PRESS F5 TO START

This presentation contains Higher past paper questions complete with solutions.

The questions are sorted into topics based on the specific outcomes for the Higher course.

To access a particular question from the main grid just simply click on the question number.

To access the formula sheet press the F button

To begin click on Main Grid.

Main Grid

Topic	2000		2001		2002		2003		2004	
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Straight Line	3		1	7abc		1	1		1	1
Functions & Graphs		3	7a		3a 7b 8a		9	2 5	4	
Basic Differentiation	2 4a	1a 6	6 9	2	4 6	3a 10	5	4a 8	8b	5 7 9
Recurrence Relations	5			3		4	4			4
Polynomials		1b		1		3bc		1	2	
Quadratics		4a	2 4	11	7ac	9	2 7 11b		8a 11a	3
Basic Integration	4b	4b		6 8		5		3	11b	11
Trig. Form & Equations	1	5	5 7b		3b 5 8b		10	10	3 10	
Circles	6	2	11	7d	1		11a	4b		8
Vectors	7	7 9	3	4	2	2	3 6	9	5	2
Further Calculus	8	8 10		10	10	6 8	8	6	6 7	
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2005 - 2009

Main Grid

Topic	2005		2006		2007		2008		2009	
	I	II	I	II	I	II	I	II	I	II
Straight Line	1		1	1	1		7 22b	1		
Functions & Graphs	4 7	8	3	7	3	4a 9	8 17 23a			
Basic Differentiation		6		3 12	9	5ab 6	21a 22a	6		
Recurrence Relations	6		4		7		1 4			
Polynomials	8	11a	9bc		8ab		21bc			
Quadratics		11b	8	2	4	10a	10 13 16			
Basic Integration		1 5	6	5	8c	10b		7		
Trig. Form & Equations	9	2	7	8	6	2 4b	6 9	3a 5		
Circles	2 11	3	2	4	5	3 5c	2 5	4		
Vectors	3	4 10	9ad	6	2	1	3 11 12 18	2		
Further Calculus	5		5	9	10	7	14 15	3c		
Logs & Exponential		7 9	10	11		8 11	19 20 23b			
Wave Function	10			10	11			3b		

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

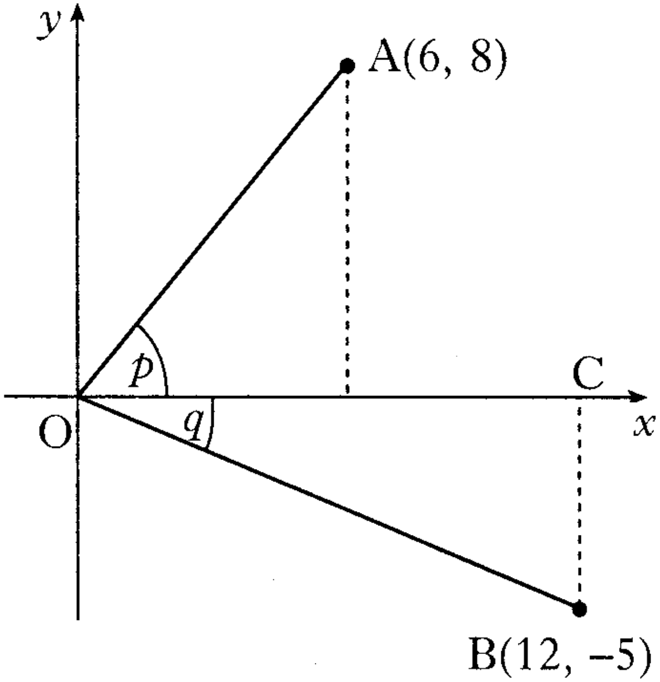
Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Main Grid

A1. On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, –5). Angle AOC = p and angle COB = q.

Find the exact value of $\sin(p + q)$.



4

F

Solution

Main Grid

1) $\sin(p+q) = \sin p \cos q + \cos p \sin q$

$OA = \sqrt{6^2+8^2} = \sqrt{100} = \underline{\underline{10}}$

$OC = \sqrt{12^2+-5^2} = \sqrt{169} = \underline{\underline{13}}$

$\sin p = \frac{8}{10} = \underline{\underline{\frac{4}{5}}}$ $\quad \cos p = \frac{6}{10} = \underline{\underline{\frac{3}{5}}}$

$\sin q = \underline{\underline{\frac{5}{13}}}$ $\quad \cos q = \underline{\underline{\frac{12}{13}}}$

$\therefore \sin(p+q) = \left(\frac{4}{5} \times \frac{12}{13}\right) + \left(\frac{3}{5} \times \frac{5}{13}\right)$

$= \frac{48}{65} + \frac{15}{65} = \underline{\underline{\frac{63}{65}}}$

Main Grid

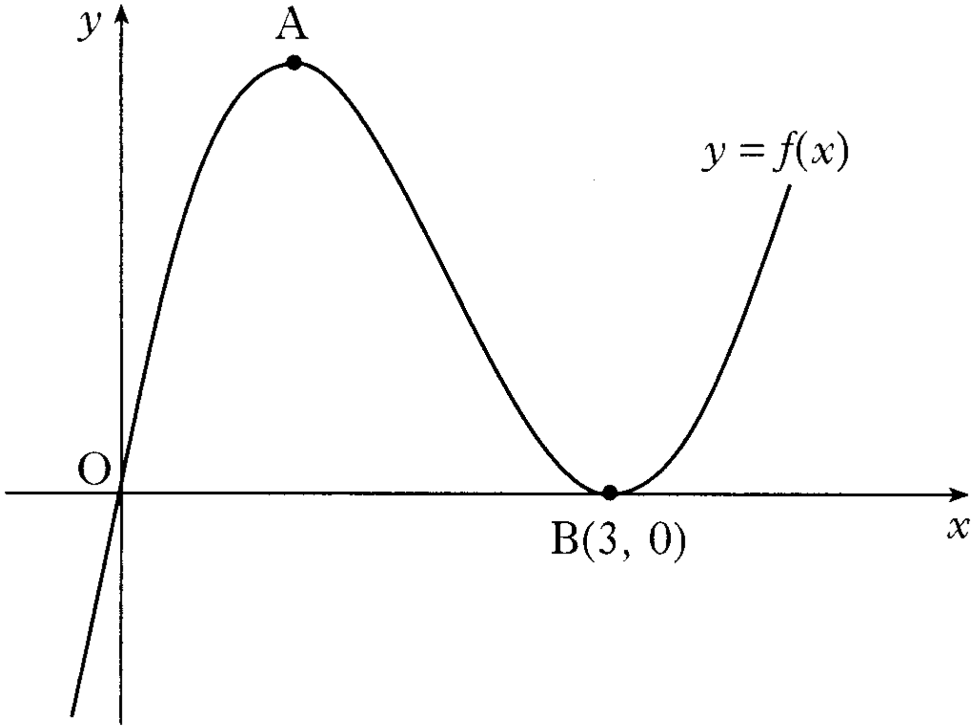
A2. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).

