac{16}{3}x^2$

(*b*) $A'(x) = 240 - \frac{32}{3}x$

$A'(x) = 0 \quad \Rightarrow x = 22\frac{1}{2}, \quad y = 20$

x	$22\frac{1}{2}^-$	$22\frac{1}{2}$	$22\frac{1}{2}^+$
$A'(x)$	+	0	−
		maximum	

$A_{max} = 2700$

Specimen 1 P1

3. (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. 3

(*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes. 1

(*c*) Find the stationary points of $y = f(x)$ and determine the nature of each. 5

(*d*) Sketch the graph of $y = f(x)$. 1

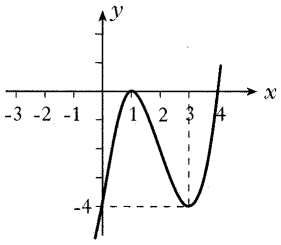
Ans

(*a*) $f(1) = 0$, $(x - 4)$, $(x - 1)$

(*b*) (1,0), (4,0), (0, −4)

(*c*) max at (1,0), min at (3, −4)

(*d*)



Specimen 1 P2

4. In the diagram below, a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, −8).

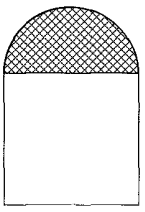
(*a*) Find the equation of the tangent at A. — 3

Ans

(*a*) $y = -5x - 3$

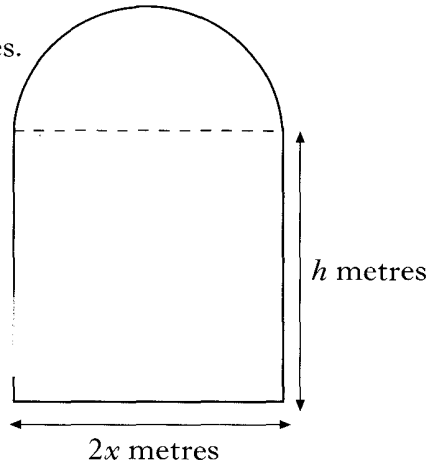
Specimen 1 P2

9. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.



The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.

The rectangle measures $2x$ metres by h metres.



(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. — 2

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. — 2

(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. — 5

Ans

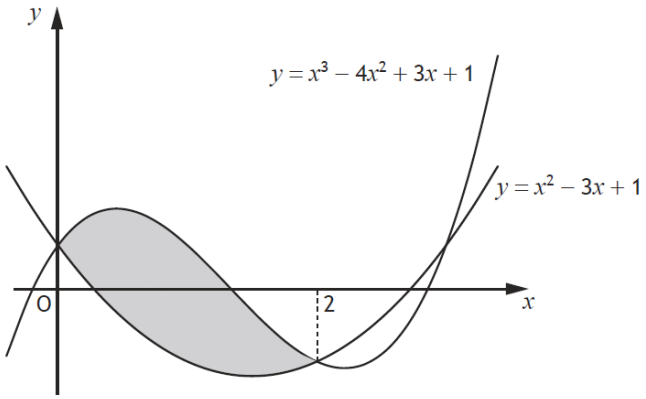
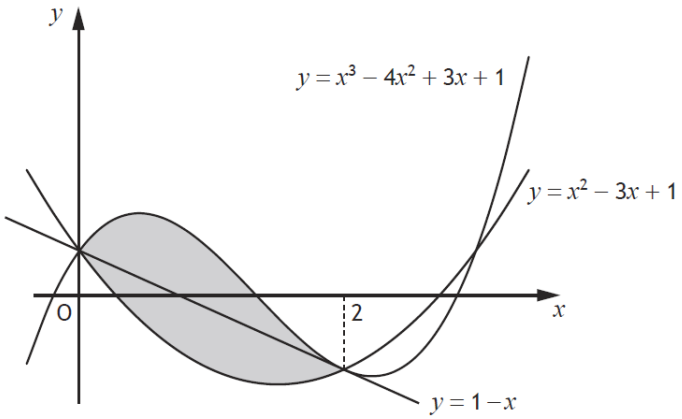
(*a*) (i) $h = \frac{1}{2}(10 - \pi x - 2x)$

(ii) $L = 2 \times 2xh + \frac{1}{2}\pi x^2$

$= 4x \times \frac{1}{2}(10 - \pi x - 2x) + \frac{1}{2}\pi x^2$

$= 20x - 2\pi^2 - 4x^2 + \frac{1}{2}\pi x^2$

(*b*) $x = \frac{20}{3\pi + 8}$, $h = \frac{5(\pi + 4)}{3\pi + 8}$

2017 P1	**10.** Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.  (a) Calculate the shaded area. The line passing through the points of intersection of the curves has equation $y = 1 - x$.  (b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$.	5 4
Ans	(a) 8/3 (b) 1/2	

2017 P1

15. A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.
The turning point is $(2, 3)$.

Diagram 2 shows part of the graph with equation $y = h(x)$.
The turning point is $(7, 6)$.

