Advanced Higher
Mathematics

HSNe21S04
Exam Solutions – 2004

Question 1

(a)

$$f(x) = \cos^2 x \, e^{\tan x}$$

$$\Rightarrow f'(x) = 2\cos x.(-\sin x)\,e^{\tan x} + \cos^2 x \, . \, \sec^2 x \, e^{\tan x}$$

Using the product rule

$$\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$$

$$= -2\sin x \cos x \, e^{\tan x} + \cos^2 x \sec^2 x \, e^{\tan x}$$

$$= -\sin 2x \, e^{\tan x} + e^{\tan x}$$

$$= e^{\tan x}(1 - \sin 2x)$$

$$\therefore f'\left(\tfrac{\pi}{4}\right) = e^{\tan \pi/4}\left(1 - \sin\tfrac{\pi}{2}\right)$$

$$= e^1(1-1) = e \times 0 = 0$$

(b)

$$g(x) = \frac{\tan^{-1} 2x}{1+4x^2}$$

Using the quotient rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$\Rightarrow g'(x) = \frac{\dfrac{1}{\cancel{1+(2x)^2}} . 2\cancel{(1+4x^2)} - \tan^{-1} 2x . 8x}{(1+4x^2)^2}$$

$$= \frac{2 - 8x\tan^{-1} 2x}{(1+4x^2)^2}$$

Question 2

Method 1

$$(a^2-3)^4$$

$$= (a^2)^4 + 4(a^2)^3(-3) + 6(a^2)^2(-3)^2 + 4(a^2)(-3)^3 + (-3)^4$$

$$= a^8 - 12a^6 + 54a^4 - 108a^2 + 81.$$

Coefficients from row 5 of Pascal's triangle...

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1 ← Row 5.

Method 2

$$(a^2-3)^4 = \sum_{k=0}^{4} \binom{4}{k} (a^2)^{4-k} (-3)^k$$

$$= \binom{4}{0}(a^2)^4(-3)^0 + \binom{4}{1}(a^2)^3(-3)^1 + \binom{4}{2}(a^2)^2(-3)^2 + \binom{4}{3}(a^2)^1(-3)^3 + \binom{4}{4}(a^2)^0(-3)^4$$

$$= 1.a^8 + 4.a^6(-3) + 6.a^4.9 + 4.a^2.(-27) + 1.81$$

$$= a^8 - 12a^6 + 54a^4 - 108a^2 + 81.$$

Question 3

$$x = 5\cos\theta \Rightarrow \frac{dx}{d\theta} = -5\sin\theta$$

$$y = 5\sin\theta \Rightarrow \frac{dy}{d\theta} = 5\cos\theta$$

$$\therefore \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{5\cos\theta}{-5\sin\theta} = -\frac{\cos\theta}{\sin\theta} = -\cot\theta.$$

When $\theta = \frac{\pi}{4}$ $\quad x = 5\cos\frac{\pi}{4} = \frac{5}{\sqrt{2}}$

$$y = 5\sin\frac{\pi}{4} = \frac{5}{\sqrt{2}}$$

$$m_{\text{tangent}} = \frac{dy}{dx} = -\cot\frac{\pi}{4} = -1.$$

$\therefore$ Equation of tangent is ...

$$y - b = m(x-a)$$

$$y - \frac{5}{\sqrt{2}} = -1\left(x - \frac{5}{\sqrt{2}}\right)$$

$$y - \frac{5}{\sqrt{2}} = -x + \frac{5}{\sqrt{2}}$$

$$\therefore \quad x + y - \frac{2.5}{\sqrt{2}} = 0$$

$$x + y - 5\sqrt{2} = 0.$$

$$\frac{5}{\sqrt{2}} + \frac{5}{\sqrt{2}}$$

$$= 2.\frac{5}{\sqrt{2}} = 5\sqrt{2}$$

Question 4

Given $z = 1+2i$ then $z^2 = (1+2i)^2$

$$= 1+4i+4i^2$$

$$= -3+4i$$

$\boxed{i^2 \equiv -1}$

$$\therefore \quad z^2(z+3) = (-3+4i)(1+2i+3)$$

$$= (-3+4i)(4+2i)$$

$$= -12-6i+16i+8i^2$$

$$= -20+10i.$$

So $z^3+3z^2-5z+25$

$$= z^2(z+3)-5z+25$$

$$= -20+10i-5(1+2i)+25$$

$$= -20+10i-5-10i+25$$

$$= 0$$

$\therefore$ $z = 1+2i$ is a root of the cubic equation.