Solution 2a

Solution 2bc

Main Grid

F



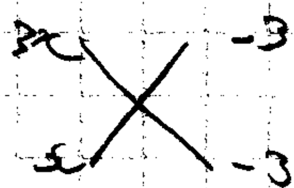
(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

2) a) $f'(x) = 3x^2 - 12x + 9 = 0$ at T.P



$-9x$
$-3x$
$-12x$

$(3x-3)(x-3) = 0$

$x = \frac{3}{3} = 1$ OR $x = 3$

$\therefore \underline{\underline{x=1}}$

at $x=1$: $y = 1^3 - (6\times1^2) + 9\times1$

$= 1 - 6 + 9 = \underline{\underline{4}}$

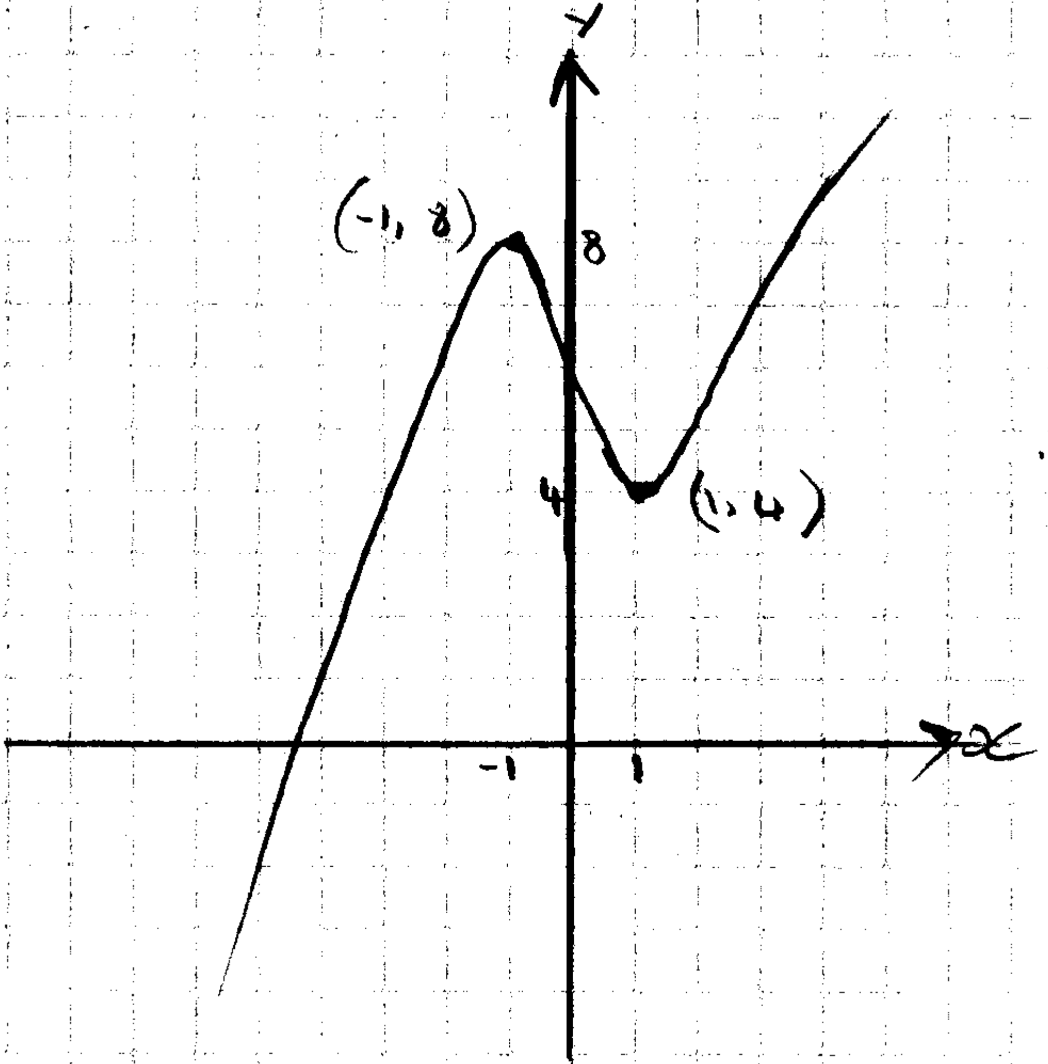
$\therefore$ T.P. at $\underline{\underline{(1,4)}}$

Solution 2bc

Main Grid

2000 - Higher Paper I

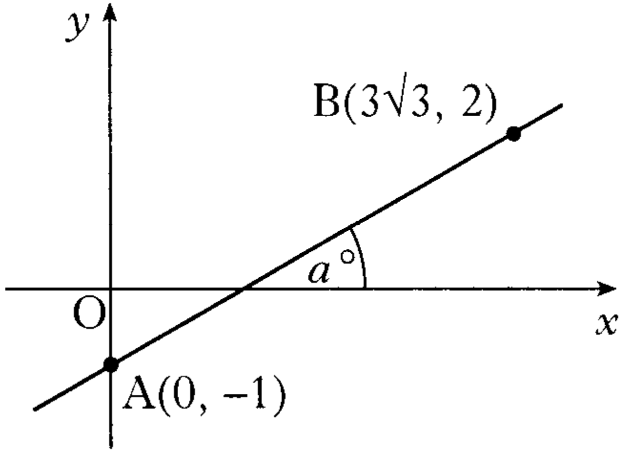
2) b)



c) [illegible] $4 < k < 8$

Main Grid

A3. Find the size of the angle $a°$ that the line joining the points A(0, –1) and B($3\sqrt{3}$, 2) makes with the positive direction of the x-axis.