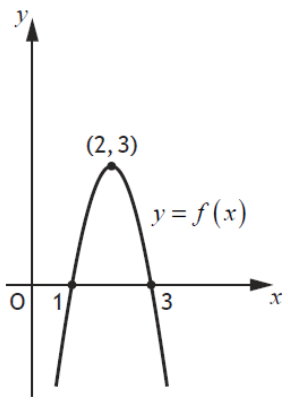


Diagram 1

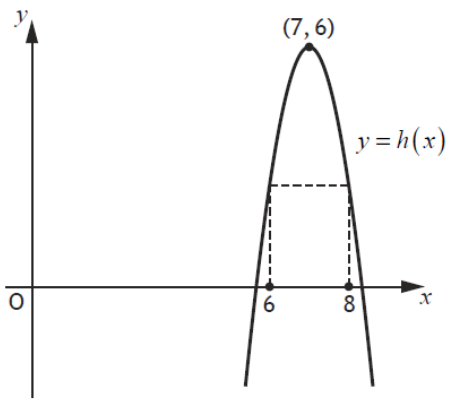


Diagram 2

(a) Given that $h(x) = f(x + a) + b$.
Write down the values of a and b. — 2

(b) It is known that $\int_1^3 f(x)\,dx = 4$.
Determine the value of $\int_6^8 h(x)\,dx$. — 1

(c) Given $f'(1) = 6$, state the value of $h'(8)$. — 1

Ans

(a) a = -5, b = 3
(b) 10
(c) -6

2016 P2

9. For a function f, defined on a suitable domain, it is known that:

- $f'(x) = \dfrac{2x+1}{\sqrt{x}}$
- $f(9) = 40$

(d) Express $f(x)$ in terms of x. — 4

Ans

(e) $f(x) = \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 2$

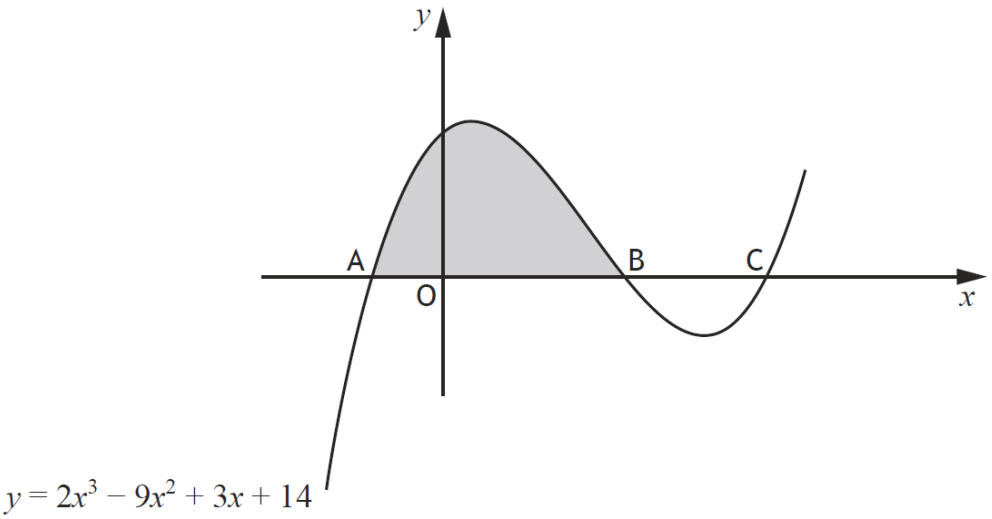
2016 P2

3. (a) (i) Show that $(x+1)$ is a factor of $2x^3 - 9x^2 + 3x + 14$. 2

(ii) Hence solve the equation $2x^3 - 9x^2 + 3x + 14 = 0$. 3

(b) The diagram below shows the graph with equation $y = 2x^3 - 9x^2 + 3x + 14$.

The curve cuts the x-axis at A, B and C.



(i) Write down the coordinates of the points A and B. 1

(ii) Hence calculate the shaded area in the diagram. 4

Ans

(a) (ii) x = -1, 2, 3.5
(b) (i) A(-1, 0), B(2, 0)
(ii) 27 units^2

2015 P1

15. The rate of change of the temperature, T °C of a mug of coffee is given by

$$\frac{dT}{dt} = \frac{1}{25}t - k \ , \ 0 \leq t \leq 50$$

- t is the elapsed time, in minutes, after the coffee is poured into the mug
- k is a constant
- initially, the temperature of the coffee is 100 °C
- 10 minutes later the temperature has fallen to 82 °C.

Express T in terms of t. 6

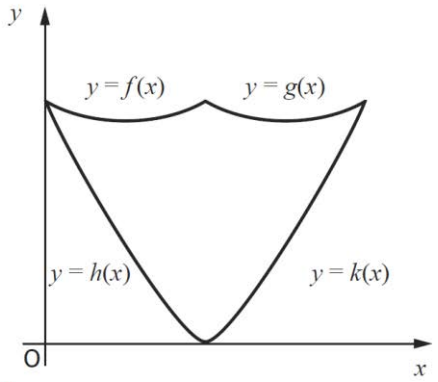
Ans

$T = \frac{1}{50}t^2 - 2t + 100$

NH 2015 P2

4. A wall plaque is to be made to commemorate the 150th anniversary of the publication of "*Alice's Adventures in Wonderland*".

The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.



