HIGHER MATHS

PRELIM REVISION

2019-20

Differentiation

Functions

Straight Line

Vectors

Polynomials

Quadratics

Differentiation

Higher Maths Exam Questions

Source: 2019 P1 Q1 Higher Maths	
(1)	Find the x-coordinates of the stationary points on the curve with equation $y=\frac{1}{2}x^4-2x^3+6$.
Answers: $x = 0 \ and \ 3$	

Source: 2019 P2 Q7b Higher Maths	
(2)	(a) Express $-6x^2+24x-25$ in the form $p(x+q)^2+r$. (b) Given that $f(x)=-2x^3+12x^2-25x+9$, show that $f(x)$ is strictly decreasing for all $x\in\mathbb{R}$.

Answers: $(a) - 6(x-2)^2 - 1$

(b)

Method 1	Method 1
•4 differentiate	•4 $-6x^2+24x-25$
•5 link with (a) and identify sign of $(x-2)^2$	•5 $f'(x)=-6(x-2)^2-1$ and $(x-2)^2\geq 0\ \forall x$
•6 communicate reason	•6 eg $\therefore -6(x-2)^2-1<0\ \forall x$ $\Rightarrow$ always strictly decreasing

Method 2	Method 2
•4 differentiate	•4 $-6x^2+24x-25$
•5 identify maximum value of $f'(x)$	•5 'maximum value is -1' or annotated sketch including x-axis
•6 communicate reason	•6 $-1<0$ or 'graph lies below x-axis' $\therefore f'(x)<0\ \forall x$ $\Rightarrow$ always strictly decreasing

Source: 2018 P2 Q3 Higher Maths	
(3)	A function, f, is defined on the set of real numbers by $f(x)=x^3-7x-6$. Determine whether f is increasing or decreasing when $x=2$.

Answer:

•[1] differentiate	•[1] $3x^2-7$
•[2] evaluate derivative at $x=2$	•[2] 5
•[3] interpret result	•[3] (f is) increasing

Source: 2018 P2 Q9 Higher Maths	
(4)	A sector with a particular fixed area has radius x cm. The perimeter, P cm, of the sector is given by $$P=2x+\frac{128}{x}.$$ Find the minimum value of P.

Answer: $Minimum\ value\ of\ P = 32$

Source: 2017 P1 Q8 Higher Maths	
(5)	Calculate the rate of change of $d(t)=\frac{1}{2t}$, $t \neq 0$, when $t=5$.

Answer: $x = -\frac{1}{50}$

Source: 2017 P2 Q4 Higher Maths

(6)

(a) Express $3x^2+24x+50$ in the form $a(x+b)^2+c$.

(b) Given that $f(x)=x^3+12x^2+50x-11$, find $f'(x)$.

(c) Hence, or otherwise, explain why the curve with equation $y=f(x)$ is strictly increasing for all values of x.

Answers: $(a)\ 3(x+4)^2+2$ $(b)\ 3x^2+24x+50$

(c)

Method 1	Method 1
•[6] link with (a) and identify sign of $(x+4)^2$	•[6] $f'(x)=3(x+4)^2+2$ and $(x+4)^2\geq 0\ \forall x$
•[7] communicate reason	•[7] $\therefore 3(x+4)^2+2>0 \Rightarrow$ always strictly increasing

Method 2	Method 2
•[6] identify minimum value of $f'(x)$	•[6] eg minimum value =2 or annotated sketch
•[7] communicate reason	•[7] $2>0 \therefore (f'(x)>0) \Rightarrow$ always strictly increasing

Source: 2017 P2 Q7 Higher Maths

(7)

(a) Find the x-coordinate of the stationary point on the curve with equation $y=6x-2\sqrt{x^3}$.

(b) Hence, determine the greatest and least values of y in the interval $1\leq x\leq 9$.

Answers: $(a)\ x=4$ $(b)\ Greatest=8,\ Least=0$

Source: 2016 P1 Q2 Higher Maths

(8)

Given that $y=12x^3+8\sqrt{x}$, where $x>0$, find $\frac{dy}{dx}$.

Answer: $\frac{dy}{dx}=36x^2+4x^{\frac{-1}{2}}$

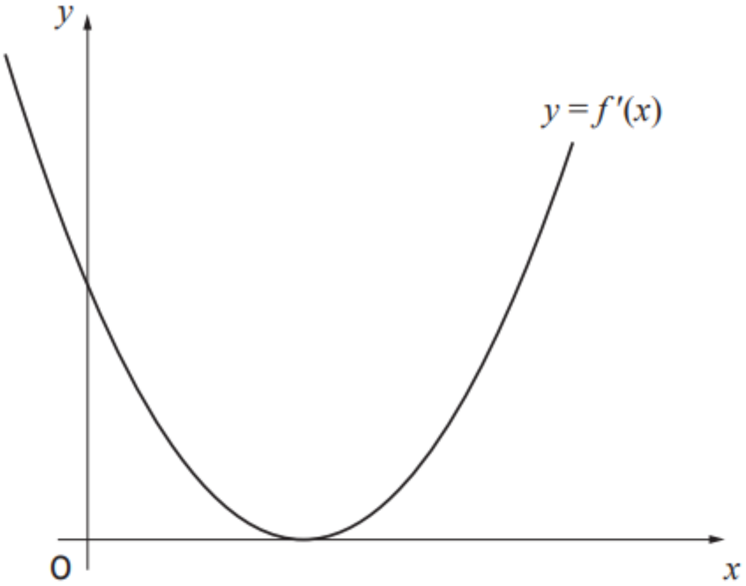
Source: 2016 P1 Q9 Higher Maths	
(9)	(a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$. (b) Hence determine the range of values of x for which the function f is strictly increasing.
Answers: $(a)\ x = -4, 2 \quad (b)\ x < -4, x > 2$	

Source: 2015 P1 Q2 Higher Maths	
(10)	Find the equation of the tangent to the curve $y = 2x^3 + 3$ at the point where $x = -2$.
Answer: $y = 24x + 35$	

Source: 2015 P1 Q7 Higher Maths	
(11)	A function f is defined on a suitable domain by $f(x) = \sqrt{x}\left(3x - \dfrac{2}{x\sqrt{x}}\right)$. Find $f'(4)$.
Answer: $9\frac{1}{8}$	

Source: Specimen P1 Q11 Higher Maths

(12) The diagram shows the graph of $y = f'(x)$. The x-axis is a tangent to this graph.

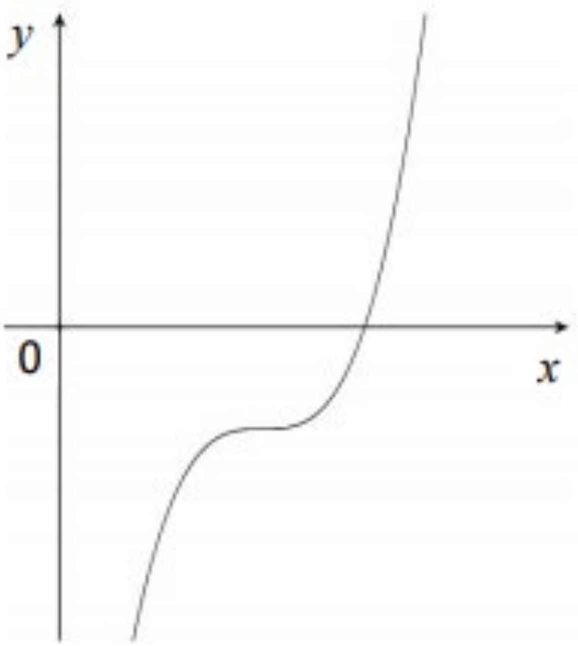


(a) Explain why the function $f(x)$ is never decreasing.

(b) On a graph of $y = f(x)$, the y-coordinate of the stationary point is negative. Sketch a possible graph for $y = f(x)$.

Answers: $(a)\ m = f'(x) \geq 0$

(b)



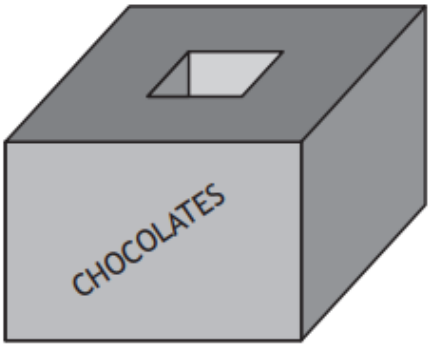
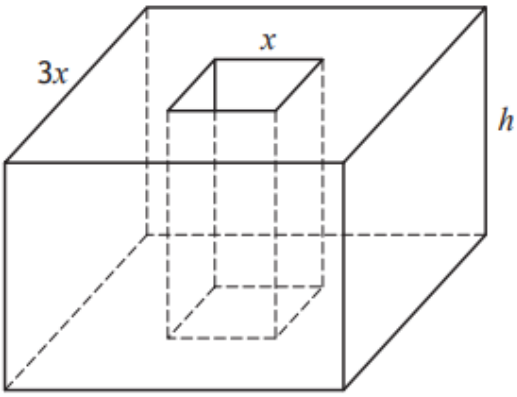
Source: Exemplar P1 Q1 Higher Maths	
(13)	The point P (5,12) lies on the curve with equation $y = x^2 - 4x + 7$. Find the equation of the tangent to this curve at P.
Answer:	$y - 12 = 6(x - 5)$

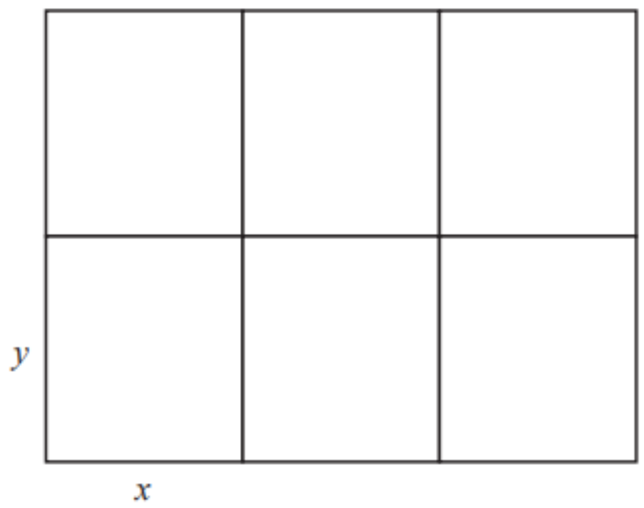
Source: Exemplar P2 Q10 Higher Maths	
(14)	Acceleration is defined as the rate of change of velocity. An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$. (a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. (b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. (c) Velocity is defined as the rate of change of displacement. Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$.
Answers:	(a) $a(t) = -16\sin(2t - \frac{\pi}{2})$ (b) $a(10) > 0$ *therefore increasing* (c) $s(t) = 4sin\left(2t - \frac{\pi}{2}\right) + 8$

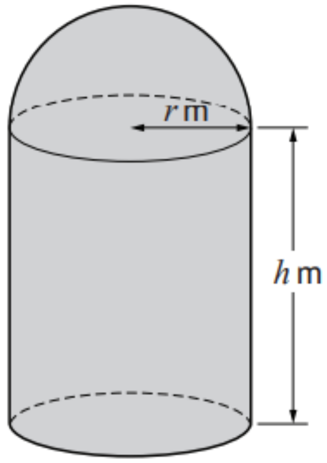
Source: 2014 P2 Q2 Higher Maths	
(15)	A curve has equation $y = x^4 - 2x^3 + 5$. Find the equation of the tangent to this curve at the point where $x = 2$.
Answer:	$y - 5 = 8(x - 2)$