3

F

Solution

Main Grid

$$m = \tan\theta$$

$$m = \frac{2 - -1}{3\sqrt{3} - 0} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

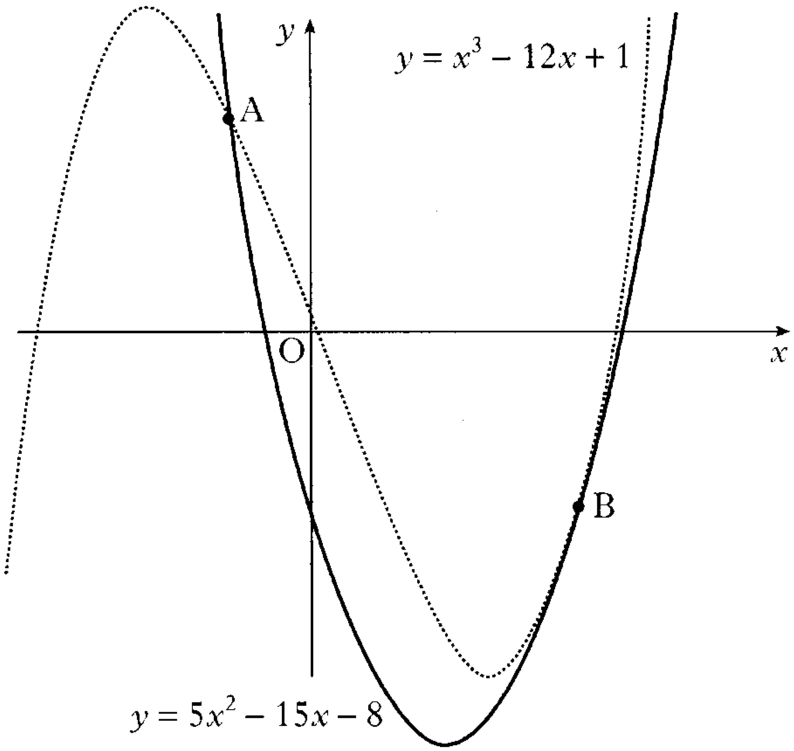
$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^{o}$$

Main Grid

A4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, ie at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the points on the curves where the gradients are equal. **4**

Solution 4a

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

Solution 4b

(*b*) The point A is (–1, 12) and B is (3, –8).

Find the area enclosed between the two curves. **5**

Main Grid

F

4) $M = \frac{dy}{dx}$ $\qquad M_A = 10x - 15$

$$M_B = 3x^2 - 12$$

at $M_A = M_B$: $\qquad 3x^2 - 12 = 10x - 15$

$$3x^2 - 10x + 3 = 0$$

$$(3x - 1)(x - 3) = 0$$

$$\therefore \underline{\underline{x = \frac{1}{3}}} \quad \text{OR} \quad \underline{\underline{x = 3}}$$

at $x = \frac{1}{3}$ the curves are parallel, at $x = 3$ the curves touch and have a common Tangent.

Solution 4b

Main Grid

b)

$$\int_{-1}^{3} (x^3-12x+1)-(5x^2-15x-8)\ dx$$

$$= \int_{-1}^{3} x^3-5x^2+3x+9\ dx$$

$$= \left[\frac{x^4}{4}-\frac{5x^3}{3}+\frac{3x^2}{2}+9x\right]_{-1}^{3}$$

$$= \left(\frac{3^4}{4}-\frac{5\times3^3}{3}+\frac{3\times3^2}{2}+9\times3\right)-\left(\frac{-1^4}{4}-\frac{5\times-1^3}{3}+\frac{3\times-1^2}{2}+9\times-1\right)$$

$$= \left(\frac{81}{4}-\frac{135}{3}+\frac{27}{2}+27\right)-\left(\frac{1}{4}+\frac{5}{3}+\frac{3}{2}-9\right)$$

$$=\left(\frac{243}{12}-\frac{540}{12}+\frac{162}{12}+\frac{324}{12}\right)-\left(\frac{3}{12}+\frac{20}{12}+\frac{18}{12}-\frac{108}{12}\right)$$

$$= \frac{189}{12}+\frac{67}{12} = \frac{256}{12} = \underline{\underline{21\tfrac{1}{3}\text{ units}^2}}$$

Main Grid

A5. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2v_n + 16$.

The two sequences approach the same limit as $n \to \infty$.

Determine the value of a and evaluate the limit. **5**

F

Solution

Main Grid

5) At the limit $L = \frac{b}{1-a}$

$\therefore\ u_{n+1} = au_n + 10 \Rightarrow L = \frac{10}{1-a}$

$\therefore\ u_{n+1} = a^2u_n + 16 \Rightarrow L = \frac{16}{1-a^2}$

Now L=L : $\frac{10}{1-a} = \frac{16}{1-a^2}$

$10(1-a^2) = 16(1-a)$

$10 - 10a^2 = 16 - 16a$

$10a^2 - 16a + 6 = 0 \quad \div 2$

$5a^2 - 8a + 3 = 0$

$(5a-3)(a-1) = 0$

$\therefore\ a = \frac{3}{5}$ or $a = 1$ Now $-1 < a < 1$

$\therefore\ a = \frac{3}{5}$

$\therefore\ L = \frac{10}{1-\frac{3}{5}} = \frac{10}{\frac{2}{5}} = \frac{10 \times 5}{2} = 25$

Main Grid

A6. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

F

Solution

Main Grid

6) For a circle $r > 0$

$2g = 4k$ $\quad 2f = -2k$ $\quad c = -k-2$

$g = 2k$ $\quad f = -k$

$\therefore \; r = \sqrt{g^2+f^2-c}$

$r = \sqrt{4K^2+k^2+K+2}$

$= \sqrt{5K^2+K+2}$

$\therefore \; 5K^2+K+2 > 0$

$\therefore$ All values of K

Main Grid

B7. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

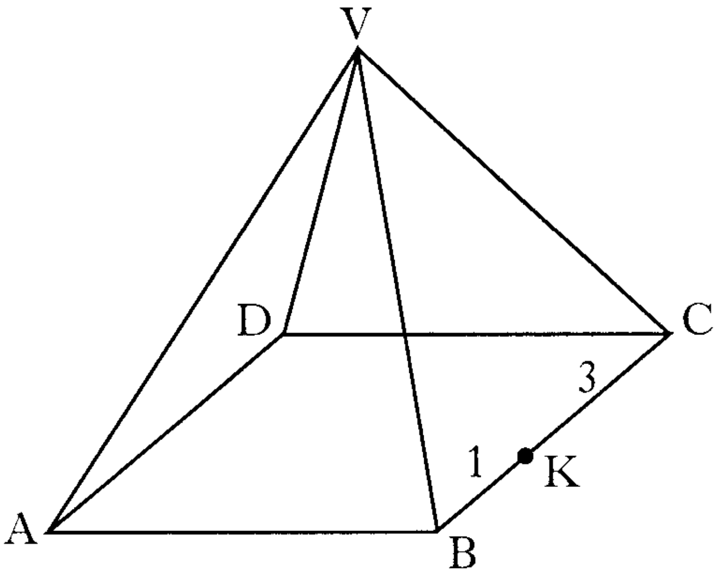
$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.

K divides BC in the ratio 1:3.

Find $\overrightarrow{VK}$ in component form.