- $f(x)=\frac{1}{4}x^2-\frac{1}{2}x+3$
- $g(x)=\frac{1}{4}x^2-\frac{3}{2}x+5$
- $h(x)=\frac{3}{8}x^2-\frac{9}{4}x+3$
- $k(x)=\frac{3}{8}x^2-\frac{3}{4}x$

(a) Find the x-coordinate of the point of intersection of the graphs with equations $y=f(x)$ and $y=g(x)$. — 2

The graphs of the functions $f(x)$ and $h(x)$ intersect on the y-axis.

The plaque has a vertical line of symmetry.

(b) Calculate the area of the wall plaque. — 7

Ans

(a) x = 2
(b) 19/3

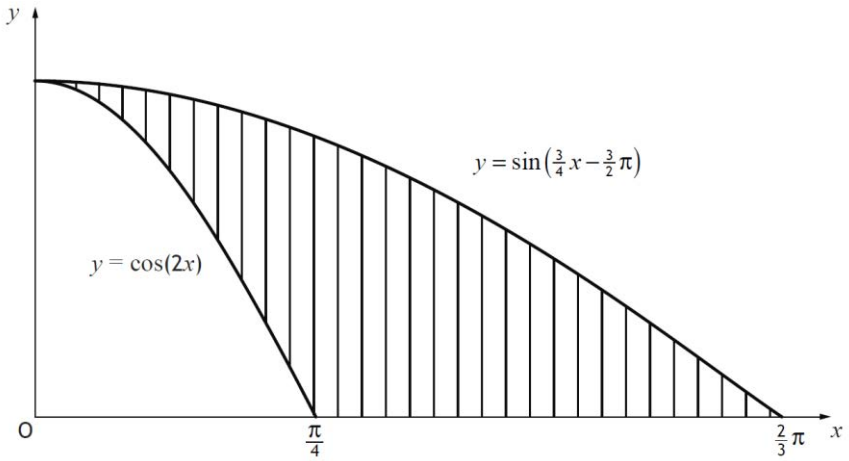
NH SP P2

9. A sea-life visitor attraction has a new logo in the shape of a shark fin.

The outline of the logo can be represented by parts of

- the x axis
- the curve with equation $y = \cos(2x)$
- the curve with equation $y = \sin\left(\frac{3}{4}x - \frac{3}{2}\pi\right)$

as shown in the diagram.



Calculate the shaded area.

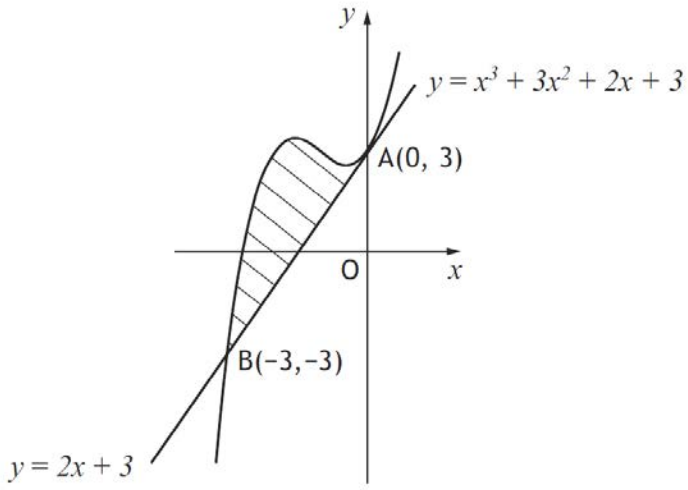
6

Ans

5/6

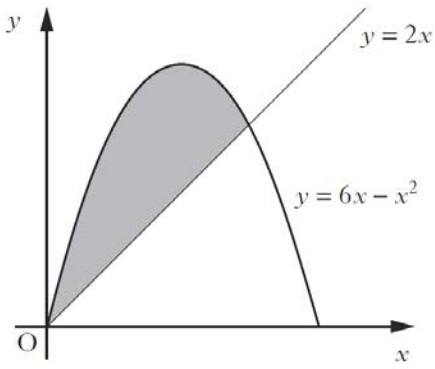
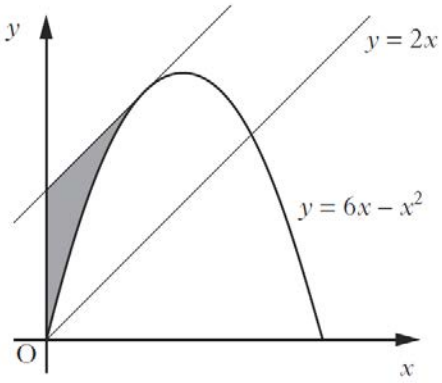
NH EX P2

4. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A (0, 3), as shown.