Optimisation

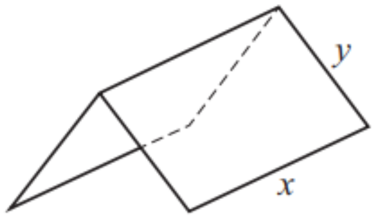
Higher Maths Exam Questions

Source: 2019 P2 Q11 Higher Maths	
(1)	A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.  The box is a cuboid with a cuboid shaped tunnel through it. • The height of the box is h centimetres • The top of the box is a square of side $3x$ centimetres • The end of the tunnel is a square of side x centimetres • The volume of the box is 2000 cm^3  (a) Show that the total surface area, A cm^2, of the box is given by $A = 16x^2 + \frac{4000}{x}$. (b) To minimise the cost of production, the surface area, A, of the box should be as small as possible. Find the minimum value of A.
Answers: $(a)\ Proof \quad (b)\ Minimum\ value\ of\ A = 1200$	

Source: 2016 P2 Q7 Higher Maths	
(2)	A council is setting aside an area of land to create six fenced plots where local residents can grow their own food. Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.  (a) The area of land being set aside is $108\ \text{m}^2$. Show that the total length of fencing, L metres, is given by $$L(x) = 9x + \frac{144}{x}.$$ (b) Find the value of x that minimises the length of fencing required.
Answers: $(a)\ Proof$ $(b)\ x = 4$	

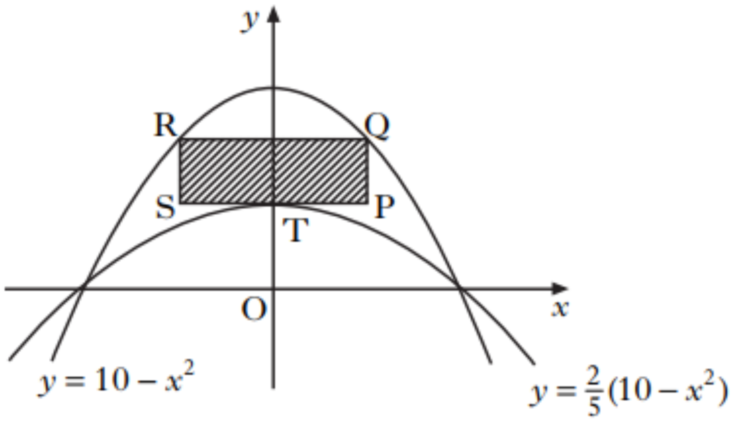
Source: Specimen P2 Q8 Higher Maths	
(3)	A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres. The volume of the **cylindrical** part of the container needs to be 100 cubic metres.  (a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by: $$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$ (b) Determine the value of r which minimises the amount of metal needed to build the container.

Answers: $(a)\ Proof$ $(b)\ r = 2.20\ m$

Source: Exemplar P2 Q9 Higher Maths	
(4)	A manufacturer is asked to design an open-ended shelter, as shown:  The frame of the shelter is to be made of rods of two different lengths: • x metres for top and bottom edges; • y metres for each sloping edge. The total length, L metres, of the rods used in a shelter is given by: $$L = 3x + \frac{48}{x}$$ To minimise production costs, the total length of rods used for a frame should be as small as possible. (a) Find the value of x for which L is a minimum. The rods used for the frame cost £8·25 per metre. The manufacturer claims that the minimum cost of a frame is less than £195. (b) Is this claim correct? Justify your answer.

Answers: $(a)\ x = 4\ cm \quad (b) \quad No\ as\ \pounds 198 > \pounds 195$

Source: 2010 P2 Q5 Higher Maths	
(5)	The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.



A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x-axis;
- T, the turning point of the lower parabola, lies on SP.

(*a*) (i) If TP = x units, find an expression for the length of PQ.

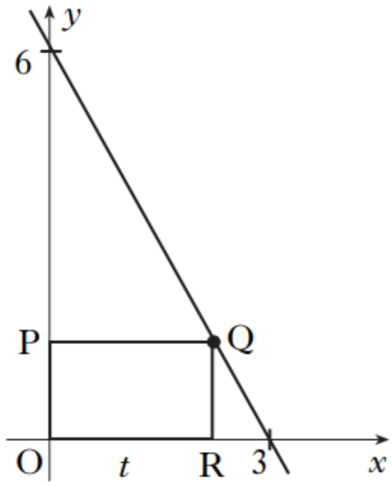
(ii) Hence show that the area, A, of rectangle PQRS is given by

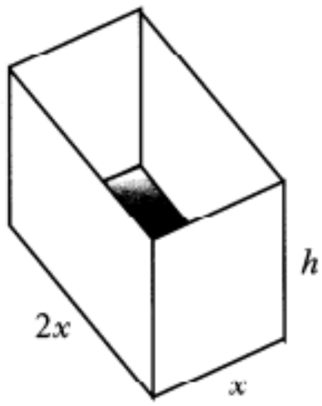
$$A(x) = 12x - 2x^3.$$

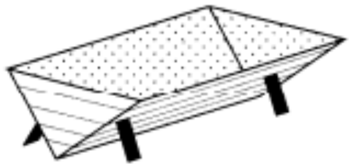
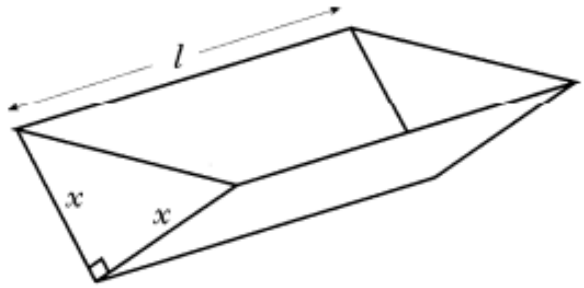
(*b*) Find the maximum area of this rectangle.

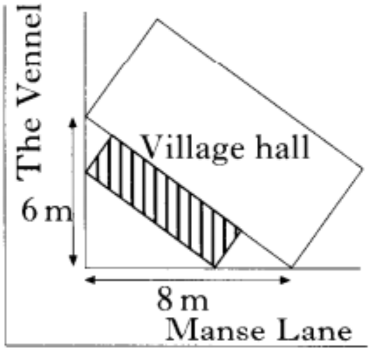
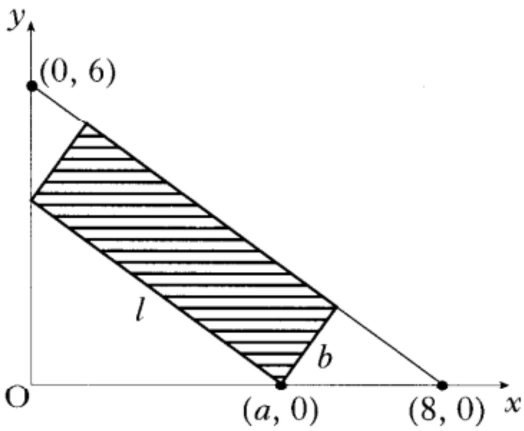
Answers: $(a)\ (i)\ PQ = 6 - x^2$ $\quad (ii)\ Proof$

$(b)\ Maximum\ area = 8\sqrt{2}\ (\approx 11.3)\ square\ units$

Source: 2008 P2 Q6 Higher Maths	
(6)	In the diagram, Q lies on the line joining (0, 6) and (3, 0). OPQR is a rectangle, where P and R lie on the axes and OR = t. (a) Show that QR = $6 - 2t$. (b) Find the coordinates of Q for which the rectangle has a maximum area. 
Answers:	(a) $Proof$ (b) $Q(\frac{3}{2}, 3)$

Source: 2004 P2 Q9 Higher Maths	
(7)	An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.  (a) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x(6 - x^2)$. ($b$) Find the exact value of x for which this volume is a maximum.
Answers:	(a) $Proof$ (b) $x = \sqrt{2}$ $units$

Source: 2003 P2 Q8 Higher Maths	
(8)	An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.  The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.  (*a*) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. (*b*) Find the value of x which minimises this surface area.
Answers: $(a)\ Proof$ $(b)\ x = 60$	

Source: 2002 P2 Q10 Higher Maths	
(9)	The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.  The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.  (a) (i) Show that $l = \frac{5}{4}a$. (ii) Express b in terms of a and hence deduce that the area, $A\,\text{m}^2$, of the extension is given by $A = \frac{3}{4}a(8-a)$. (b) Find the value of a which produces the largest area of the extension.

Answers:

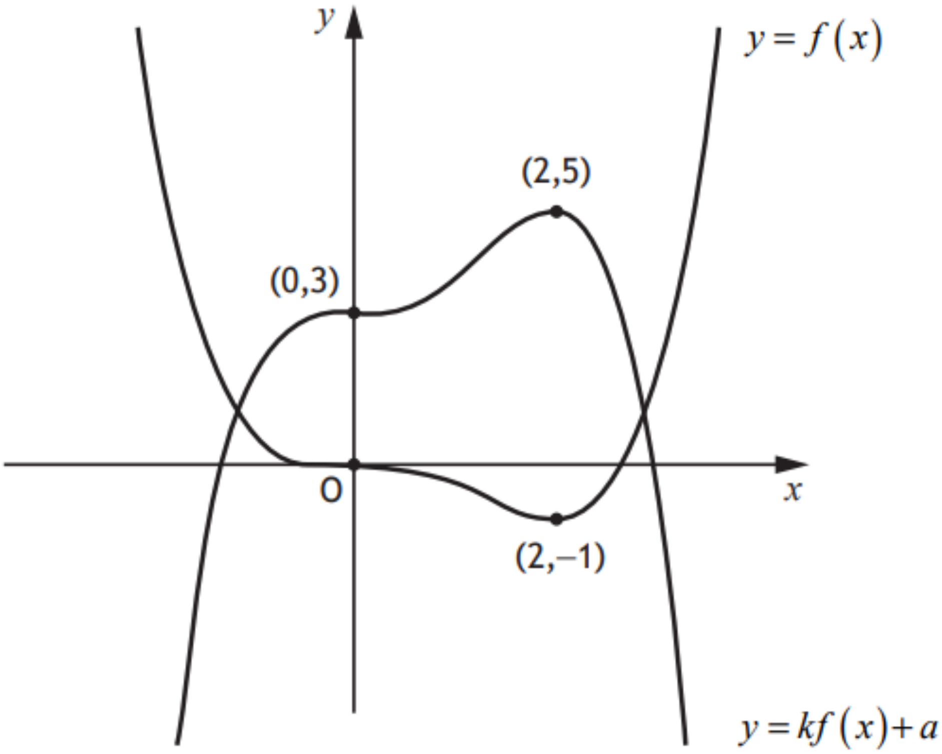
(a) (i) *Proof*

(ii) $b = \frac{3}{5}(8-a)$ *leading to* $A = \frac{3}{4}a(8-a)$

(b) $a = 4$

Functions & Graphs

Higher Maths Exam Questions

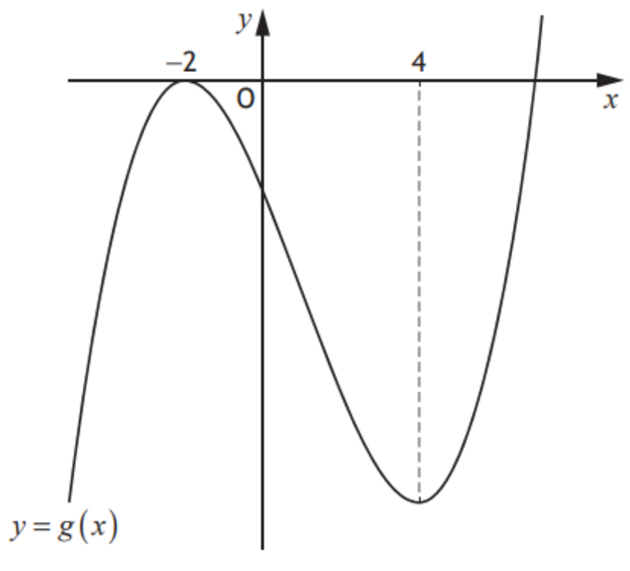
Source: 2019 P1 Q10 Higher Maths	
(1)	The diagram shows the graphs with equations $y = f(x)$ and $y = kf(x) + a$.  (a) State the value of a. (b) Find the value of k.
Answers: $(a)\ a = 3 \quad (b)\ k = -2$	