3

F

Solution

Main Grid

7) $\overrightarrow{VK} = \overrightarrow{UA} + \overrightarrow{AB} + \overrightarrow{BK}$

If $\overrightarrow{AB} = \begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix}$ than $\overrightarrow{BK} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$$\therefore \overrightarrow{VK} = \begin{pmatrix} -7 \\ -13 \\ -11 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}}}$$

Main Grid

B8. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$, express y in terms of x. **4**

F

Solution

Main Grid

8) $f'(x) = \sin(3x)$ then

$$y = \int \sin(3x) = -\frac{1}{3}\cos 3x + c$$

at $\left(\frac{\pi}{9}, 1\right)$: $1 = -\frac{1}{3}\cos\frac{\pi}{3} + c$

$$c = 1 + \frac{1}{3}\cos\frac{\pi}{3}$$

$$c = 1 + \frac{1}{3} \times \frac{1}{2} = 1 + \frac{1}{6} = \underline{\underline{\frac{7}{6}}}$$

$$\therefore \quad \underline{\underline{y = -\frac{1}{3}\cos 3x + \frac{7}{6}}}$$

Main Grid

B9. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

F

Solution

Main Grid

a)

$$\log_5 2 + \log_5 50 - \log_5 4$$

$$= \log_5 100 - \log_5 4$$

$$= \log_5 25$$

$$= \underline{\underline{2}}$$

Main Grid

B10. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \leq x \leq 2\pi$. **6**

F

Solution

Main Grid

10) $\cos x - \sin x = R\cos(x - \alpha)$

$= R(\cos x \cos\alpha - \sin x \sin\alpha)$

$\cos x - \sin x = R\cos x\cos\alpha + R\sin x\sin\alpha$

$R\cos\alpha = 1 \qquad R\sin\alpha = -1$

Squaring and adding: $R^2\cos^2\alpha + R^2\sin^2\alpha = 1^2 + -1^2$

$R^2(\cos^2\alpha + \sin^2\alpha) = 2$

$R^2 = 2$

$\underline{\underline{R = \sqrt{2}}}$

More Solution
Main Grid

NOW: $\frac{\cancel{k}\sin\alpha}{\cancel{k}\cos\alpha} = \frac{-1}{1} = \tan\alpha$

$$\therefore \underline{\underline{\tan\alpha = -1}}$$

Sin is -ve, cos α +ve ∴ α is a 4th quadrant.

$R.A. = \tan^{-1}(1) = \pi/4$

$$\therefore \alpha = 2\pi - \pi/4 = \underline{\underline{\frac{7\pi}{4}}}$$

$$\therefore \cos x - \sin x = \underline{\underline{\sqrt{2}\cos\left(x - \frac{7\pi}{4}\right)}}$$

More Solution Main Grid

Max at : $\sqrt{2}\cos\left(x - \frac{7\pi}{4}\right) = \sqrt{2}$

$$\cos\left(x - \frac{7\pi}{4}\right) = 1$$

$$x - \frac{7\pi}{4} = 0$$

$$x = \frac{7\pi}{4}$$

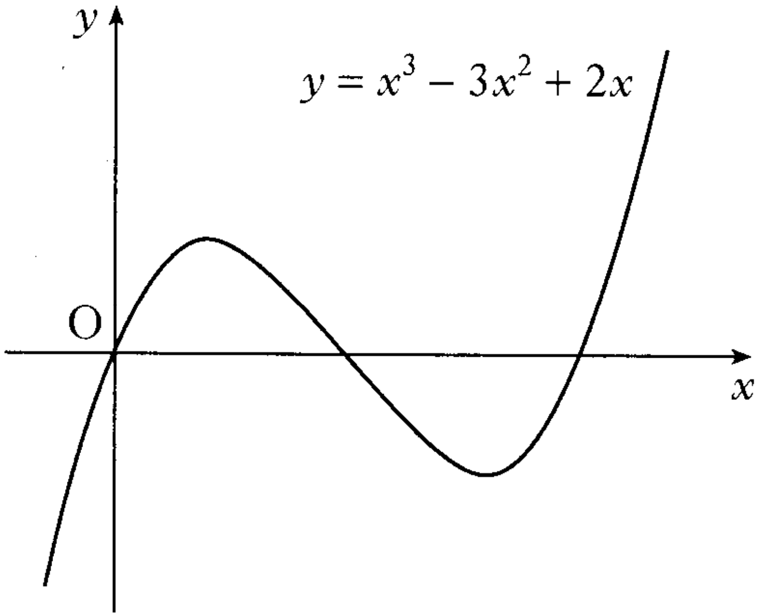
$\therefore$ Maximum Value of $\sqrt{2}$ at $x = \frac{7\pi}{4}$

Main Grid

A1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.

(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**



F

Solution

Main Grid

1) at $x=1$: $y = 1 - 3 + 2 = 0$ $\underline{\underline{(1,0)}}$

$M = \frac{dy}{dx} = 3x^2 - 6x + 2$

at $x=1$: $M = 3 - 6 + 2 = \underline{\underline{-1}}$

$$y - b = M(x-a)$$
$$y - 0 = -1(x-1)$$
$$\underline{\underline{y = 1 - x}}$$

b) $y = y$

$$x^3 - 3x^2 + 2x = 2x - 4$$
$$x^3 - 3x^2 + 4 = 0$$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & 0 & 4 \\ & & 2 & -2 & -4 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$$(x-2)(x^2 - x - 2) = 0$$
$$(x-2)(x-2)(x+1) = 0$$
$$\therefore \underline{\underline{x = -1}}$$

$y = 2x-1 \quad -4 = \underline{\underline{-6}}$

$\therefore \underline{\underline{(-1,-6)}}$

Main Grid

A2. (*a*) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9). **4**

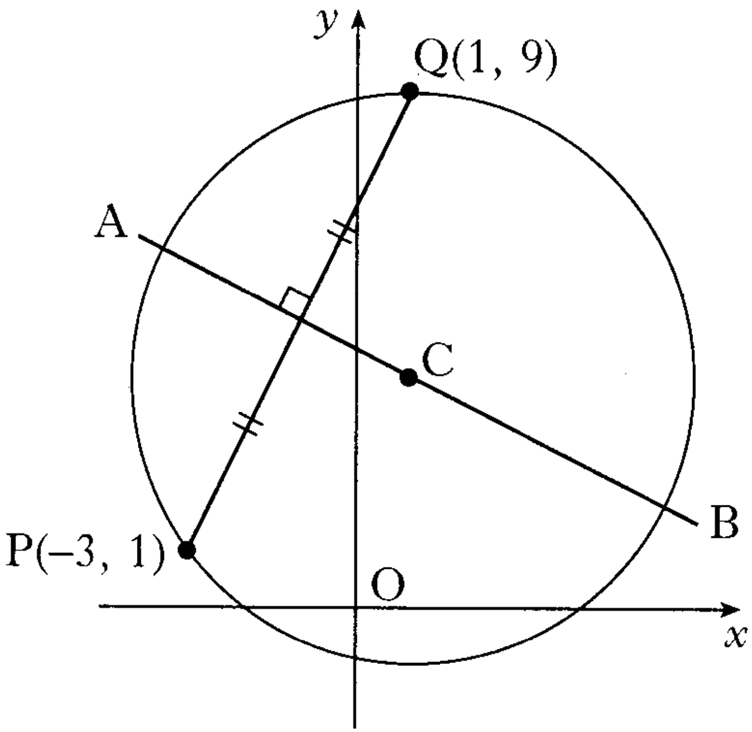
(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. **3**