Consequently $z = 1-2i$ is also a root.

So a factor is $[(z-1)-2i][(z-1)+2i]$

$$= (z-1)^2-4i^2$$

$$= z^2-2z+1+4$$

$$= z^2-2z+5$$

$$\begin{array}{r|l} & \quad\quad\quad\quad\quad\quad\quad z+5 \\ \hline z^2-2z+5 & z^3+3z^2-5z+25 \\ & \underline{z^2-2z^2+5z} \\ & \quad\quad 5z^2-10z+25 \\ & \quad\quad \underline{5z^2-10z+25} \\ & \quad\quad\quad\quad - \end{array}$$

$$\therefore \quad z^3+3z^2-5z+25 = (z^2-2z+5)(z+5)$$

So the roots are $1+2i$, $1-2i$ and -5.

Question 5

$$\frac{1}{x^2-x-6} = \frac{1}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$\therefore \quad A(x+2) + B(x-3) = 1$$

Using the cover-up rule...

Let $x=3$ $\quad 5A = 1$ so $A = \frac{1}{5}$.

Let $x=-2$ $\quad -5B = 1$ so $B = -\frac{1}{5}$

$$\therefore \quad \frac{1}{x^2-x-6} = \frac{1/5}{x-3} - \frac{1/5}{x+2}$$

$$= \frac{1}{5(x-3)} - \frac{1}{5(x+2)}$$

$$\int_0^1 \frac{1}{x^2-x-6}\,dx = \int_0^1 \frac{1}{5(x-3)} - \frac{1}{5(x+2)}\,dx.$$

$$= \frac{1}{5}\int_0^1 \frac{1}{x-3} - \frac{1}{x+2}\,dx$$

$$= \frac{1}{5}\Big[\ln|x-3| - \ln|x+2|\Big]_0^1$$

$$= \frac{1}{5}\left[\ln\left|\frac{x-3}{x+2}\right|\right]_0^1$$

$$= \frac{1}{5}\left[\ln\left|\frac{1-3}{1+2}\right| - \ln\left|\frac{0-3}{0+2}\right|\right]$$

$$= \frac{1}{5}\left[\ln\left|-\frac{2}{3}\right| - \ln\left|-\frac{3}{2}\right|\right] = \frac{1}{5}\left[\ln\frac{2}{3} - \ln\frac{3}{2}\right] = \frac{1}{5}\ln\frac{4}{9}.$$

Alternatively ...

from... $\frac{1}{5}\int_0^1 \frac{1}{x-3} - \frac{1}{x+2}\,dx$

Writing $\frac{1}{x-3}$ as $-\frac{1}{3-x}$.

$$= \frac{1}{5}\int_0^1 -\frac{1}{3-x} - \frac{1}{x+2}\,dx$$

$$= \frac{1}{5}\Big[\ln|3-x| - \ln|x+2|\Big]_0^1$$

$$= \frac{1}{5}\left[\ln\left|\frac{3-x}{x+2}\right|\right]_0^1$$

$$= \frac{1}{5}\left[\ln\left|\frac{2}{3}\right| - \ln\left|\frac{3}{2}\right|\right]$$

$$= \frac{1}{5}\ln\left(\frac{2/3}{3/2}\right)$$

$$= \frac{1}{5}\ln\frac{4}{9}.$$

Question 6

$$M_1 = \begin{pmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$M_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \leftarrow$ Reflection in x-axis ie $(x,y) \mapsto (x,-y)$

$$\therefore M_2M_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

This is a reflection in the line $y = -x$.