Source: 2019 P1 Q12 Higher Maths	
(2)	Functions f and g are defined by • $f(x)=\frac{1}{\sqrt{x}}$, where $x>0$ • $g(x)=5-x$, where $x\in\mathbb{R}$. (a) Determine an expression for $f(g(x))$. (b) State the range of values of x for which $f(g(x))$ is undefined.
Answers: $(a)\ \frac{1}{\sqrt{5-x}}$ $(b)\ x\geq 5$	

Source: 2019 P2 Q5 Higher Maths

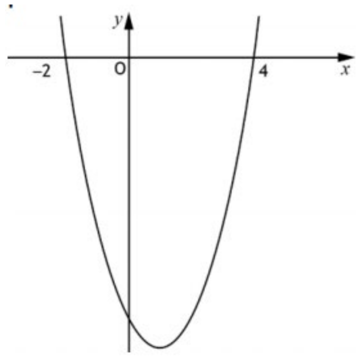
(3)

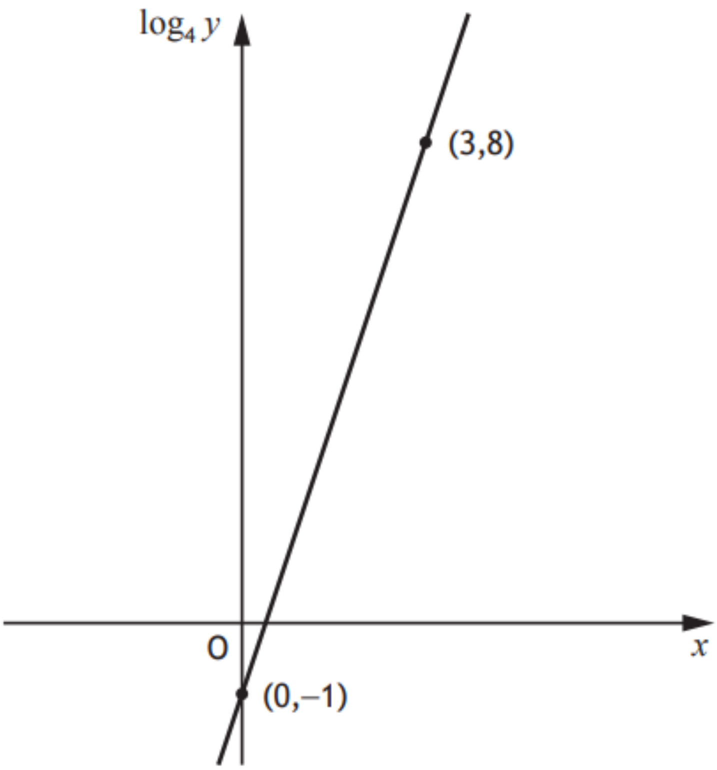
The diagram below shows the graph of a cubic function $y=g(x)$, with stationary points at $x=-2$ and $x=4$.



On the diagram in your answer booklet, sketch the graph of $y=g'(x)$.

Answer:



Source: 2019 P2 Q12 Higher Maths	
(4)	Two variables, x and y, are connected by the equation $y = ab^x$. The graph of $\log_4 y$ against x is a straight line as shown.  Find the values of a and b.
Answers: $a = \frac{1}{4}$ $b = 64$	

Source: 2018 P1 Q2 Higher Maths	
(5)	A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by $$g(x) = \frac{1}{5}x - 4.$$ Find the inverse function, $g^{-1}(x)$.
Answer: $g^{-1}(x) = 5(x + 4)$	

Source: 2018 P1 Q15 Higher Maths	
(6)	A cubic function, f, is defined on the set of real numbers. • $(x+4)$ is a factor of $f(x)$ • $x=2$ is a repeated root of $f(x)$ • $f'(-2)=0$ • $f'(x)>0$ where the graph with equation $y=f(x)$ crosses the y-axis Sketch a possible graph of $y=f(x)$ on the diagram in your answer booklet.

Answer:

•[1] root at $x=-4$ identifiable from graph	•[1]
•[2] stationary point touching x-axis when $x=2$ identifiable from graph	•[2]
•[3] stationary point when $x=-2$ identifiable from graph	•[3]
•[4] identify orientation of the cubic curve and $f'(0)>0$ identifiable from graph	•[4] 

Source: 2018 P2 Q6 Higher Maths	
(7)	Functions, f and g, are given by $f(x)=3+\cos x$ and $g(x)=2x$, $x\in\mathbb{R}$. (a) Find expressions for (i) $f(g(x))$ and (ii) $g(f(x))$. (b) Determine the value(s) of x for which $f(g(x))=g(f(x))$ where $0\leq x<2\pi$.

Answers: $(a)\ (i)\ 3+cos2x\quad (ii)\ 2(3+cosx)\qquad (b)\ x=\pi$

Source: 2017 P1 Q1 Higher Maths	
(8)	Functions f and g are defined on suitable domains by $f(x) = 5x$ and $g(x) = 2\cos x$. (a) Evaluate $f(g(0))$. (b) Find an expression for $g(f(x))$.
Answers: $(a)\ 10 \quad (b)\ 2cos5x$	

Source: 2017 P1 Q6 Higher Maths	
(9)	A function, h, is defined by $h(x) = x^3 + 7$, where $x \in \mathbb{R}$. Determine an expression for $h^{-1}(x)$.
Answer: $h^{-1}(x) = \sqrt[3]{x-7} \quad or \quad (x-7)^{\frac{1}{3}}$	

Source: 2016 P1 Q6 Higher Maths	
(10)	Functions f and g are defined on $\mathbb{R}$, the set of real numbers. The inverse functions f^{-1} and g^{-1} both exist. (a) Given $f(x) = 3x + 5$, find $f^{-1}(x)$. (b) If $g(2) = 7$, write down the value of $g^{-1}(7)$.
Answers: $(a)\ g^{-1}(x) = \frac{x-5}{3} \quad (b)\ g^{-1}(7) = 2$	

Source: 2017 P1 Q15a Higher Maths

(11) A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.

The turning point is $(2, 3)$.

Diagram 2 shows part of the graph with equation $y = h(x)$.

The turning point is $(7, 6)$.

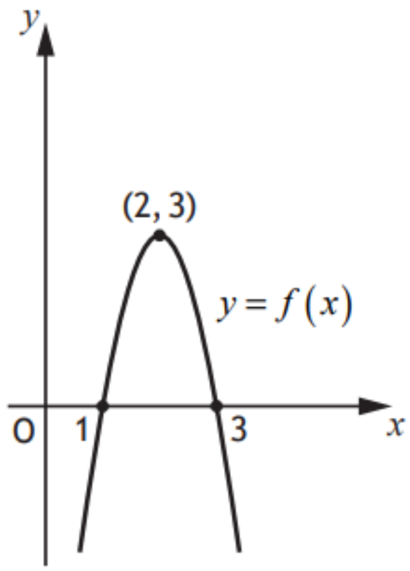


Diagram 1

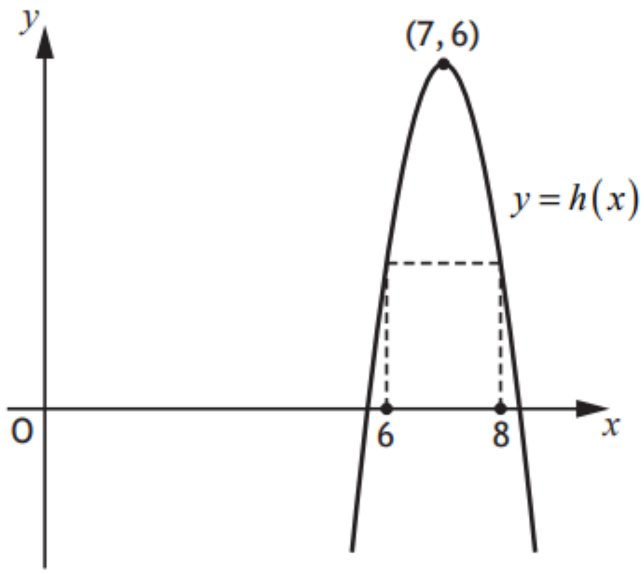


Diagram 2

(a) Given that $h(x) = f(x + a) + b$.

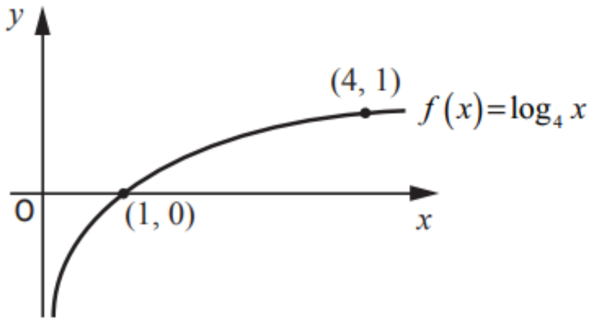
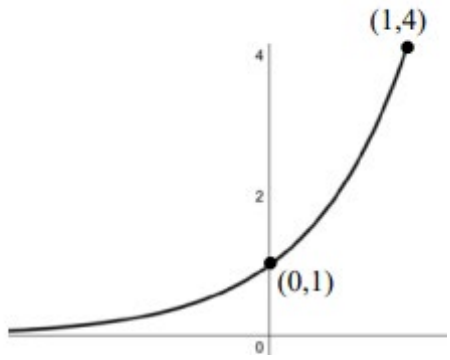
Write down the values of a and b.

(b) It is known that $\int_1^3 f(x)\,dx = 4$.