(*c*) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T. **2**



F

Solution

Main Grid

2) a) Mid Point of PQ $(-1, 5)$

$M_{PQ} = \frac{8}{4} = \underline{\underline{2}}$

$\therefore M_{PQ} M_{AB} = -1 \qquad M_{AB} = \underline{\underline{\frac{-1}{2}}}$

$y - b = M(x - a)$

$y - 5 = -\frac{1}{2}(x + 1)$

$2y - 10 = -x - 1$

$\underline{\underline{2y + x - 9 = 0}}$

b) $x_c = 1 \qquad 2y = 9 - x$

$2y = 8$

$\underline{\underline{y = 4}} \qquad \therefore C(1, 4)$

$|CQ| = r = 9 - 4 = \underline{\underline{5}}$

$\therefore (x - a)^2 + (y - b)^2 = r^2$

$\underline{\underline{(x - 1)^2 + (y - 4)^2 = 25}}$

c) i) $\underline{\underline{y = 9}}$

ii) $2 \times 9 + x - 9 = 0$

$18 + x - 9 = 0$

$\underline{\underline{x = -9}} \qquad \underline{\underline{T(-9, 9)}}$

Main Grid

A3. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$.

(*a*) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(*b*) If $q(x) = \frac{3}{3 - x}$, $x \neq 3$, find $p(q(x))$ in its simplest form. **3**

F

Solution

Main Grid

3) a) $p(x) = \underline{\underline{3 - \frac{3}{x}}}$

b) $p\left(q(x)\right) = 3 - \dfrac{3}{\frac{3}{3-x}}$

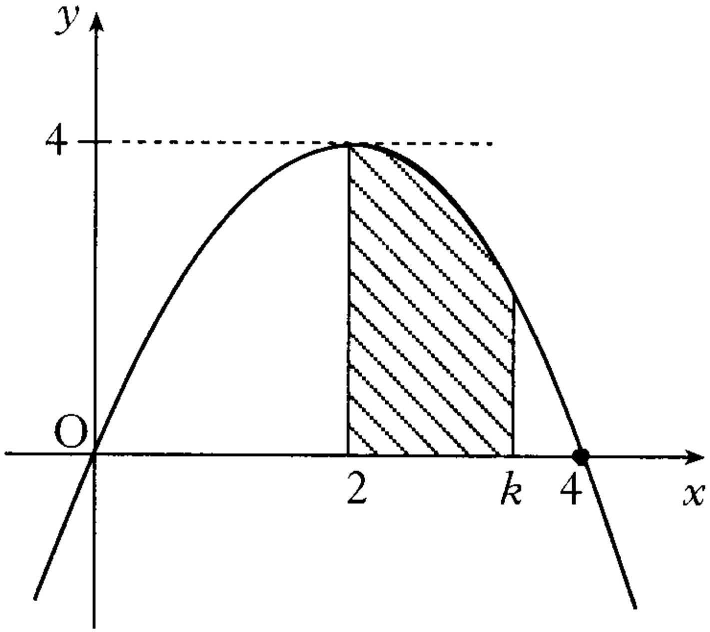
$= 3 - \dfrac{3(3-x)}{3} = 3 - 3 + x$

$= \underline{\underline{x}}$

Main Grid

A4. The parabola shown crosses the x-axis at (0, 0) and (4, 0), and has a maximum at (2, 4).

The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.



(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$ **3**

F

Solution

Main Grid

4) a) $y = Kx(x-4)$

at $(2,4)$: $4 = 2K(-2)$

$4 = -4K$

$K = -1 \quad \therefore \quad y = -x(x-4)$

$$y = \underline{4x - x^2}$$

b) $$A = \int_2^K 4x - x^2 \, dx = \left[2x^2 - \frac{x^3}{3}\right]_2^K$$

$$= \left(2K^2 - \frac{K^3}{3}\right) - \left(8 - \frac{8}{3}\right)$$

$$= \underline{\underline{-\frac{1}{3}K^3 + 2K^2 - \frac{16}{3}}}$$

Main Grid

A5. Solve the equation $3\cos 2x° + \cos x° = -1$ in the interval $0 \le x \le 360$. **5**

F

Solution

Main Grid

5) $3\cos 2x + \cos x = -1$

$3(2\cos^2 x - 1) + \cos x = -1$

$6\cos^2 x - 3 + \cos x + 1 = 0$

$6\cos^2 x + \cos x - 2 = 0$

$a = 6$, $b = 1$, $c = -2$ $\nearrow$

$\therefore \cos x = 0.5$

$x = 60°$

$x = 300°$

$\cos x = -0.666$

$RA = 48.2$

$x = 180 - 48.2$

$= 131.8°$

$x = 180 + 48.2$

$= 228.2°$

$\therefore \underline{\underline{x = 60°, 131.8°, 228.2°, 300°}}$

Main Grid

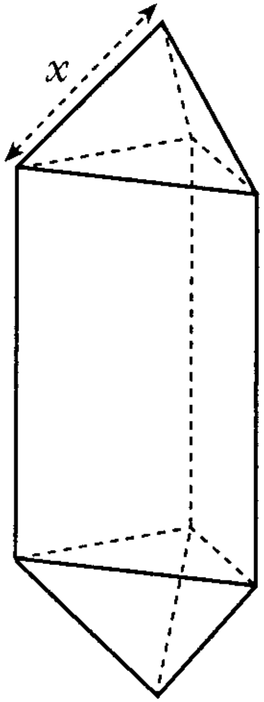
A6. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid.