Question 7

Method 1

$f(x) = e^x \sin x.$ so $f(0) = e^0 \sin 0 = 0.$

$\Rightarrow f'(x) = e^x \sin x + e^x \cos x$

$= e^x(\sin x + \cos x)$ so $f'(0) = e^0(\sin 0 + \cos 0)$

$= 1.$

$\Rightarrow f''(x) = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$

$= 2e^x \cos x$ so $f''(0) = 2e^0 \cos 0$

$= 2$

$\Rightarrow f'''(x) = 2e^x \cos x + 2e^x(-\sin x)$

$= 2e^x \cos x - 2e^x \sin x$

$= 2e^x(\cos x - \sin x)$ so $f'''(0) = 2e^0(\cos 0 - \sin 0)$

$= 2$

$\therefore$ Maclaurin expansion is

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots$$

So $e^x \sin x = 0 + 1.x + \frac{2}{2}x^2 + \frac{2}{6}x^3 + \ldots$

$= x + x^2 + \frac{1}{3}x^3 + \ldots$

Method 2 From knowledge of simple Maclaurin expansions

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \ldots$$

$$\sin x = x - \frac{1}{6}x^3 + \ldots$$

So $e^x \sin x = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \ldots\right)\left(x - \frac{1}{6}x^3 + \ldots\right)$

$$= x - \frac{1}{6}x^3 + x^2 \cancel{- \frac{1}{6}x^4} + \frac{1}{2}x^3 + \ldots$$

$$= x + x^2 + \frac{2}{6}x^3 + \ldots$$

$$= x + x^2 + \frac{1}{3}x^3 + \ldots$$

Question 8

$231 = 13(17) + 10.$

$17 = 1(10) + 7$

$10 = 1(7) + 3.$

$7 = 2(3) + 1$ — $*$

$3 = 3(1) + 0$ $\therefore (231, 17) = 1.$

Working backwards from — $*$

$$7 - 2(3) = 1$$

So $7 - 2(10 - 1(7)) = 7 - 2(10) + 2(7) = 3(7) - 2(10) = 1$

and $3(17 - 1(10)) - 2(10) = 3(17) - 3(10) - 2(10) = 3(17) - 5(10) = 1$

and $3(17) - 5(231 - 13(17)) = 3(17) - 5(231) + 65(17) = 68(17) - 5(231) = 1$

So $17 \times 68 + 231 \times (-5) = 1$

$\therefore x = -5$ and $y = 68.$

Question 9

$$\int \frac{1}{(1+\sqrt{x})^3}\,dx \qquad x=(u-1)^2 \Rightarrow u-1=\sqrt{x} \Rightarrow u=1+\sqrt{x}.$$

$$\Rightarrow \frac{dx}{du} = 2(u-1) \quad \text{so} \quad dx = 2(u-1)\,du$$

$$= \int \frac{1}{u^3}\cdot 2(u-1)\,du$$

$$= 2\int \frac{u-1}{u^3}\,du = 2\int \frac{u}{u^3} - \frac{1}{u^3}\,du$$

$$= 2\int u^{-2} - u^{-3}\,du$$

$$= 2\left[-u^{-1} + \frac{1}{2}u^{-2}\right] + c.$$

$$= 2\left(-\frac{1}{u} + \frac{1}{2u^2}\right) + c$$

$$= \frac{1}{u^2} - \frac{2}{u} + c.$$

$$= \frac{1}{(1+\sqrt{x})^2} - \frac{2}{1+\sqrt{x}} + c.$$

Question 10

$$f(x) = x^4 \sin 2x$$

$$\therefore \quad f(-x) = (-x)^4 \sin 2(-x)$$

$$= x^4 \sin(-2x)$$

$$= x^4 . -\sin 2x$$

$$= -x^4 \sin 2x = -f(x)$$

Since $f(-x) = -f(x)$ then $f(x)$ is odd.