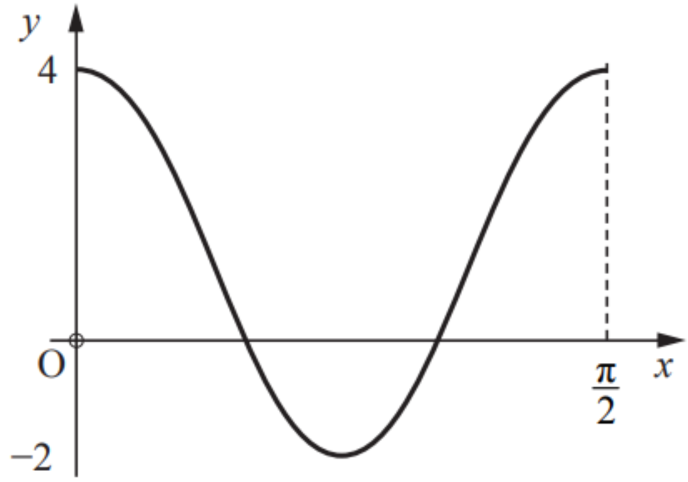
Determine the value of $\int_6^8 h(x)\,dx$.

(c) Given $f'(1) = 6$, state the value of $h'(8)$.

Answers: $(a)\ a = -5 \ \ b = 3 \quad (b)\ 10 \quad (c) - 6$

Source: 2016 P1 Q10 Higher Maths	
(12)	The diagram below shows the graph of the function $f(x)=\log_4 x$, where $x>0$.  The inverse function, f^{-1}, exists. On the diagram in your answer booklet, sketch the graph of the inverse function.
Answer: 	

Source: 2016 P1 Q12 Higher Maths	
(13)	The functions f and g are defined on $\mathbb{R}$, the set of real numbers by $f(x)=2x^2-4x+5$ and $g(x)=3-x$. (a) Given $h(x)=f(g(x))$, show that $h(x)=2x^2-8x+11$. (b) Express $h(x)$ in the form $p(x+q)^2+r$.
Answers: $(a)\ Proof \quad (b)\ 2(x-2)^2+3$	

Source: 2015 P1 Q4 Higher Maths	
(14)	The diagram shows part of the graph of the function $y = p\cos qx + r$  Write down the values of p, q and r.
Answers: $p = 3, \quad q = 4, \quad r = 1$	

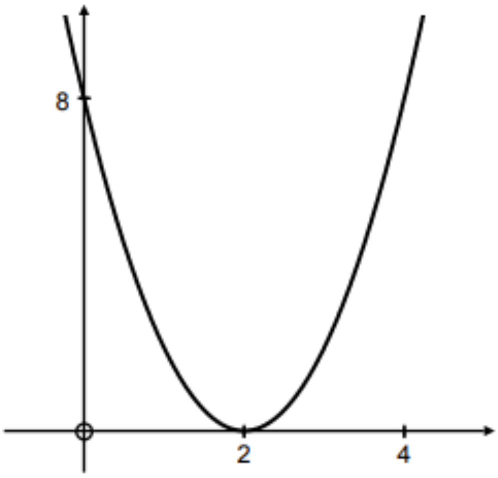
Source: 2015 P1 Q5 Higher Maths	
(15)	A function g is defined on $\mathbb{R}$, the set of real numbers, by $g(x) = 6 - 2x$. (a) Determine an expression for $g^{-1}(x)$. (b) Write down an expression for $g(g^{-1}(x))$.
Answers: $(a)\ g^{-1}(x) = \frac{6-x}{2} \quad = 3 - \frac{x}{2} \quad = \frac{x-6}{-2}$ $(b)\ x$	

Source: 2015 P2 Q2 Higher Maths	
(16)	Functions f and g are defined on suitable domains by $f(x) = 10 + x$ and $g(x) = (1 + x)(3 - x) + 2$. (a) Find an expression for $f(g(x))$. (b) Express $f(g(x))$ in the form $p(x+q)^2 + r$. (c) Another function h is given by $h(x) = \frac{1}{f(g(x))}$. What values of x cannot be in the domain of h?
Answers: $(a) 10 + (1+x)(3-x) + 2$ $(b) -x^2 + 2x + 15$ $(c)\ 5\ \&\ -3$	

Source: Specimen P1 Q8 Higher Maths	
(17)	$f(x)$ and $g(x)$ are functions, defined on the set of real numbers, such that $f(x) = 1 - \frac{1}{2}x$ and $g(x) = 8x^2 - 3$. (a) Given that $h(x) = g(f(x))$, show that $h(x) = 2x^2 - 8x + 5$. (b) Express $h(x)$ in the form $a(x+p)^2 + q$. (c) Hence, or otherwise, state the coordinates of the turning point on the graph of $y = h(x)$. (d) Sketch the graph of $y = h(x) + 3$, showing clearly the coordinates of the turning point and the y-axis intercept.

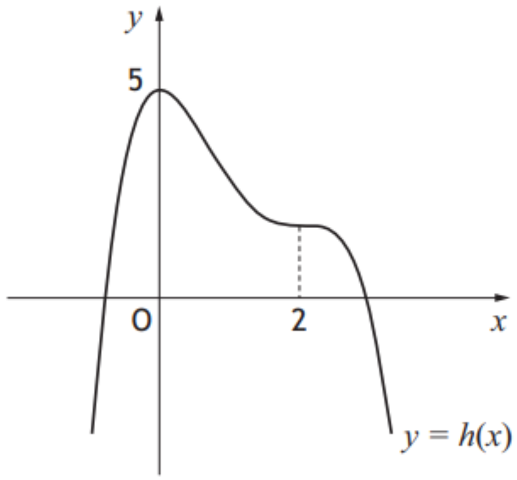
Answers: $(a)\ Proof$ $(b)\ 2(x-2)^2 - 3$ $(c)\ (2, -3)$

(d)



Source: Exemplar P1 Q8 Higher Maths

(18) The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



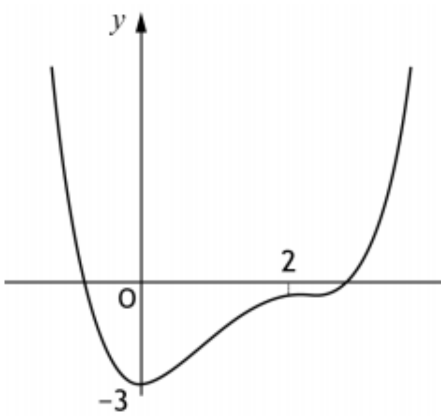
On separate diagrams sketch the graphs of:

(a) $y = 2 - h(x)$.

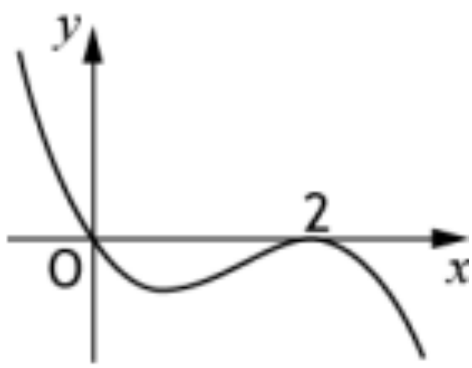
(b) $y = h'(x)$.

Answers:

(a)



(b)



Polynomials & Quadratics

Higher Maths Exam Questions

Source: 2019 P1 Q2 Higher Maths	
(1)	The equation $x^2+(k-5)x+1=0$ has equal roots. Determine the possible values of k.
Answers: $k=3,\ 7$	

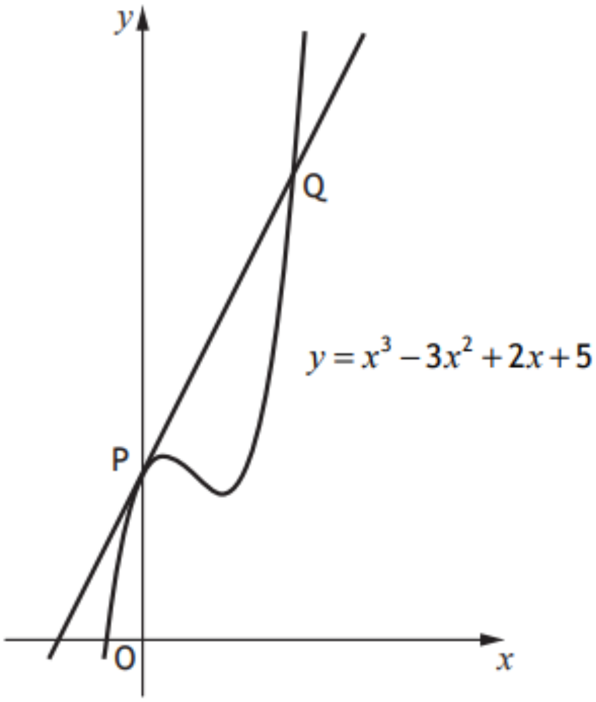
Source: 2019 P2 Q7 Higher Maths	
(2)	(a) Express $-6x^2+24x-25$ in the form $p(x+q)^2+r$. (b) Given that $f(x)=-2x^3+12x^2-25x+9$, show that $f(x)$ is strictly decreasing for all $x\in\mathbb{R}$.

Answers: $(a) -6(x-2)^2-1$

(b)

Method 1	Method 1
•[4] differentiate	•[4] $-6x^2+24x-25$
•[5] link with (a) and identify sign of $(x-2)^2$	•[5] $f'(x)=-6(x-2)^2-1$ and $(x-2)^2\geq 0\ \forall x$
•[6] communicate reason	•[6] eg $\therefore -6(x-2)^2-1<0\ \forall x$ $\Rightarrow$ always strictly decreasing
Method 2	**Method 2**
•[4] differentiate	•[4] $-6x^2+24x-25$
•[5] identify maximum value of $f'(x)$	•[5] 'maximum value is -1' or annotated sketch including x-axis
•[6] communicate reason	•[6] $-1<0$ or 'graph lies below x-axis' $\therefore f'(x)<0\ \forall x$ $\Rightarrow$ always strictly decreasing

Source: 2019 P2 Q10 Higher Maths	
(3)	(a) Show that $(x+3)$ is a factor of $3x^4+10x^3+x^2-8x-6$. (b) Hence, or otherwise, factorise $3x^4+10x^3+x^2-8x-6$ fully.
Answers: (a) *Use synthetic division to show the remainder equals zero* (b) $(x+3)(x-1)(3x^2+4x+2)$	

Source: 2018 P1 Q7 Higher Maths	
(4)	The curve with equation $y=x^3-3x^2+2x+5$ is shown on the diagram.  (a) Write down the coordinates of P, the point where the curve crosses the y-axis . (b) Determine the equation of the tangent to the curve at P. (c) Find the coordinates of Q, the point where this tangent meets the curve again.
Answers: (a) $P(0,5)$ (b) $y=2x+5$ (c) $Q(3,11)$	

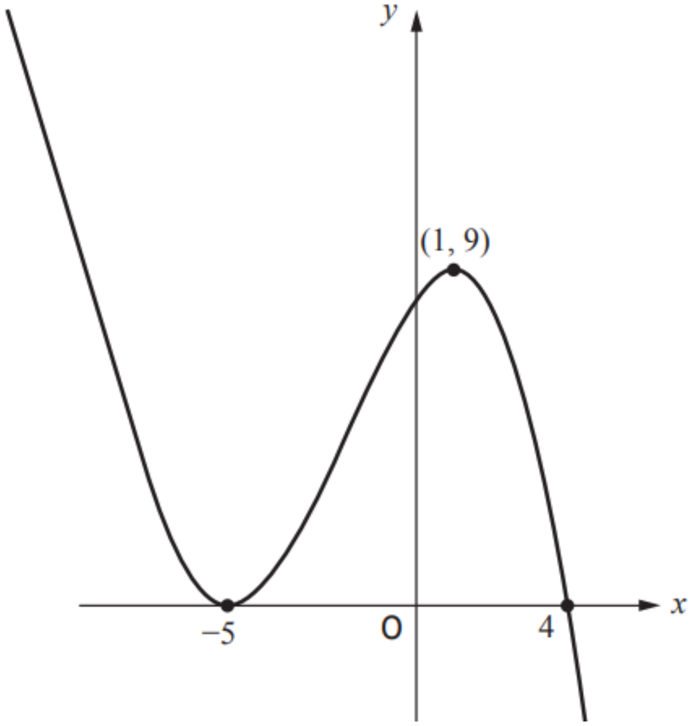
Source: 2018 P2 Q4 Higher Maths	
(5)	Express $-3x^2-6x+7$ in the form $a(x+b)^2+c$.
Answer: $-3(x+1)^2+10$	