6

F

Solution

Main Grid

6) $A(x) = \frac{3\sqrt{3}}{2}x^2 + 24\sqrt{3}\,x^{-1}$

Min at $A'(x)=0$: $3\sqrt{3}\,x - \frac{24\sqrt{3}}{x^2} = 0 \quad \times x^2$

$$3\sqrt{3}\,x^3 - 24\sqrt{3} = 0$$

$$x^3 = \frac{24\cancel{\sqrt{3}}}{3\cancel{\sqrt{3}}}$$

$$x^3 = 8$$

$$\underline{\underline{x = 2}}$$

x	2^-	2	2^+
$A'(x)$	$-ve$	0	$+ve$
SHAPE	\	—	/

$\therefore$ $\underline{\underline{\text{Min at } x=2}}$

Main Grid

B7. For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? **2**

F

Solution

Main Grid

7) If Perpendicular $a \cdot b = 0$

$$u \cdot v = 2t - 20 + 3t = 0$$

$$5t = 20$$

$$\underline{t = 4}$$

Main Grid

B8. Given that $f(x) = (5x-4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

F

Solution

Main Grid

8) $f'(x) = \frac{1}{2}(5x-4)^{-1/2} \times 5$

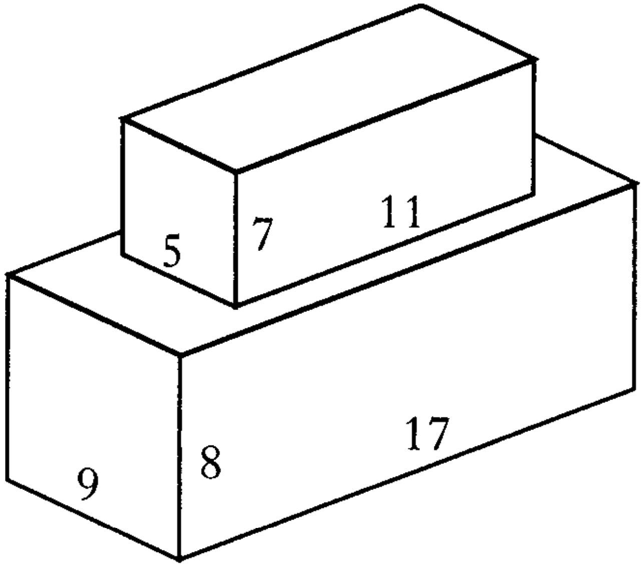
$= \frac{5}{2\sqrt{5x-4}}$

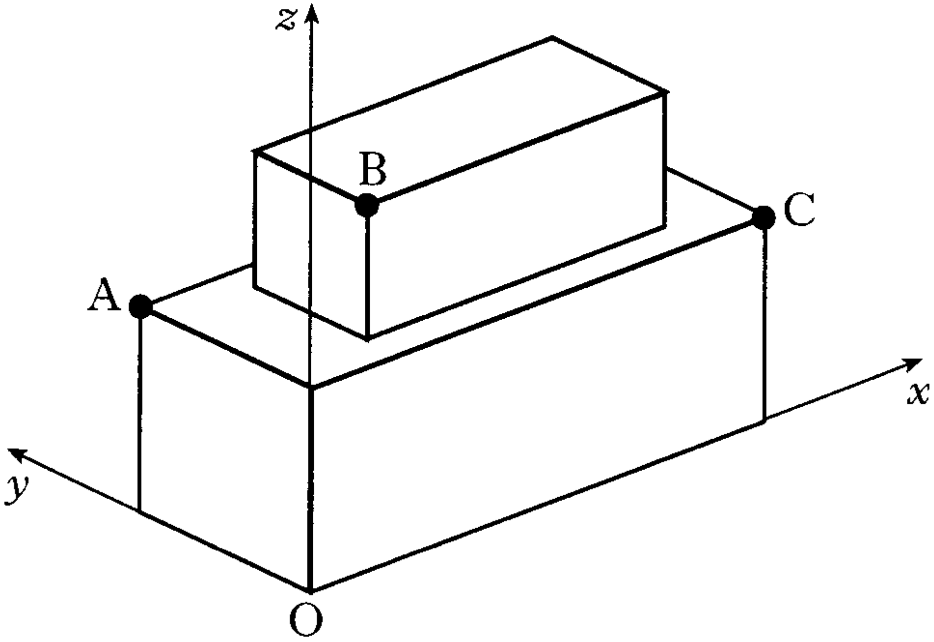
$f'(4) = \frac{5}{2 \times \sqrt{16}} = \underline{\underline{\frac{5}{8}}}$

Main Grid

B9. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinate axes are taken as shown.





(*a*) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8).
Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

F

Solution

Main Grid

9) a) $(3, 2, 15)$

b) $\overrightarrow{BA} = \begin{pmatrix} 0 \\ 9 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 15 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \\ -7 \end{pmatrix}$ $\qquad \overrightarrow{BC} = \begin{pmatrix} 17 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 15 \end{pmatrix} = \begin{pmatrix} 14 \\ -2 \\ -7 \end{pmatrix}$

$$\cos \theta = \frac{a \cdot b}{|a|\,|b|}$$

$$|\overrightarrow{BA}| = \sqrt{9 + 49 + 49}$$

$$= \underline{\underline{\sqrt{107}}}$$

$$|\overrightarrow{BC}| = \sqrt{196 + 4 + 49}$$

$$= \underline{\underline{\sqrt{249}}}$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = (-3 \times 14) + (7 \times -2) + (-7 \times -7)$$

$$= \underline{\underline{-7}}$$

$$\therefore \cos \theta = \frac{-7}{\sqrt{107} \times \sqrt{249}} = \underline{\underline{-0.043}}$$

$$\therefore \angle ABC = \cos^{-1}(-0.043) = \underline{\underline{92.5^{\circ}}}$$

Main Grid

Marks
2

B10. Find $\int \frac{1}{(7-3x)^2}\,dx$.

F

Solution

Main Grid

10)

$$\int (7-3x)^{-2}\,dx = \frac{1}{-3(-2+1)}(7-3x)^{-2+1} + C$$

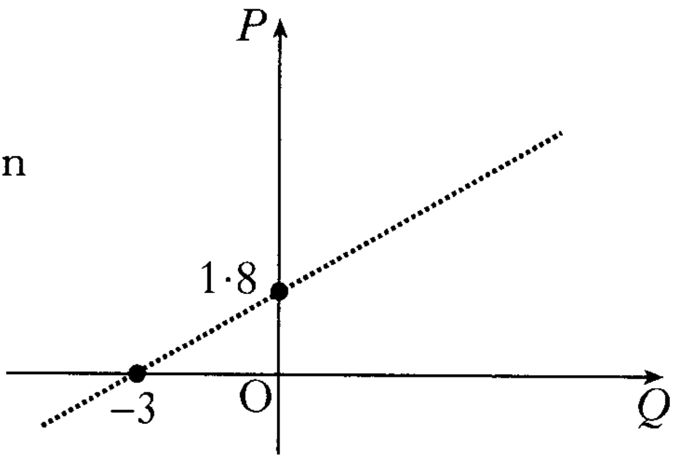
$$= \frac{1}{3}(7-3x)^{-1} + C$$

$$= \frac{1}{3(7-3x)} + C$$

Main Grid

B11. The results of an experiment give rise to the graph shown.