Question 11

$$\text{Volume} = \pi \int_0^1 y^2 \, dx \quad \text{where} \quad y = e^{-2x}$$

$$\text{so } y^2 = (e^{-2x})^2$$

$$= e^{-4x}.$$

$$\text{So} \quad V = \pi \int_0^1 e^{-4x} \, dx$$

$$= -\frac{\pi}{4} \left[e^{-4x} \right]_0^1$$

$$= -\frac{\pi}{4} \left(e^{-4} - e^0 \right)$$

$$= -\frac{\pi}{4} \left(\frac{1}{e^4} - 1 \right)$$

$$= \frac{\pi}{4} - \frac{\pi}{4e^4} \approx 0.7710 \text{ (to 4dp)}$$

Question 12

$$\frac{d^n}{dx^n}.(xe^x) = (x+n)e^x.$$

When $\underline{n=1}$. L.H.S. $= \frac{d}{dx}(xe^x) = 1.e^x + x.e^x = (1+x)e^x.$

$= (x+1)e^x.$

R.H.S. $(x+1)e^x$

$\therefore$ L.H.S. = R.H.S.

So true for $n=1$.

Assume true for $\underline{n=k}$

$$\frac{d^k}{dx^k}(xe^x) = (x+k)e^x.$$

For $\underline{n=k+1}$

$$\frac{d^{k+1}}{dx^{k+1}}(xe^x) = \frac{d}{dx}\left(\frac{d^k}{dx^{k+1}}(xe^x)\right)$$

$$= \frac{d}{dx}\left[(x+k)e^x\right].$$

Using the product rule!

$$= 1.e^x + (x+k)e^x.$$

$$= e^x(1+x+k)$$

$$= e^x(x+k+1)$$

$$= [x+(k+1)]e^x.$$

$\therefore$ True for $n=k+1$

Since true for $n=1$ and $n=k+1$, then by the Principle of Mathematical induction, it is true for all integers $n \geqslant 1$.

Question 13

(a)

$$f(x) = \frac{x-3}{x+2}$$

$$\begin{array}{r|l} & \ \ 1 \\ x+2 & \overline{x-3} \\ & \ \underline{x+2} \\ & \ \ -5 \end{array}$$

$$= 1 - \frac{5}{x+2}$$

Vertical Asymptote : $x = -2$ ← occurs when denominator $= 0$

Non-vertical Asymptote : $y = 1$.

(b)

$$f'(x) = 5(x+2)^{-2} = \frac{5}{(x+2)^2}$$

Stationary points when $f'(x) = 0$

but $\frac{5}{(x+2)^2} \neq 0$

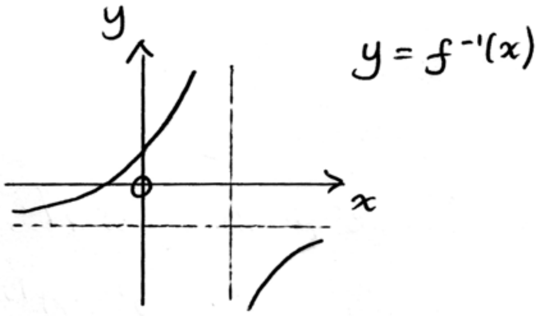
$\therefore$ There are no stationary points

(c)

$$f''(x) = -10(x+2)^{-3} = -\frac{10}{(x+2)^3} \neq 0$$

$\therefore$ There are no points of inflexion

(d)



graph of $y = f^{-1}(x)$ is the graph of $y = f(x)$ reflected in the line $y = x$.

Asymptotes : Vertical $x = 1$

Non-vertical $y = -2$.

Domain of f^{-1} : $x \in \mathbb{R} - \{1\}$

ie must not include $x = 1$.

Question 14

(a)

$$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix}$$

$$\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ -1 & 2 & -4 \\ 0 & 1 & -3 \end{vmatrix}$$

$$= \underset{\sim}{i} \begin{vmatrix} 2 & -4 \\ 1 & -3 \end{vmatrix} - \underset{\sim}{j} \begin{vmatrix} -1 & -4 \\ 0 & -3 \end{vmatrix} + \underset{\sim}{k} \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= \underset{\sim}{i}(-6+4) - \underset{\sim}{j}(3+4) + \underset{\sim}{k}(-1-0)$$

$$= -2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$$

$\therefore$ Eq$^{\text{n}}$ of plane is of the form

$$-2x - 3y - z = d.$$

$$-2 + 0 - 3 = d \qquad \text{Using } A(1,0,3).$$

$$d = -5$$

$\therefore$ An equation of the plane π_1 is

$$2x + 3y + z = 5.$$

Normal to plane π_1 is $\underset{\sim}{n}_1 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