Source: 2018 P2 Q7a Higher Maths	
(6)	(a) (i) Show that $(x-2)$ is a factor of $2x^3-3x^2-3x+2$. (ii) Hence, factorise $2x^3-3x^2-3x+2$ fully. The fifth term, u_5, of a sequence is $u_5=2a-3$. The terms of the sequence satisfy the recurrence relation $u_{n+1}=au_n-1$. (b) Show that $u_7=2a^3-3a^2-a-1$. For this sequence, it is known that • $u_7=u_5$ • a limit exists. (c) (i) Determine the value of a. (ii) Calculate the limit.
Answers: $(a)\ (i)\ \ Use\ synthetic\ division\ to\ show\ remainder\ equals\ zero$ $(ii)\ (x-2)(2x-1)(x+1)$ $(b)\ \ 2a^3-3a^2-a-1$ $(c)\ (i)\ a=\frac{1}{2}\ \ (ii)\ L=-2$	

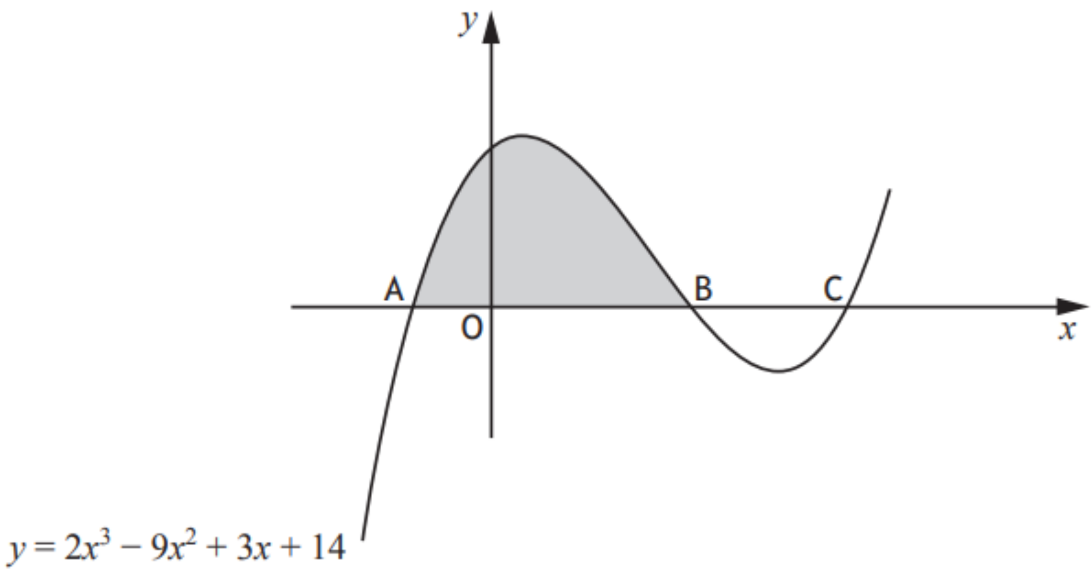
Source: 2018 P2 Q10 Higher Maths	
(7)	The equation $x^2+(m-3)x+m=0$ has two real and distinct roots. Determine the range of values for m.
Answers: $m < 1, m > 9$ *with eg sketch or table of signs*	

Source: 2017 P1 Q4 Higher Maths	
(8)	Find the value of k for which the equation $x^2+4x+(k-5)=0$ has equal roots.
Answer: $k = 9$	

Source: 2017 P2 Q2 Higher Maths	
(9)	(a) Show that $(x-1)$ is a factor of $f(x)=2x^3-5x^2+x+2$. (b) Hence, or otherwise, solve $f(x)=0$.
Answers: (a) *Use synthetic division to show remainder equals zero* $(b)\ x = -\frac{1}{2}, 1, 2$	

Source: 2016 P1 Q15 Higher Maths	
(10)	The diagram below shows the graph with equation $y = f(x)$, where $f(x) = k(x-a)(x-b)^2$.  (a) Find the values of a, b and k. (b) For the function $g(x) = f(x) - d$, where d is positive, determine the range of values of d for which $g(x)$ has exactly one real root.
Answers: $(a)\ a = 4,\ b = -5,\ k = -\frac{1}{12}$ $(b)\ d > 9$	

Source: 2016 P2 Q2 Higher Maths	
(11)	Find the range of values for p such that $x^2 - 2x + 3 - p = 0$ has no real roots.
Answer: $p < 2$	

Source: 2016 P2 Q3 Higher Maths	
(12)	(a) (i) Show that $(x+1)$ is a factor of $2x^3 - 9x^2 + 3x + 14$. (ii) Hence solve the equation $2x^3 - 9x^2 + 3x + 14 = 0$. (b) The diagram below shows the graph with equation $y = 2x^3 - 9x^2 + 3x + 14$. The curve cuts the x-axis at A, B and C.  (i) Write down the coordinates of the points A and B. (ii) Hence calculate the shaded area in the diagram.

Answers:

(a) (i) *Use synthetic division to show remainder equals zero*

(ii) $x = -1, 2, 3.5$

(b) (i) $A\ (-1, 0)$ $B\ (2, 0)$

(ii) $Area = 27\ units^2$

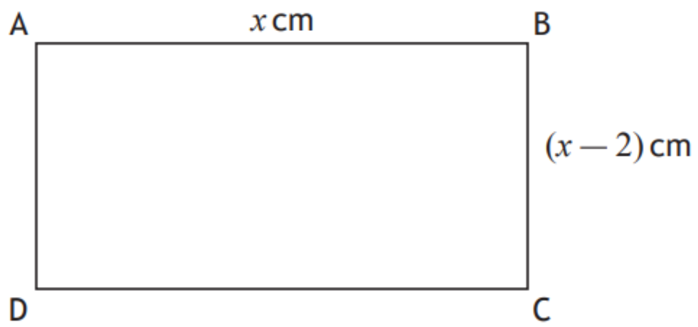
Source: 2015 P1 Q3 Higher Maths	
(13)	Show that $(x + 3)$ is a factor of $x^3 - 3x^2 - 10x + 24$ and hence factorise $x^3 - 3x^2 - 10x + 24$ fully.

Answer: *Use synthetic division to show remainder equals zero*

$(x + 3)(x - 4)(x - 2)$

Source: 2015 P1 Q8 Higher Maths

(14) ABCD is a rectangle with sides of lengths x centimetres and $(x-2)$ centimetres, as shown.



If the area of ABCD is less than $15\,\text{cm}^2$, determine the range of possible values of x.

Answer: $2 < x < 5$

Source: Specimen P1 Q4 Higher Maths

(15) Given that $2x^2 + px + p + 6 = 0$ has no real roots, find the range of values for p, where $p \in \mathbb{R}$.

Answer: $-4 < p < 12$

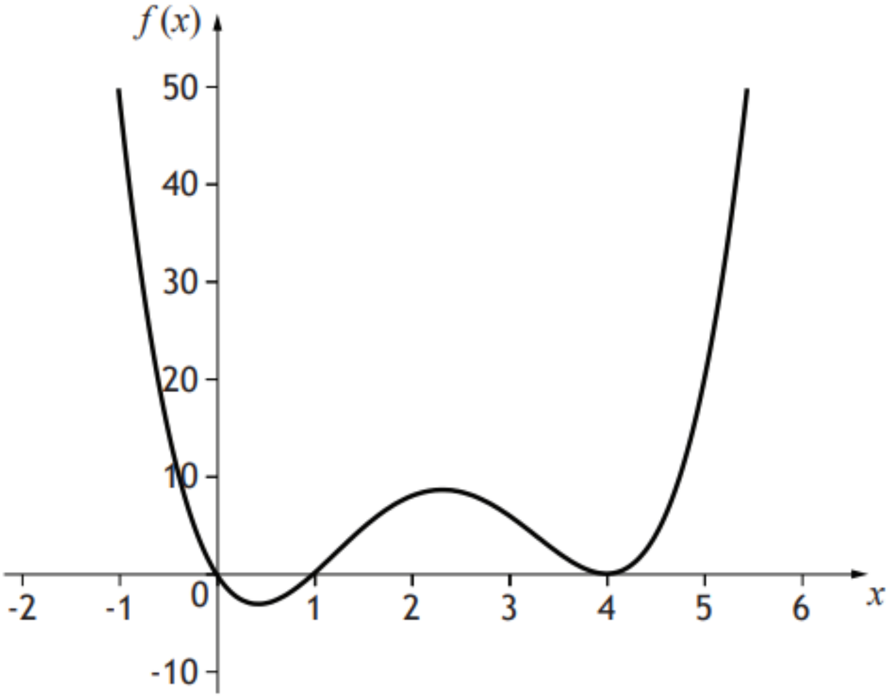
Source: Specimen P1 Q7 Higher Maths

(16) (a) Show that $(x+1)$ is a factor of $x^3 - 13x - 12$.

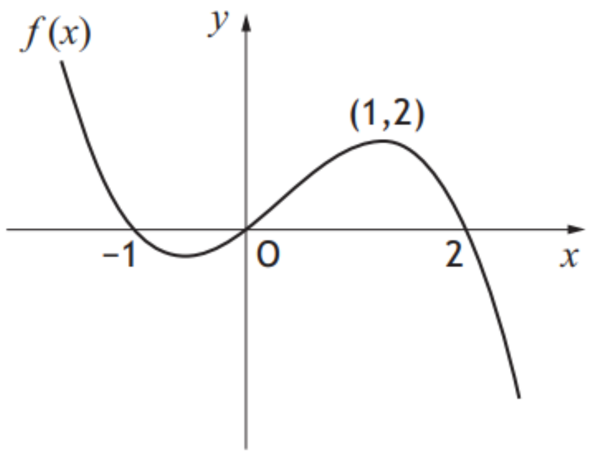
(b) Factorise $x^3 - 13x - 12$ fully.