(*a*) Write down the equation of the line in terms of P and Q. **2**



It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

F

Solution
Main Grid

11) a) $\underline{P = 0.6Q + 1.8}$

$M = \frac{1.8}{3} = 0.6$

b) $p = aq^b$

$\log_e p = \log_e aq^b$

$\log_e p = \log_e a + \log_e q^b$

$\log_e p = \log_e a + b \log_e q$

$\log_e p = b\log_e q + \log_e a$

$P = bQ + \log_e a$

and $P = 0.6Q + 1.8$

$\therefore \underline{b = 0.6}$

$\log_e a = 1.8$

$a = e^{1.8}$

$a = \underline{6.05}$

Main Grid

1. Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point $(2, -1)$. **3**

F

Solution

Main Grid

1) $2x + 3Y = 5$

$\therefore \; Y = -\frac{2}{3}x + \frac{5}{3}$ $\qquad \therefore \; M = -\frac{2}{3}$

$Y - b = M(x-a)$ $\qquad (a,b) = (2,-1)$

$Y + 1 = \frac{-2}{3}(x-2)$ $\times 3$

$3Y + 3 = -2(x-2)$

$3Y + 3 = -2x + 4$

$\therefore \; \underline{\underline{2x + 3Y - 1 = 0}}$

Main Grid

2. For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots? **3**

F

Solution

Main Grid

2) $b^2 - 4ac = 0$ $a = 1$, $b = -5$ $c = k+6$

$25 - 4(k+6) = 0$

$25 - 4k - 24 = 0$

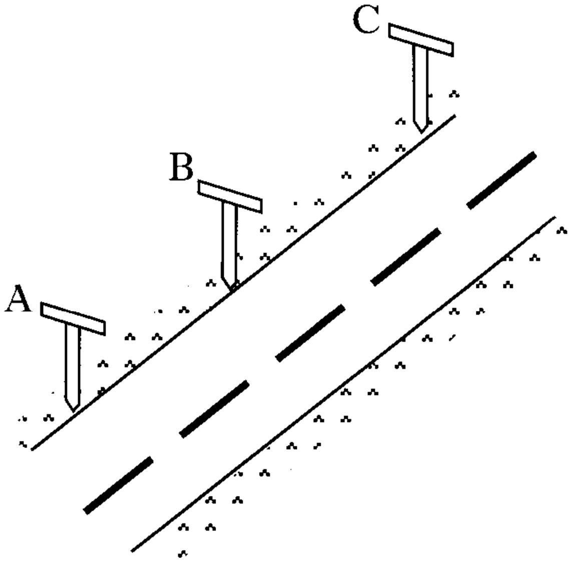
$1 = 4k$

$\underline{\underline{k = \frac{1}{4}}}$

Main Grid

3. (*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points A(–8, –10, –2), B(–2, –1, 1) and C(6, 11, 5).

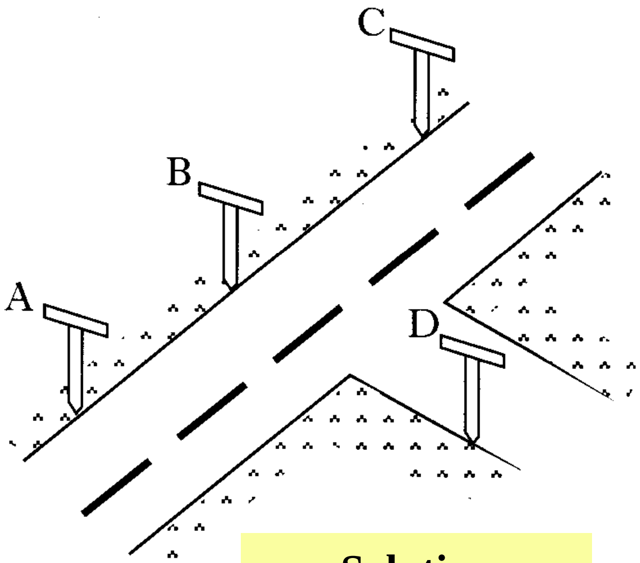
Determine whether or not the section of road ABC has been built in a straight line.



3

(*b*) A further T-rod is placed such that D has coordinates (1, –4, 4).

Show that DB is perpendicular to AB.



3

F

Solution

Main Grid

3) a) $\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix}$ $\overrightarrow{BC} = \underline{c} - \underline{b} = \begin{pmatrix} 8 \\ 12 \\ 4 \end{pmatrix}$

$\overrightarrow{AB} = \frac{3}{4} \overrightarrow{BC}$ B is a common point

$\therefore$ Straight line

b) $\overrightarrow{BA} = \underline{a} - \underline{b} = \begin{pmatrix} -6 \\ -9 \\ -3 \end{pmatrix}$ $\overrightarrow{BD} = \underline{d} - \underline{b} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

$\overrightarrow{BA} \cdot \overrightarrow{BD} = -18 + 27 - 9 = 0$

$\therefore$ Perpendicular.

Main Grid

4. Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$. **2**

F

Solution

Main Grid

4)

$$
\begin{aligned}
f(x) &= x^2 + 2x - 8 \\
&= (x^2 + 2x + 1^2) - 8 - 1^2 \\
&= (x^2 + 2x + 1) - 9 \\
&= (x+1)(x+1) - 9 \\
&= \underline{\underline{(x+1)^2 - 9}}
\end{aligned}
$$

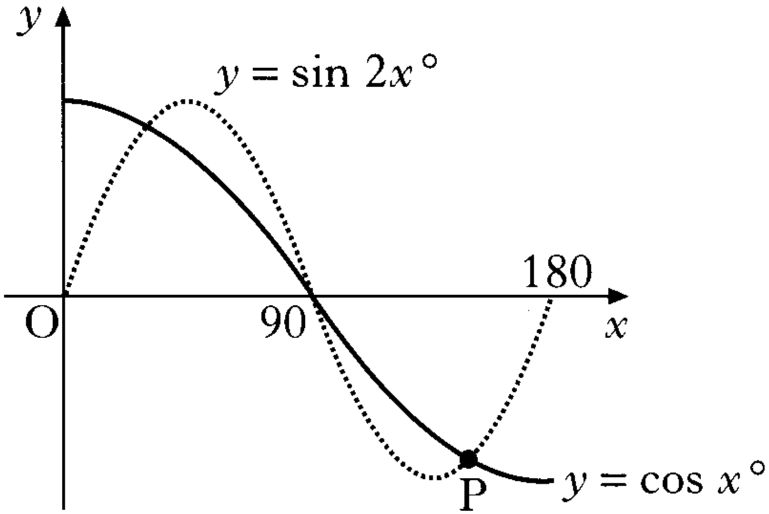
Main Grid

Marks

5. (*a*) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \leq x \leq 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**