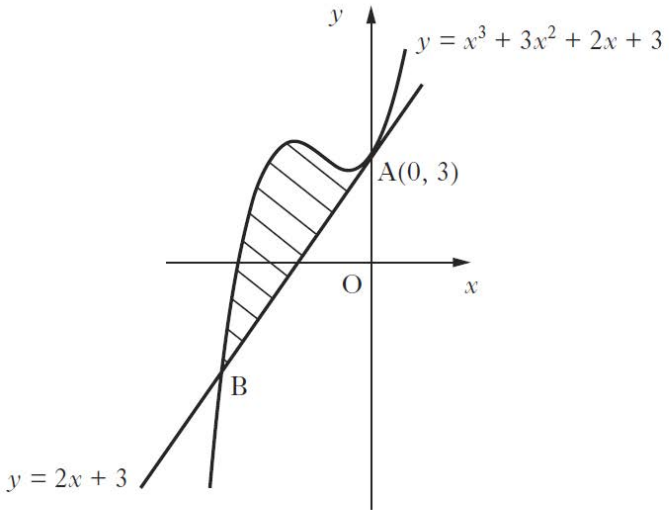
The line meets the curve again at B (−3,−3). Find the area enclosed by the line and the curve.

5

Ans	27/4	
NH EX P1	**3.** Evaluate $\int_1^2 \frac{1}{6}x^{-2}\,dx$.	3
Ans	1/12	
2014 P2	**7.** Land enclosed between a path and a railway line is being developed for housing. This land is represented by the shaded area shown in Diagram 1. • The path is represented by a parabola with equation $y = 6x - x^2$. • The railway is represented by a line with equation $y = 2x$. • One square unit in the diagram represents 300 m^2 of land.  Diagram 1 (*a*) Calculate the area of land being developed. (*b*) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.  Diagram 2 It is decided that the land, represented by the shaded area in Diagram 2, will become a car park. Calculate the area of the car park.	5 5
Ans	(a) 3200m^2 (b) 800m^2	

2013 P2

4. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram.



The line meets the curve again at B.

Show that B is the point (–3, –3) and find the area enclosed by the line and the curve. 6

Ans $\frac{27}{4} units^2$

201 3 P1

7. Find $\int x(3x + 2)\,dx$.

Ans $x^3 + x^2 + c$

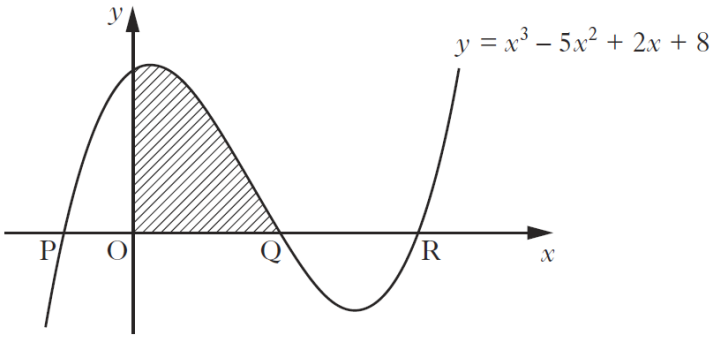
2012 P1

21. (*a*) (i) Show that $(x – 4)$ is a factor of $x^3 – 5x^2 + 2x + 8$.

(ii) Factorise $x^3 – 5x^2 + 2x + 8$ fully.

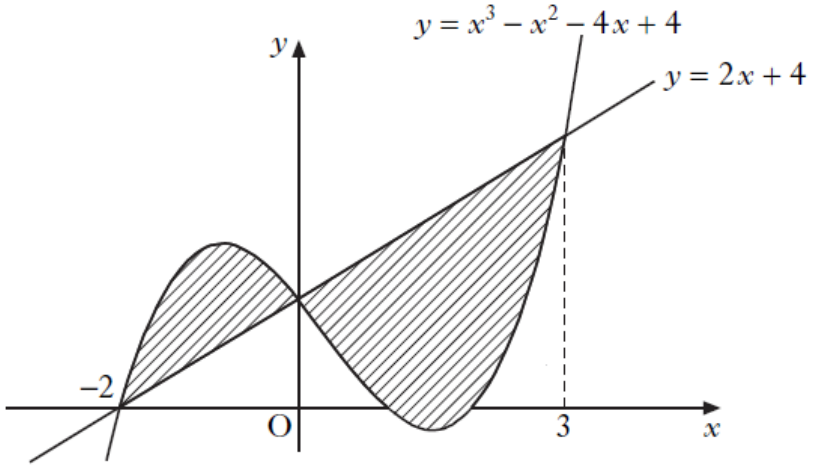
(iii) Solve $x^3 – 5x^2 + 2x + 8 = 0$.

(*b*) The diagram shows the curve with equation $y = x^3 – 5x^2 + 2x + 8$.



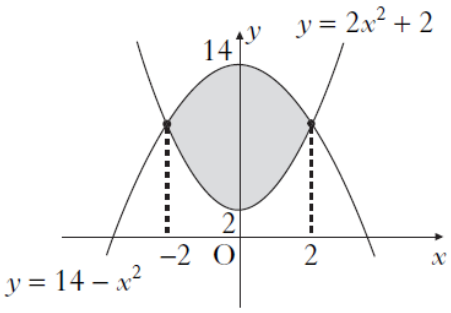
The curve crosses the x-axis at P, Q and R.

Determine the shaded area.