Answer: (a) *Use synthetic division to show remainder equals zero*

$(b)\ (x+1)(x+3)(x-4)$

Source: Specimen P2 Q3 Higher Maths	
(17)	The diagram shows the graph of $f(x) = x(x-p)(x-q)^2$.  (a) Determine the values of p and q. (b) Find the equation of the tangent to the curve when $x = 1$.
Answers: $(a)\ p = 1,\ q = 4$ $\quad (b)\ y = 9(x-1)$	

Source: Exemplar P2 Q3 Higher Maths	
(18)	Find the value of p such that the equation $x^2 + (p+1)x + 9 = 0$ has no real roots.
Answer: $-7 < p < 5$	

Source: Exemplar P1 Q2 Higher Maths	
(19)	The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$. The curve passes through (-1,0), (0,0), (1,2) and (2,0).  Find the values of a, b and k.
Answers: $a = 1, b = -2,\ k = -1$	

Source: Exemplar P1 Q5 Higher Maths	
(20)	For the polynomial, $x^3 - 4x^2 + ax + b$ • $x - 1$ is a factor • -12 is the remainder when it is divided by $x - 2$ (a) Determine the values of a and b. (b) Hence solve $x^3 - 4x^2 + ax + b = 0$.
Answers: $(a)\ a = -7,\ b = 10 \quad (b)\ x = 1, x = 5, x = -2$	

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Straight Lines

Higher Maths Exam Questions

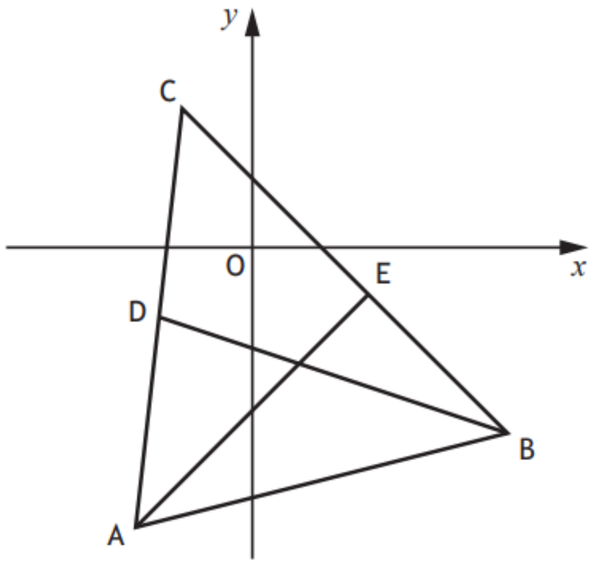
Source: 2019 P1 Q5 Higher Maths	
(1)	(a) Show that the points A(1,5,–3), B(4,–1,0) and C(8,–9,4) are collinear. (b) State the ratio in which B divides AC.
Answers: $(a)\ \overrightarrow{AB} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}\ \overrightarrow{BC} = \begin{pmatrix} 4 \\ -8 \\ 4 \end{pmatrix}\quad \overrightarrow{AB} = \frac{3}{4}\overrightarrow{BC}\ .$ $AB\ is\ parallel\ to\ BC\ (common\ diection)\ and\ B\ is\ a\ common\ point.$ $Therefore\ A, B\ and\ C\ are\ collinear.$ $(b)\ Ratio\ 3:4$	

Source: 2019 P1 Q7 Higher Maths	
(2)	The line, L, makes an angle of 30° with the positive direction of the x-axis. Find the equation of the line perpendicular to L, passing through (0,–4).
Answer: $y = -\sqrt{3}x - 4$	

Source: 2019 P2 Q1 Higher Maths

(3)

Triangle ABC has vertices A(–5,–12), B(11,–8) and C(–3,6).



(a) Find the equation of the median BD.

(b) Find the equation of the altitude AE.

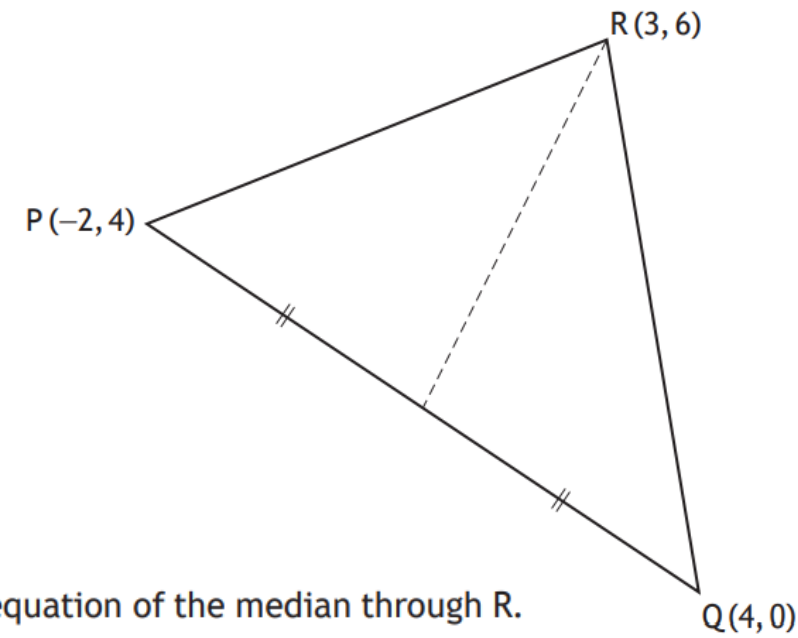
(c) Find the coordinates of the point of intersection of BD and AE.

Answers: $(a)\ 3y = -x - 13$ $(b)\ y = x - 7$ $(c)\ (2, -5)$

Source: 2018 P1 Q1 Higher Maths

(4)

PQR is a triangle with vertices P (–2, 4), Q (4, 0) and R (3, 6).



Find the equation of the median through R.

Answer: $y = 2x$

Source: 2018 P1 Q8 Higher Maths	
(5)	A line has equation $y - \sqrt{3}x + 5 = 0$. Determine the angle this line makes with the positive direction of the x-axis.
Answer: $60° \quad or \quad \frac{\pi}{3}$	

Source: 2017 P1 Q7 Higher Maths	
(6)	A $(-3, 5)$, B $(7, 9)$ and C $(2, 11)$ are the vertices of a triangle. Find the equation of the median through C.
Answer: $x = 2$	

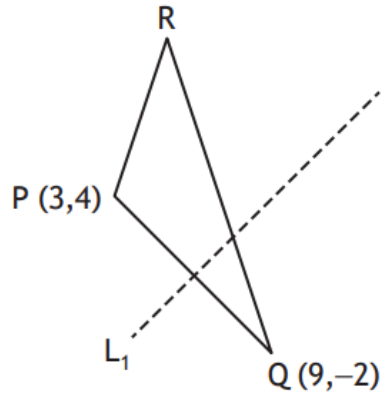
Source: 2017 P1 Q11 Higher Maths	
(7)	A and B are the points $(-7, 2)$ and $(5, a)$. AB is parallel to the line with equation $3y - 2x = 4$. Determine the value of a.
Answer: $a = 10$	

Source: 2015 P1 Q9 Higher Maths	
(8)	A, B and C are points such that AB is parallel to the line with equation $y + \sqrt{3}x = 0$ and BC makes an angle of $150°$ with the positive direction of the x-axis. Are the points A, B and C collinear?
Answer: $m_{ab} = -\sqrt{3} \quad and \quad m_{bc} = -\frac{1}{\sqrt{3}} \quad thereore\ points\ are\ not\ collinear$	

Source: 2018 P2 Q5a,b Higher Maths

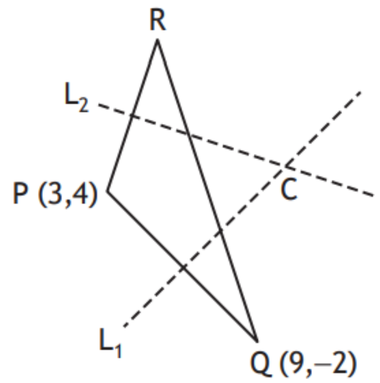
(9)

PQR is a triangle with P(3,4) and Q(9,–2).



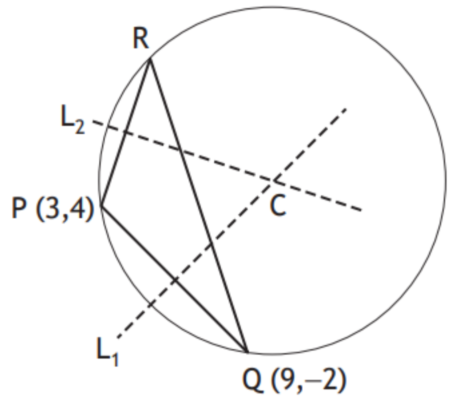
(a) Find the equation of L_1, the perpendicular bisector of PQ.

The equation of L_2, the perpendicular bisector of PR is $3y + x = 25$.



(b) Calculate the coordinates of C, the point of intersection of L_1 and L_2.

C is the centre of the circle which passes through the vertices of triangle PQR.



(c) Determine the equation of this circle.

Answers: $(a)\ y = x - 5$ $(b)\ C(10, 5)$ $(c)\ (x - 10)^2 + (y - 5)^2 = 50$

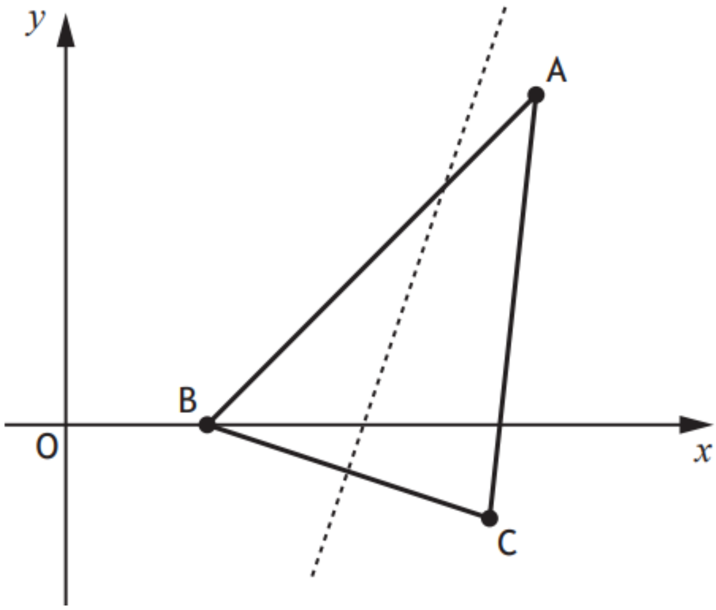
Source: 2017 P2 Q1 Higher Maths

(10)

Triangle ABC is shown in the diagram below.

The coordinates of B are (3,0) and the coordinates of C are (9,–2).

The broken line is the perpendicular bisector of BC.



(a) Find the equation of the perpendicular bisector of BC.

(b) The line AB makes an angle of 45° with the positive direction of the x-axis.

Find the equation of AB.

(c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC.

Answers: $(a)\ y = 3x - 19$ $(b)\ y = x - 3$ $(c)\ (8, 5)$

Source: 2016 P1 Q1 Higher Maths

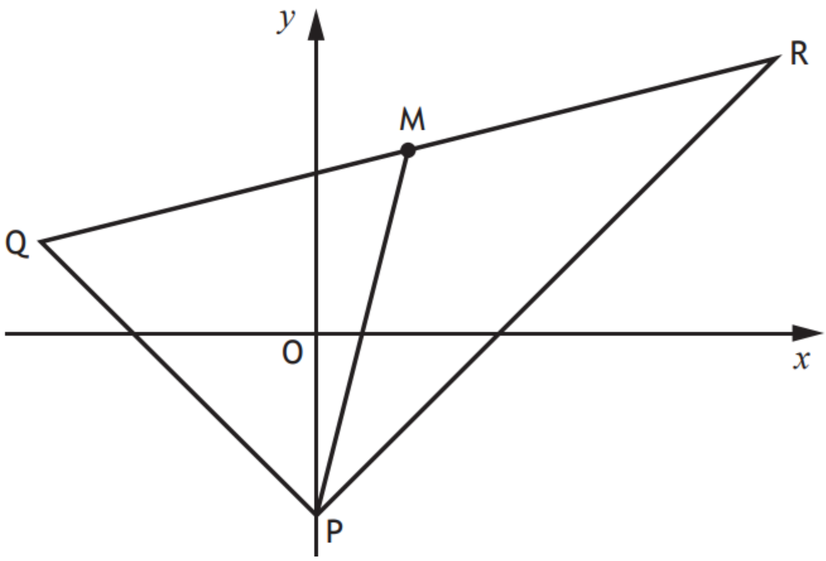
(11)

Find the equation of the line passing through the point $(-2, 3)$ which is parallel to the line with equation $y + 4x = 7$.