Normal to plane π_2 is $\underset{\sim}{n}_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

Let the angle between the planes π_1 and π_2 be θ

$$\therefore \quad \cos\theta = \frac{\underset{\sim}{n}_1 \cdot \underset{\sim}{n}_2}{|\underset{\sim}{n}_1||\underset{\sim}{n}_2|}$$

$$\underset{\sim}{n}_1 \cdot \underset{\sim}{n}_2 = (2\times1)+(3\times1)+(1\times(-1))$$
$$= 2+3-1$$
$$= 4$$

$$|\underset{\sim}{n}_1| = \sqrt{2^2+3^2+1^2} = \sqrt{14}$$
$$|\underset{\sim}{n}_2| = \sqrt{1^2+1^2+(-1)^2} = \sqrt{3}$$

So $$\cos\theta = \frac{4}{\sqrt{14}\sqrt{3}}$$
$$= \frac{4}{\sqrt{42}}$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{42}}\right)$$

$= 51 \cdot 9°$ (to 1dp) OR $0 \cdot 906$ rads (to 3dp)

(b)

$$\frac{x-11}{4} = \frac{y-15}{5} = \frac{z-12}{2} = t$$

Writing these in parametric form ...

$$x-11 = 4t \Rightarrow x = 4t+11$$
$$y-15 = 5t \Rightarrow y = 5t+15$$
$$z-12 = 2t \Rightarrow z = 2t+12$$

Substituting these into equation of plane π_2 ...

$$4t+11+5t+15-(2t+12) = 0$$
$$7t+14 = 0$$
$$7t = -14$$
$$t = -2$$

When $t=-2$: $x = -8+11 = 3$

$y = -10+15 = 5$

$z = -4+12 = 8$

$\therefore$ Point of intersection is $(3, 5, 8)$

Question 15

(a)

$$x\frac{dy}{dx} - 3y = x^4$$

$\div x$

$$\frac{dy}{dx} - \frac{3}{x}y = x^3$$

Linear first order d.e, $\frac{dy}{dx} + p(x)y = q(x)$

$\therefore$ Integrating factor is

$$e^{\int -\frac{3}{x}dx} = e^{-3\ln x} = e^{\ln x^{-3}} = x^{-3} = \frac{1}{x^3}$$

So

$$\frac{1}{x^3}\frac{dy}{dx} - \frac{3}{x^4}y = 1$$

multiplying through by $\frac{1}{x^3}$

$$\therefore \frac{d}{dx}\left(\frac{y}{x^3}\right) = 1$$

Alternatively...

$$\frac{1}{x^3}y = \int \frac{1}{x^3}.x^3 dx.$$

$$\frac{1}{x^3}y = \int 1\,dx$$

etc...

$$\therefore \frac{y}{x^3} = x + c$$

$$y = x^4 + x^3c.$$

When $\left.\begin{array}{l} y=2 \\ x=1 \end{array}\right\}$ $2 = 1 + C$

$\therefore C = 1$

So $y = x^4 + x^3.$ or $y = x^3(x+1)$

(b)

$$y\frac{dy}{dx} - 3x = x^4$$

$$\therefore \quad y\frac{dy}{dx} = x^4 + 3x$$

This is a variables separable first order d.e.

$$\therefore \int y \, dy = \int x^4 + 3x \, dx$$

$$\tfrac{1}{2}y^2 = \tfrac{1}{5}x^5 + \tfrac{3}{2}x^2 + c$$

When $\left.\begin{matrix} y = 2 \\ x = 1 \end{matrix}\right\}$ $\tfrac{1}{2} \times 2^2 = \tfrac{1}{5} + \tfrac{3}{2} + c$

$$2 = \tfrac{17}{10} + c$$

$$\therefore \quad c = 2 - \tfrac{17}{10} = \tfrac{3}{10}$$

So $\tfrac{1}{2}y^2 = \tfrac{1}{5}x^5 + \tfrac{3}{2}x^2 + \tfrac{3}{10}$

× 2 $\quad y^2 = \tfrac{2}{5}x^5 + 3x^2 + \tfrac{3}{5}$

$$\therefore \quad y = \left(\tfrac{2}{5}x^5 + 3x^2 + \tfrac{3}{5}\right)^{1/2}$$