Ans	(ii) $(x-4)(x-2)(x+1)$ (iii) $x = -1, 2, 4$ (b) Area = $\frac{32}{3}$ units^2	
2012 P1	**11.** Find $\int \left(\frac{1}{6x^2}\right) dx,\ x \neq 0.$	
Ans	$-\frac{1}{6}x^{-1} + c$	
2011 P2	**4.** The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$. The curve and the line intersect at the points (–2, 0), (0, 4) and (3, 10).  Calculate the total shaded area.	**10**
Ans	$21\frac{1}{12}$ unit^2	
2011 P1	**16.** Find $\int \frac{1}{3x^4} dx$, where $x \neq 0$.	
Ans	$-\frac{1}{9x^3} + c$	

2010 P1

14. The diagram shows graphs with equations $y = 14 - x^2$ and $y = 2x^2 + 2$.



Which of the following represents the shaded area?

(Write an integral which)

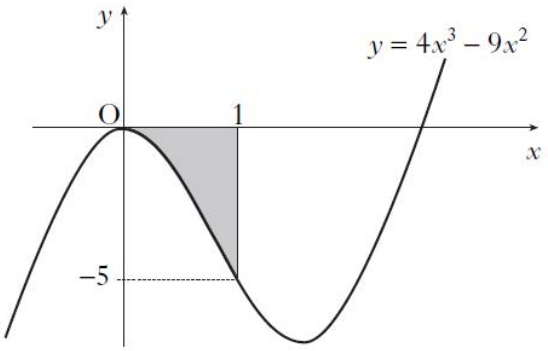
Ans

$$\int_{-2}^{2} (12 - 3x^2)\,dx$$

2009 P1

16. The graph of $y = 4x^3 - 9x^2$ is shown in the diagram.

Which of the following gives the area of the shaded section?



A $\left[x^4 - 3x^3\right]_{-5}^{0}$

B $-\left[x^4 - 3x^3\right]_{0}^{1}$

C $\left[12x^2 - 18x\right]_{-5}^{0}$

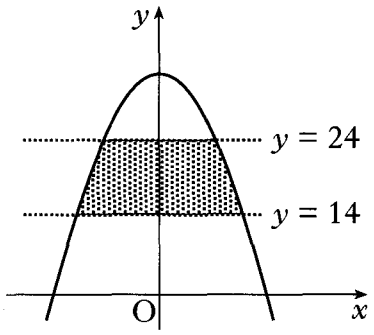
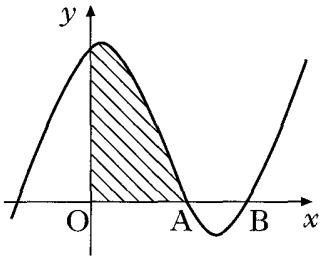
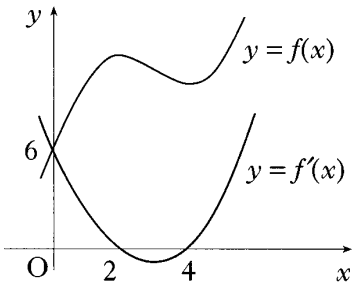
D $-\left[12x^2 - 18x\right]_{0}^{1}$

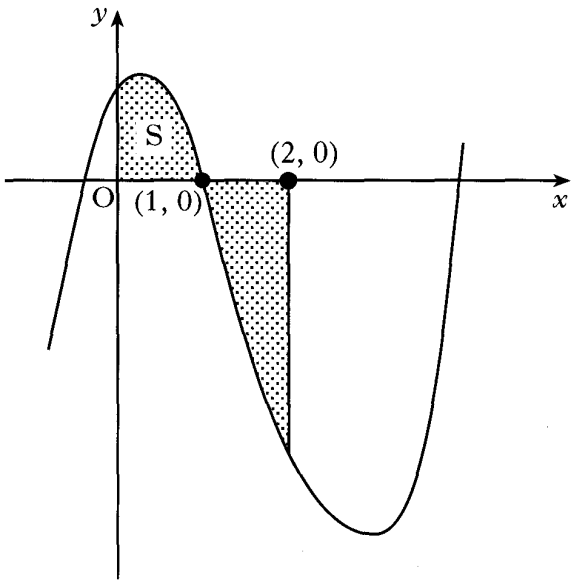
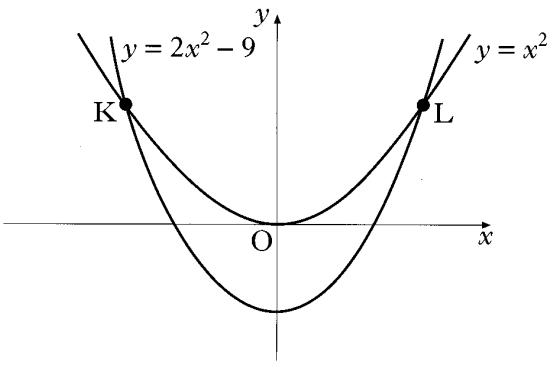
Ans