Answer: $y + 4x = -5$

Source: 2016 P2 Q1 Higher Maths

(12) PQR is a triangle with vertices P$(0,-4)$, Q$(-6,2)$ and R$(10,6)$.



(a) (i) State the coordinates of M, the midpoint of QR.

(ii) Hence find the equation of PM, the median through P.

(b) Find the equation of the line, *L*, passing through M and perpendicular to PR.

(c) Show that line *L* passes through the midpoint of PR.

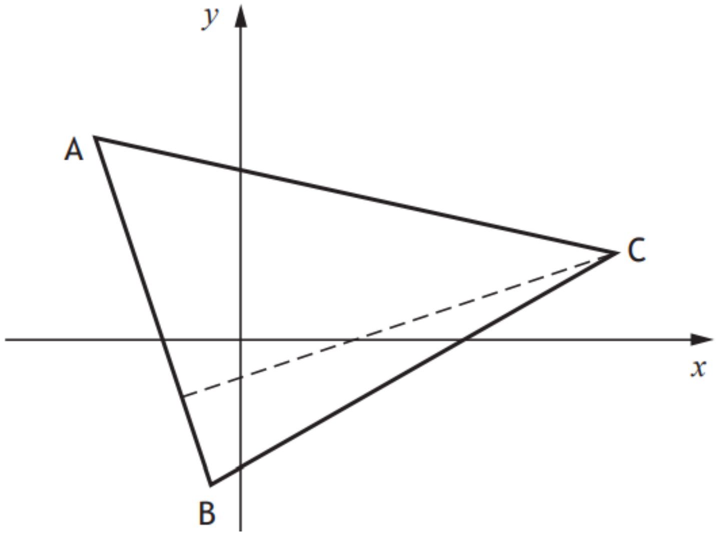
Answers: $(a)\ (i)\ M(2,4)$ $(ii)\ y = 4x - 6$

$(b)\ y = -x + 6$

(c)

•[1] find the midpoint of PR	•[1] $(5,1)$
•[2] substitute x-coordinate into equation of L.	•[2] $y = -5 + 6\ \ (1 = -x + 6)$
•[3] verify y-coordinate and communicate conclusion	•[3] $y = 1\ (x = 5)\ \therefore$ L passes through the midpoint of PR

Other methods valid – see marking scheme

Source: 2015 P2 Q1 Higher Maths	
(13)	The vertices of triangle ABC are A(-5, 7), B(-1, -5) and C(13, 3) as shown in the diagram. The broken line represents the altitude from C.  (a) Show that the equation of the altitude from C is $x - 3y = 4$. (b) Find the equation of the median from B. (c) Find the coordinates of the point of intersection of the altitude from C and the median from B.
Answers: $(a)\ Proof$ $(b)\ y = 2x - 3$ $(c)\ (1,\ -1)$	

Source: Specimen P1 Q2 Higher Maths	
(14)	Find the coordinates of the points of intersection of the curve $y = x^3 - 2x^2 + x + 4$ and the line $y = 4x + 4$.
Answers: $(-1, 0)$ $(0, 4)$ $(3, 16)$	

Source: Specimen P1 Q5 Higher Maths

(15) Line l_1 has equation $\sqrt{3}y - x = 0$.

(a) Line l_2 is perpendicular to l_1. Find the gradient of l_2.

(b) Calculate the angle l_2 makes with the positive direction of the x-axis.

Answers: $(a)\ m = -\sqrt{3}$ $(b)\ Angle = \frac{2\pi}{3}\ or\ 120°$

Source: Specimen P1 Q9 Higher Maths

(16) (a) AB is a line parallel to the line with equation $y + 3x = 25$.

A has coordinates $(-1, 10)$.

Find the equation of AB.

(b) $3y = x + 11$ is the perpendicular bisector of AB.

Determine the coordinates of B.

Answers: $(a)\ y - 10 = -3(x + 1)$ $(b)\ B\ (3,\ -2)$

Source: Exemplar P1 Q6 Higher Maths

(17) (a) Find the equation of l_1, the perpendicular bisector of the line joining P $(3,-3)$ and Q $(-1,9)$.

(b) Find the equation of l_2 which is parallel to PQ and passes through R $(1,-2)$.

(c) Find the point of intersection of l_1 and l_2.

(d) Hence find the shortest distance between PQ and l_2.

Answers: $(a)\ y - 3 = \frac{1}{3}(x - 1)$ $(b)\ y + 2 = -3(x - 1)$

$(c)\ x = -\frac{1}{2},\quad y = \frac{5}{2}$ $(c)\ \sqrt{\frac{5}{2}} = \frac{\sqrt{10}}{2} = \sqrt{2.5}$

Vectors

Higher Maths Exam Questions

Source: 2019 P1 Q9 Higher Maths

(1)

Vectors **u** and **v** have components $\begin{pmatrix} p \\ -2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 2p+16 \\ -3 \\ 6 \end{pmatrix}$, $p \in \mathbb{R}$.

(a) (i) Find an expression for **u.v**.

(ii) Determine the values of p for which **u** and **v** are perpendicular.

(b) Determine the value of p for which **u** and **v** are parallel.

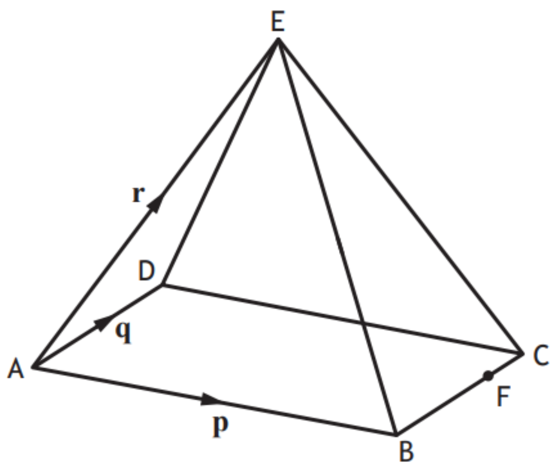
Answers: $(a)(i)\ 2p^2 + 16p + 30$ $(ii)\ p = -5\ \&\ -3$ $(b)\ p = -32$

Source: 2019 P2 Q3 Higher Maths

(2)

E,ABCD is a rectangular based pyramid.

$\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AD} = \mathbf{q}$ and $\overrightarrow{AE} = \mathbf{r}$.



(a) Express $\overrightarrow{BE}$ in terms of **p** and **r**.

Point F divides BC in the ratio 3:1.

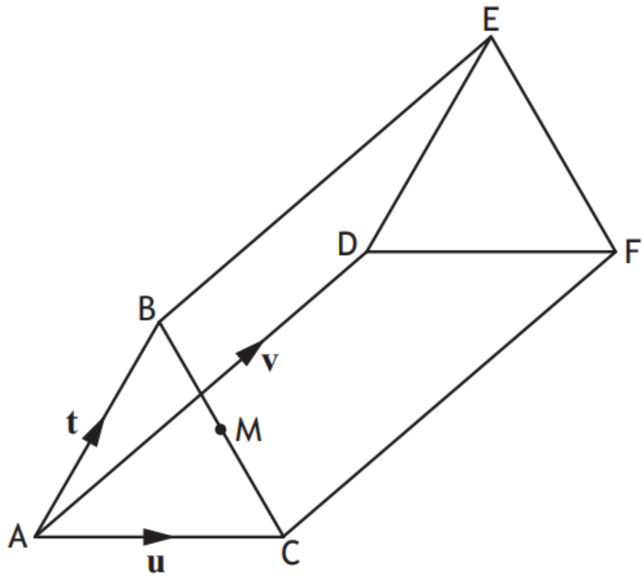
(b) Express vector $\overrightarrow{EF}$ in terms of **p**, **q** and **r**.

Answers: $(a)\ \overrightarrow{BE} = -p + r$ (b) $\overrightarrow{EF} = p - r + \frac{3}{4}q$ $or\ equivalent$

Source: 2019 P2 Q14 Higher Maths	
(3)	The vectors **u** and **v** are such that • $\lvert\mathbf{u}\rvert = 4$ • $\lvert\mathbf{v}\rvert = 5$ • $\mathbf{u}.(\mathbf{u}+\mathbf{v}) = 21$ Determine the size of the angle between the vectors **u** and **v**.
Answer: $Angle = 75.5°\ \ or\ \ 1.31\, radians$	

Source: 2018 P1 Q5 Higher Maths	
(4)	A$(-3, 4, -7)$, B$(5, t, 5)$ and C$(7, 9, 8)$ are collinear. (a) State the ratio in which B divides AC. (b) State the value of t.
Answers: $(a)\ Ratio\ 4:1\quad (b)\ t = 8$	

Source: 2018 P1 Q12 Higher Maths	
(5)	Vectors **a** and **b** are such that $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}$. (a) Express $2\mathbf{a} + \mathbf{b}$ in component form. (b) Hence find the values of p for which $\lvert 2\mathbf{a}+\mathbf{b}\rvert = 7$.
Answers: $(a)\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}\quad (b)\ p = -2,\ -6$	

Source: 2018 P1 Q9 Higher Maths	
(6)	The diagram shows a triangular prism ABC,DEF. $\overrightarrow{AB} = \mathbf{t}$, $\overrightarrow{AC} = \mathbf{u}$ and $\overrightarrow{AD} = \mathbf{v}$.  (a) Express $\overrightarrow{BC}$ in terms of $\mathbf{u}$ and $\mathbf{t}$. M is the midpoint of BC. (b) Express $\overrightarrow{MD}$ in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$.
Answers: $(a)\ \overrightarrow{BC} = -t + u$ $(b)\ \overrightarrow{MD} = -\frac{1}{2}t - \frac{1}{2}u + v$	

Source: 2018 P2 Q2 Higher Maths	
(7)	Vectors $\mathbf{u}$ and $\mathbf{v}$ are defined by $\mathbf{u} = \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -7 \\ 8 \\ 5 \end{pmatrix}$. (a) Find $\mathbf{u}.\mathbf{v}$. (b) Calculate the acute angle between $\mathbf{u}$ and $\mathbf{v}$.
Answers: $(a)\ u.v = 24$ $(b)\ 66.38°\ or\ 1.16\,radians$	

Source: 2017 P1 Q5 Higher Maths

(8)

Vectors **u** and **v** are $\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$ respectively.

(a) Evaluate **u**.**v**.

(b)

Vector **w** makes an angle of $\frac{\pi}{3}$ with **u** and $|\mathbf{w}| = \sqrt{3}$.

Calculate **u**.**w**.