or $y = \sqrt{\tfrac{2}{5}x^5 + 3x^2 + \tfrac{3}{5}}$

Question 16

(a) Using $l = a + (n-1)d$ where $a = 8$, $d = 3$, $l = 56$

$$\therefore \quad 56 = 8 + (n-1) \times 3$$

ie $3n + 5 = 56$

$3n = 51$

$n = 17$

Method 1

Using $S_n = \frac{n}{2}(a+l)$ where $a = 8$, $l = 56$, $n = 17$

$$\therefore \quad S_{17} = \frac{17}{2} \times (8+56) = \frac{17}{2} \times 64 = 17 \times 32$$
$$= 544$$

Method 2

Using $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ where $a = 8$, $d = 3$, $n = 17$

$$\therefore \quad S_{17} = \frac{17}{2}\left[2 \times 8 + (17-1) \times 3\right]$$
$$= \frac{17}{2}(16 + 16 \times 3)$$
$$= \frac{17}{2} \times 64 = 17 \times 32 = 544.$$

(b) **Method 1** First 3 terms are a, ar and ar^2

$\therefore \quad S_3 = a + ar + ar^2$ where $a = 2$, $S_3 = 266$

so $2 + 2r + 2r^2 = 266$

$r^2 + r + 1 = 133 \qquad \div 2$

$r^2 + r - 132 = 0$

$(r + 12)(r - 11) = 0$

$r = -12$ and $r = 11$

but $r > 0 \quad \therefore \quad r = 11.$

Method 2 Using $S_n = \dfrac{a(1-r^n)}{1-r}$, $r \neq 1$ where $a = 2$, $S_3 = 266$

$$\therefore \quad S_3 = \frac{2(1-r^3)}{1-r}$$

$$266 = \frac{2(1-r^3)}{1-r}$$

$\times \dfrac{1-r}{2}$

$$133(1-r) = 1 - r^3$$

$$\therefore \quad r^3 - 133r + 133 - 1 = 0.$$

$$r^3 - 133r + 132 = 0$$

1	1	0	−133.	132
		1	1	−132
	1	1	−132	0

$$\therefore \quad (r-1)(r^2 + r - 132) = 0$$

$$(r-1)(r+12)(r-11) = 0$$

$\therefore \quad r = 1$ or $r = -12$ or $r = 11$

but $r > 0$ but $r \neq 1$ so $r = 11.$

(c) Arithmetic $S_4 = \frac{4}{2}\left[2a + (4-1)d\right]$ where $d = 2$

$= 2(2a + 3 \times 2) = 4a + 12$

Geometric method 1 $S_4 = a + ar + ar^2 + ar^3$ where $r = 2$

$= a(1 + 2 + 2^2 + 2^3)$

$= a(1 + 2 + 4 + 8)$

$= 15a.$

method 2 $S_4 = \frac{a(1-r^4)}{1-2}$ or $\frac{a(r^4-1)}{2-1}$

$= \frac{a(2^4-1)}{1} = 15a$

$\therefore \quad 15a = 4a + 12$

$11a = 12$

$a = \frac{12}{11}.$

Consequently...

Arithmetic Sequence : $S_A = \frac{n}{2}\left[2.\frac{12}{11} + 2(n-1)\right]$

$= \frac{n}{2}\left[\frac{24}{11} + 2(n-1)\right]$

Geometric Sequence : $S_B = \frac{12}{11}(2^n - 1)$

We require n such that

$$S_B > 2S_A$$

Since equal when $n=4$ try $n>4$

n	5	6	7
S_A	$\frac{280}{11}$	$\frac{402}{11}$	$\frac{546}{11}$
S_B.	$\frac{372}{11}$	$\frac{756}{11}$	$\frac{1524}{11}$
Condition $S_B>2S_A$	X	x	✓

$\therefore$ Smallest value of n is 7.