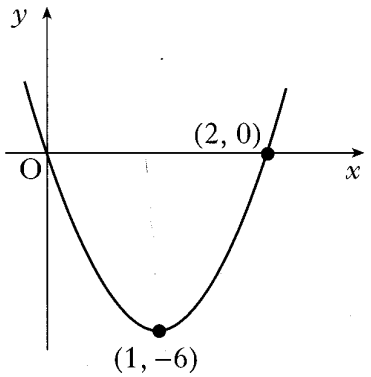
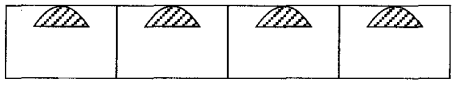
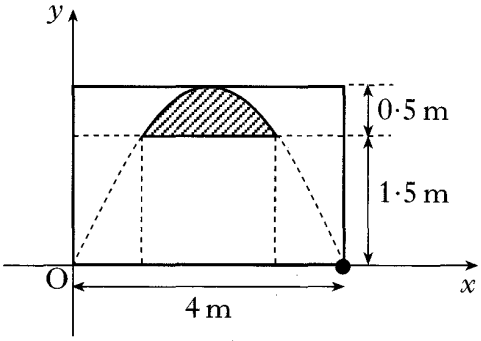
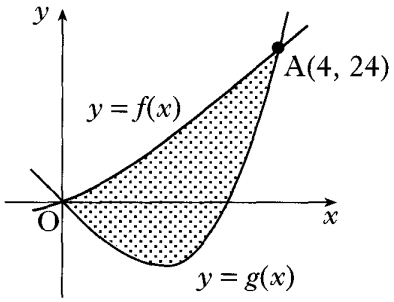
A

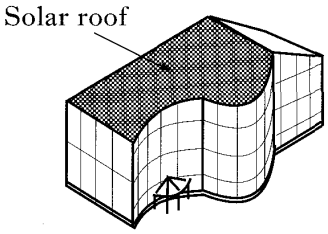
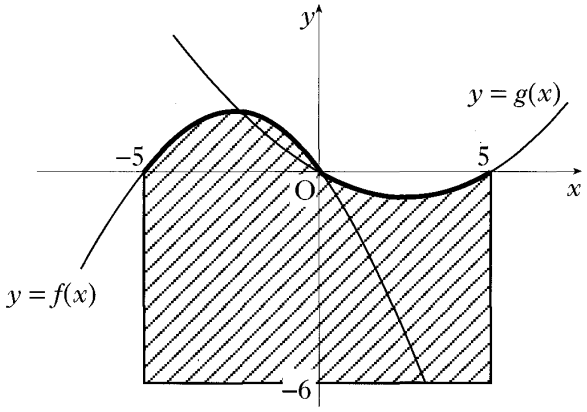
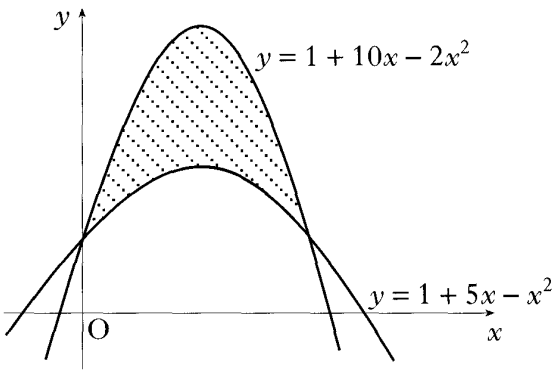
2008 P1

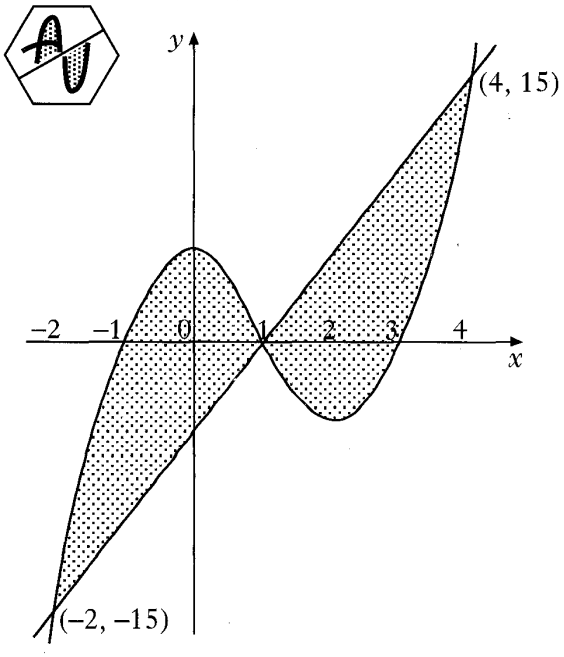
14. Find $\int 4\sin(2x + 3)\,dx$. 2

Ans	$-2\cos(2x+3)+c$	
2008 P2	**7.** The parabola shown in the diagram has equation $y = 32 - 2x^2$. The shaded area lies between the lines $y = 14$ and $y = 24$. Calculate the shaded area. 	8
Ans	$50\frac{2}{3}$	
2007 P1	**8.** The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$. (*a*) Show that the graph cuts the x-axis at (3, 0). (*b*) Hence or otherwise find the coordinates of A. (*c*) Find the shaded area. 	1 3 5
Ans	(*a*) To cut the x-axis, $y = 0$. So $0 = x^3 - 4x^2 + x + 6$ $= (x-3)(x^2 - x - 2)$ $= (x-3)(x-2)(x+1)$ So graph cuts x-axis at $x = -1, 3, 2$. (*b*) (2,0) (*c*) $\frac{22}{3}$	
2007 P2	**10.** The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$. Both graphs pass through the point (0, 6). The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).  (*a*) Given that $f'(x)$ is of the form $k(x-a)(x-b)$: (i) write down the values of a and b; (ii) find the value of k. (*b*) Find the equation of the graph of the cubic function $y = f(x)$.	3 4