Answers: $(a)\ u.v = 1 \quad (b)\ u.w = 4.5$

Source: 2016 P1 Q7 Higher Maths

(9)

Three vectors can be expressed as follows:

$\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$

$\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$

$\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

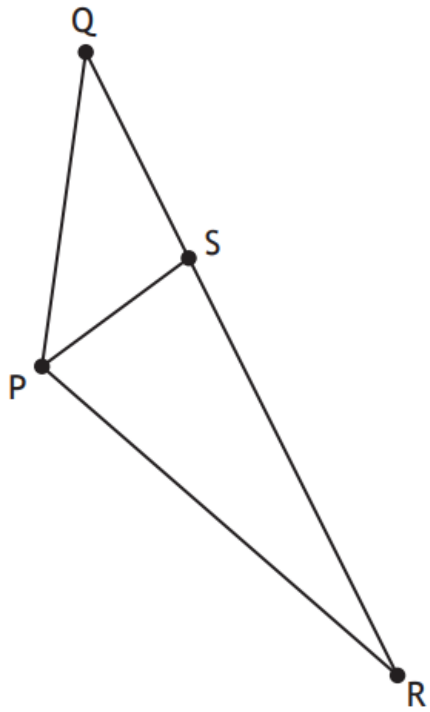
(a) Find $\overrightarrow{FH}$.

(b) Hence, or otherwise, find $\overrightarrow{FE}$.

Answers: $(a)\ u.v = 24 \quad (b)\ 66.38° \ or \ 1.16\, radians$

Source: 2017 P2 Q5 Higher Maths

(10) In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.



(a) Express $\overrightarrow{PQ}$ in terms of **i**, **j** and **k**.

The point S divides QR in the ratio 1:2.

(b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$.

(c) Hence, find the size of angle QPS.

Answers: $(a) -3i - 4j + 5k$ $(b) Proof$ $(c)\ 45.6°\ or\ 0.795\ radians$

Source: 2016 P1 Q11 Higher Maths

(11) (a) A and C are the points $(1, 3, -2)$ and $(4, -3, 4)$ respectively.

Point B divides AC in the ratio $1 : 2$.

Find the coordinates of B.

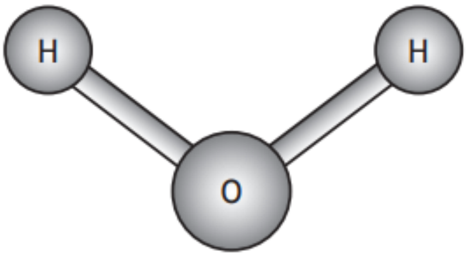
(b) $k\overrightarrow{AC}$ is a vector of magnitude 1, where $k > 0$.

Determine the value of k.

Answers: $(a)\ B\ (2, 1, 0)$ $(b)\ k = \frac{1}{9}$

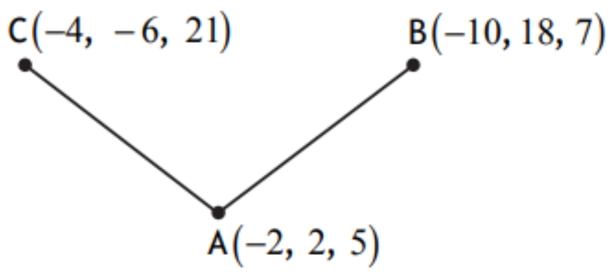
Source: 2016 P2 Q5 Higher Maths

(12) The picture shows a model of a water molecule.



Relative to suitable coordinate axes, the oxygen atom is positioned at point A$(-2, 2, 5)$.

The two hydrogen atoms are positioned at points B$(-10, 18, 7)$ and C$(-4, -6, 21)$ as shown in the diagram below.



(a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form.

(b) Hence, or otherwise, find the size of angle BAC.

Answers: $(a)\ \overrightarrow{AB} = \begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}\ \overrightarrow{BC} = \begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$ $\quad (b)\ 104.3° \ \ or \ \ 1.82\ radians$

Source: 2015 P1 Q1 Higher Maths

(13) Vectors $\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ are perpendicular.

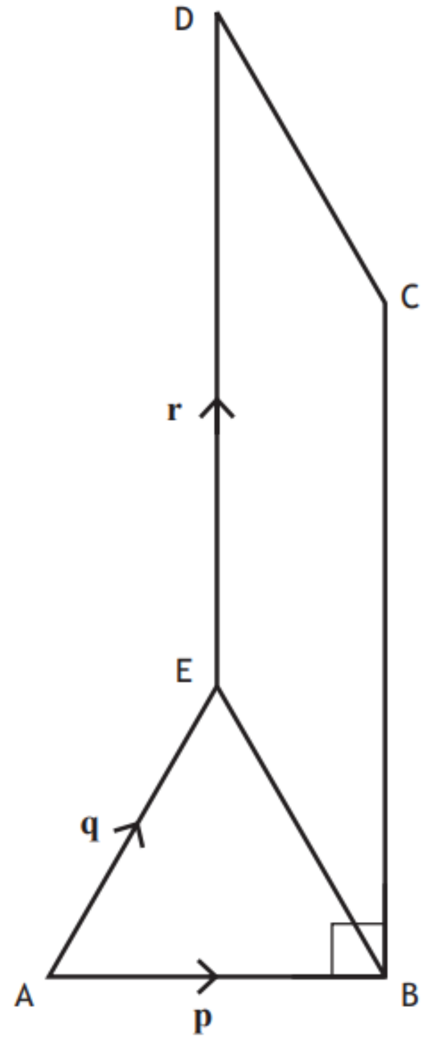
Determine the value of t.

Answer: $t = 9$

Source: 2015 P2 Q6 Higher Maths

(14) Vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are represented on the diagram as shown.

- BCDE is a parallelogram
- ABE is an equilateral triangle
- $|\mathbf{p}| = 3$
- Angle ABC = 90°



(a) Evaluate $\mathbf{p}.(\mathbf{q}+\mathbf{r})$.

(b) Express $\overrightarrow{EC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$.

(c) Given that $\overrightarrow{AE}.\overrightarrow{EC} = 9\sqrt{3} - \frac{9}{2}$, find $|\mathbf{r}|$.

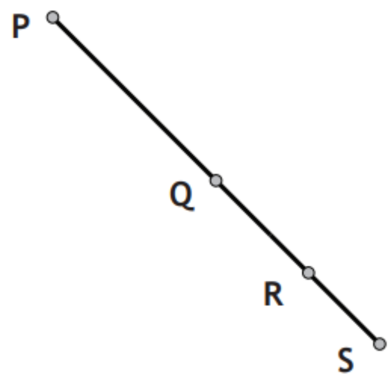
Answers: $(a)\ p.(q + r) = 4.5$ $(b)\ \overrightarrow{EC} = -q + p + r$ $(c)\ \frac{3\sqrt{3}}{cos30}$

Source: Specimen P1 Q3 Higher Maths

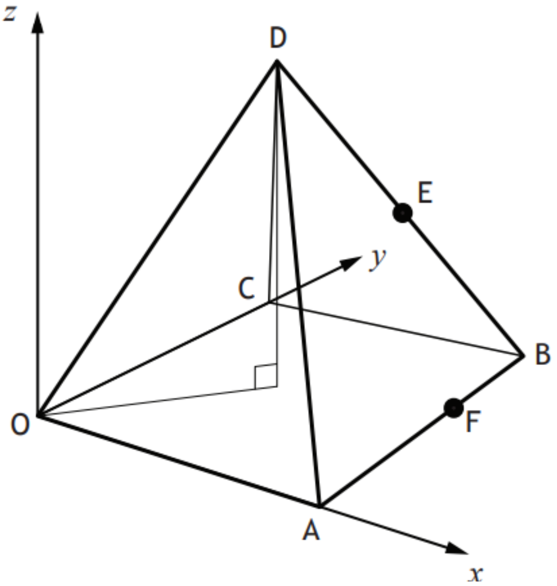
(15) In the diagram, P has coordinates (–6, 3, 9),

$\overrightarrow{PQ} = 6\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{PQ} = 2\overrightarrow{QR} = 3\overrightarrow{RS}$.

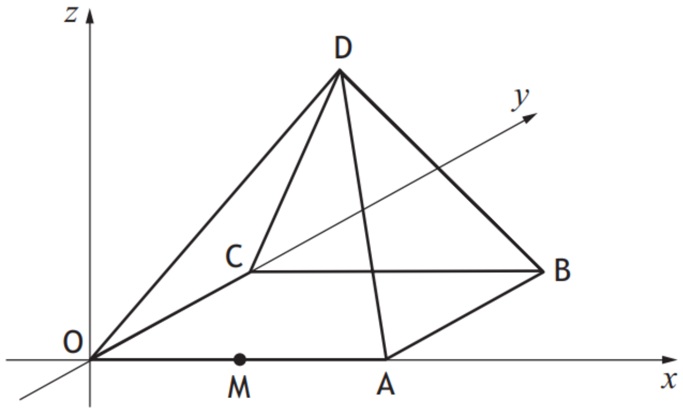
Find the coordinates of S.

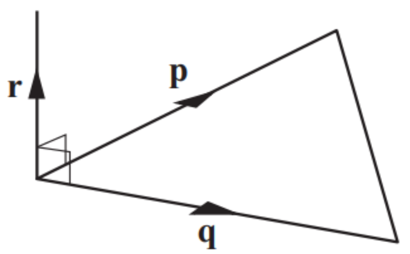


Answer: $S\ (5,\ 25,\ -2)$

Source: Specimen P2 Q1 Higher Maths	
(16)	 A square based right pyramid is shown in the diagram. Square OABC has a side length of 60 units with edges OA and OC lying on the x-axis and y-axis respectively. The coordinates of D are (30, 30, 80). E is the midpoint of BD and F divides AB in the ratio 2:1. (a) Find the coordinates of E and F. (b) Calculate $\overrightarrow{ED}.\overrightarrow{EF}$. (c) Hence, or otherwise, calculate the size of angle DEF.
Answers: $(a)\ E\ (45, 45, 40)\ \ F\ (60, 40, 0)\quad (b)\ \overrightarrow{ED}.\overrightarrow{EF} = -1750\quad (c)\ 154°$	

Source: Specimen P2 Q6 Higher Maths	
(17)	The points A(0, 9, 7), B(5, –1, 2), C(4, 1, 3) and D(x, –2, 2) are such that AB is perpendicular to CD. Determine the value of x.
Answer: $x = -3$	

Source: Exemplar P2 Q5 Higher Maths	
(18)	D,OABC is a square-based pyramid as shown.  O is the origin and OA = 4 units. M is the mid-point of OA. $\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ (a) Express $\overrightarrow{OB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$ and $\mathbf{k}$. (b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. (c) Find the size of angle BDM.
Answers: $(a)\ \overrightarrow{OB} = 4i + 4j\ \ (b)\ \overrightarrow{DB} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}\ \ \overrightarrow{DM} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}\ (c)\ 40.3°\ or\ 0.703\ radians$	

Source: Exemplar P2 Q6 Higher Maths	
(19)	An equilateral triangle with sides of length 3 units is shown.  Vector $\mathbf{r}$ is 2 units long and is perpendicular to both vectors $\mathbf{p}$ and $\mathbf{q}$. Calculate the value of the scalar product $\mathbf{p.(p+q+r)}$.
Answer: $p.(p+q+r) = \frac{27}{2}$	