Ans	(a) (i) $a = 2, b = 4$ (ii) $k = \frac{3}{4}$ (b) $y = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + 6$	
2006 P1	**6.** The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$. The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$. (a) Calculate the shaded area labelled S. (b) Hence find the total shaded area. 	4 3
Ans	(a) $\frac{5}{4}$ or equivalent (b) $\frac{9}{2}$ or equivalent	
2006 P2	**5.** The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x.	4
Ans	$y = 2x^2 - 2x^3 + 5$	
2005 P2	**1.** Find $\int \frac{4x^3 - 1}{x^2}\,dx, \quad x \neq 0.$	4
Ans	$2x^2 + x^{-1} + c$	
2005 P2	**5.** The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown. Calculate the area enclosed between the curves. 	8

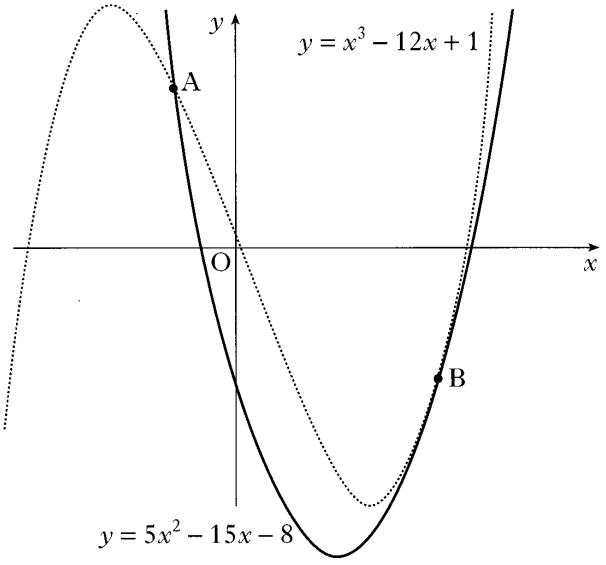
Ans	36	
2004 P1	**11.** The diagram shows a parabola passing through the points (0, 0), (1, −6) and (2, 0). (*a*) The equation of the parabola is of the form $y = ax(x - b)$. Find the values of a and b. (*b*) This parabola is the graph of $y = f'(x)$. Given that $f(1) = 4$, find the formula for $f(x)$. 	3 5
Ans	(*a*) $a = 6, b = 2$ (*b*) $f(x) = 2x^3 - 6x^2 + 8$	
2004 P2	**11.** An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic. The second diagram shows one such window. The shaded part represents the glass. The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$. Find the area in square metres of the glass in one window.  	8
Ans	2/3	
2003 P2	**3.** The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin. Find the shaded area enclosed between the curves. 	5
Ans	$A = \int_{42\frac{2}{3}} [(x^2 + 2x) - (x^3 - x^2 - 6x)]\,dx$	
2002W P1	**7.** Find $\int \left(\sqrt[3]{x} - \frac{1}{\sqrt{x}} \right) dx$.	4

Ans	$\frac{3}{4}x^{\frac{4}{3}} - 2x^{\frac{1}{2}} + c$	
2002W P2	**11.** An energy efficient building is designed with solar cells covering the whole of its south facing roof. The energy generated by the solar cells is directly proportional to the area, in square units, of the solar roof.   The shape of the solar roof can be represented on the coordinate plane as the shaded area bounded by the functions $f(x) = \frac{1}{4}\left(-x^2 - 5x\right)$, $g(x) = \frac{1}{12}\left(x^2 - 5x\right)$ and the lines $x = -5$, $x = 5$ and $y = -6$. (*a*) Find the area of the solar roof. (*b*) Ten square units of solar cells generate a maximum of 1 kilowatt. What is the maximum energy the solar roof can generate in kilowatts (to the nearest kilowatt)?	7 1
Ans	$\int_{-5}^{0}(f(x)-(-6))dx + \int_{0}^{5}(g(x)-(-6))dx$ $63\frac{17}{36}$ (*b*) 6 kilowatts	
2002 P2	**5.** Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.  	6

Ans	Area $= \int_0^5 ((1+10x-2x^2)-(1+5x-x^2))\,dx$ $20\frac{5}{6}$	
2002 P2	**8.** A point moves in a straight line such that its acceleration a is given by $a = 2(4-t)^{\frac{1}{2}}$, $0 \le t \le 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$.	4
Ans	$v = \int (2(4-t)^{\frac{1}{2}})\,dt$ use $v = 0$, $t = 0$ to find c $v = -\frac{4}{3}(4-t)^{\frac{3}{2}} + \frac{32}{3}$	
2001 P2	**6.** Find $\int \frac{(x^2-2)(x^2+2)}{x^2}dx$, $x \neq 0$	4
Ans	$\frac{1}{3}x^3 + 4x^{-1} + c$	
2001 P2	**8.** A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon. A mathematical representation of the final logo is shown in the coordinate diagram. The curve has equation $y = (x+1)(x-1)(x-3)$ and the straight line has equation $y = 5x - 5$. The point (1, 0) is the centre of half-turn symmetry. Calculate the total shaded area.  	7
Ans	$40\frac{1}{2}$ units^2	

2000 P1

4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.
The two curves intersect at A and touch at B, ie at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the points on the curves where the gradients are equal. 4

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). 1

(*b*) The point A is (–1, 12) and B is (3, –8).
Find the area enclosed between the two curves. 5

Ans

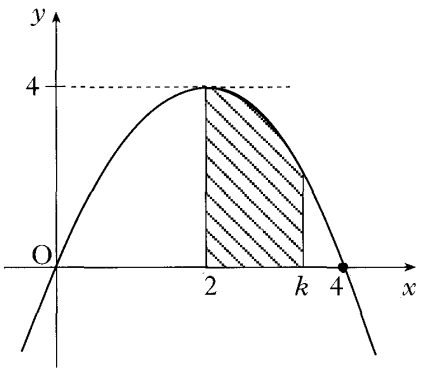
(*a*) (i) $x = \frac{1}{3}$ and $x = 3$

(ii) parallel and coincident

(*b*) $21\frac{1}{3}$

2000 P2

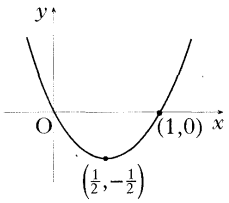
4. The parabola shown crosses the x-axis at (0, 0) and (4, 0), and has a maximum at (2, 4).
The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.



(*a*) Find the equation of the parabola. 2

(*b*) Hence show that the shaded area, A, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$$ 3

Ans	(a) $y = 4x - x^2$ (b) $\int_2^k (\text{function from } (a))$ $= -\frac{1}{3}x^3 + 2x^2$ $= -\frac{1}{3}k^3 + 2k^2 - \left(\frac{-8}{3} + 8\right)$	
Specimen 2 P1	**8.** Functions f and g are defined on the set of real numbers by $f(x) = x - 1$ $g(x) = x^2$. (a) Find formulae for (i) $f(g(x))$ (ii) $g(f(x))$. (b) The function h is defined by $h(x) = f(g(x)) + g(f(x))$. Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h. (c) Find the area enclosed between this graph and the x-axis.	3 3 4
Ans	(a) $f(g(x)) = x^2 - 1,\ g(f(x)) = (x-1)^2$ (b) $h(x) = x^2 - 1 + x^2 - 2x + 1 = 2x^2 - 2x$  (c) $Area = \int_0^1 \left(2x^2 - 2x\right)dx = \frac{1}{3}$	
Specimen 2 P1	**9.** Find $\int \frac{x^2 - 5}{x\sqrt{x}}dx$.	4
Ans	$\int \left(x^{\frac{1}{2}} - 5x^{-\frac{3}{2}}\right)dx = \frac{2}{3}x^{\frac{3}{2}} + 10x^{-\frac{1}{2}} + c$	
Specimen 1 P1	**7.** Find the value of $\int_1^2 \frac{u^2 + 2}{2u^2}du$.	5
Ans	1	

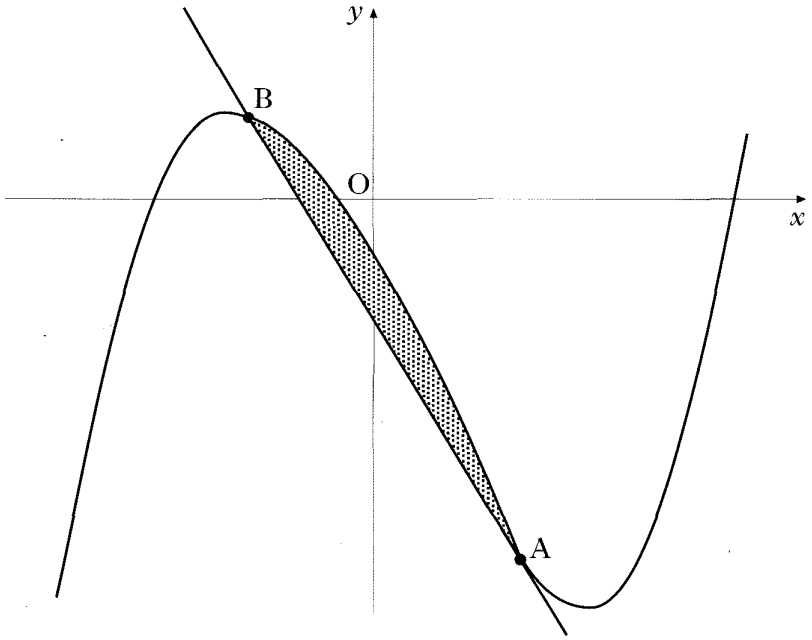
Specimen 1 P2

4. In the diagram below, a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, −8).

(*a*) Find the equation of the tangent at A. 3

(*b*) Hence find the coordinates of B. 4

(*c*) Find the area of the shaded part which represents the land bounded by the river and the road. 3



Ans

(*a*) $y = -5x - 3$

(*b*) B = (−1,2)

(*c*) area = $1\frac{1}{3}$