F

Solution

Main Grid

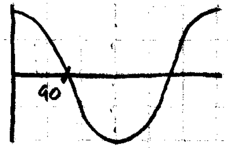
5) a) $\sin 2x - \cos x = 0$

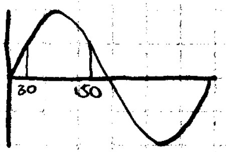
$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ OR $2\sin x - 1 = 0$

$\cos x = 0$ OR $\sin x = \frac{1}{2}$





$x = 90°$ OR $x = 30°, 150°$

$x = 30°; 90°, 150°$

b) At P: $\sin 2x = \cos x$

$2\sin x\cos x = \cos x$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

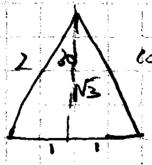
$x = 30°$ OR $150°$

At P $x = 150°$

$y = \cos x = \cos 150°$

$y = \cos 150°$

$y = \frac{-\sqrt{3}}{2}$



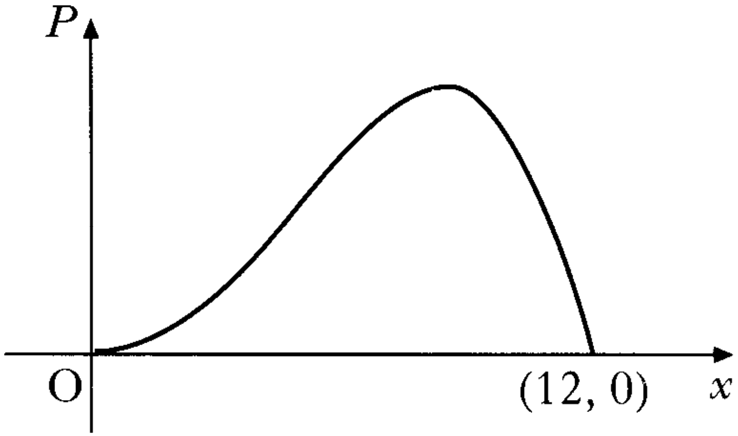
$\cos 30 = \frac{\sqrt{3}}{2}$

$\therefore P\left(150; \frac{-\sqrt{3}}{2}\right)$

Main Grid

6. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \le x \le 12$.

Find the value of x which gives the maximum profit.



5

F

Solution

Main Grid

6) Find stationary points. $f'(x) = 0$

$\therefore \quad 36x^2 - 4x^3 = 0$

$4x^2(9 - x) = 0$

$4x^2 = 0$ OR $9 - x = 0$

$x = 0$ OR $x = 9$

by inspection Max T.P. $x = 9$.

$\therefore \quad P = 12 \times 9^3 - 9^4$

$= 9^2(12\times 9 - 9^2)$

$= 81(108 - 81)$

$= 81 \times 27$

$= 2187$

$$\begin{array}{r} 27 \\ 81 \\ \hline {}^{5}27 \\ 2160 \\ \hline 2187 \end{array}$$

Main Grid

$\therefore$ Max $P = £\ 2187000$

7. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$. **5**

F

Solution 7a, bi

Solution 7bii

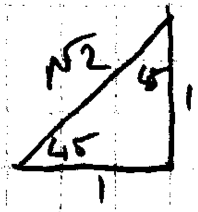
Main Grid

7) a) i) $f(h(x)) = \sin\left(x + \frac{\pi}{4}\right)$

ii) $g(h(x)) = \cos\left(x + \frac{\pi}{4}\right)$

b) $f(h(x)) = \sin\left(x + \frac{\pi}{4}\right) = \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$

$\frac{\pi}{4} = 45^\circ$



$\therefore \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

So: $\underline{\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x}$

Solution 7bii

Main Grid

7) b) ii) $g(h(x)) = \cos\left(x + \frac{\pi}{4}\right)$

$$= \cos x \cos \tfrac{\pi}{4} - \sin x \sin \tfrac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x$$

$$f(h(x)) - g(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x$$

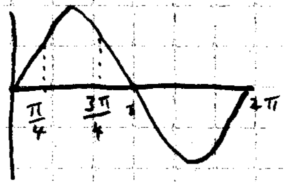
$$= \frac{2}{\sqrt{2}} \sin x = 1$$

$$= \sin x = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin x = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\sin x = \frac{1}{\sqrt{2}}$$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$



$$\therefore\ x = \frac{\pi}{4}, \frac{3\pi}{4}$$

8. Find x if $4 \log_x 6 - 2 \log_x 4 = 1$. **3**

F

Solution

Main Grid

8) $4\log_x 6 - 2\log_x 4 = 1$

$\log_x 6^4 - \log_x 4^2 = 1$

$\log_x \frac{6^4}{16} = 1$

$\log_x \frac{1296}{16} = 1$

$\log_x 81 = 1$

$Y = a^x \quad x = \log_a Y \quad \therefore \quad 81 = x^1 \quad \therefore \underline{\underline{x = 81}}$

$$\begin{array}{r} 36 \\ \underline{{}^{3}6} \\ 216 \\ \underline{{}^{3}6} \\ 1296 \end{array}$$

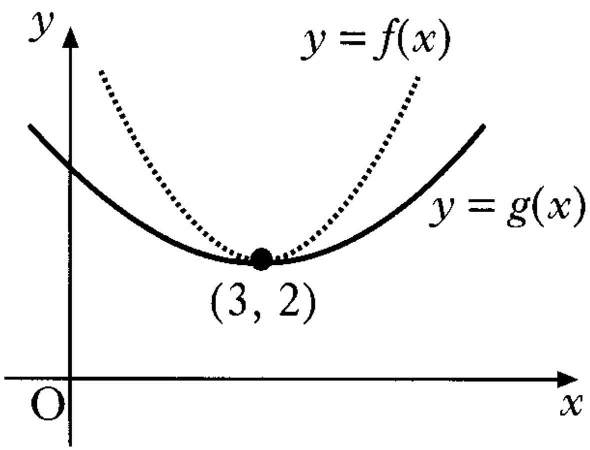
$\therefore 6^4 = 1296$

$$\begin{array}{r} 81 \\ 16\overline{)1296} \end{array}$$

Main Grid

9. The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at $(3, 2)$.

Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$.



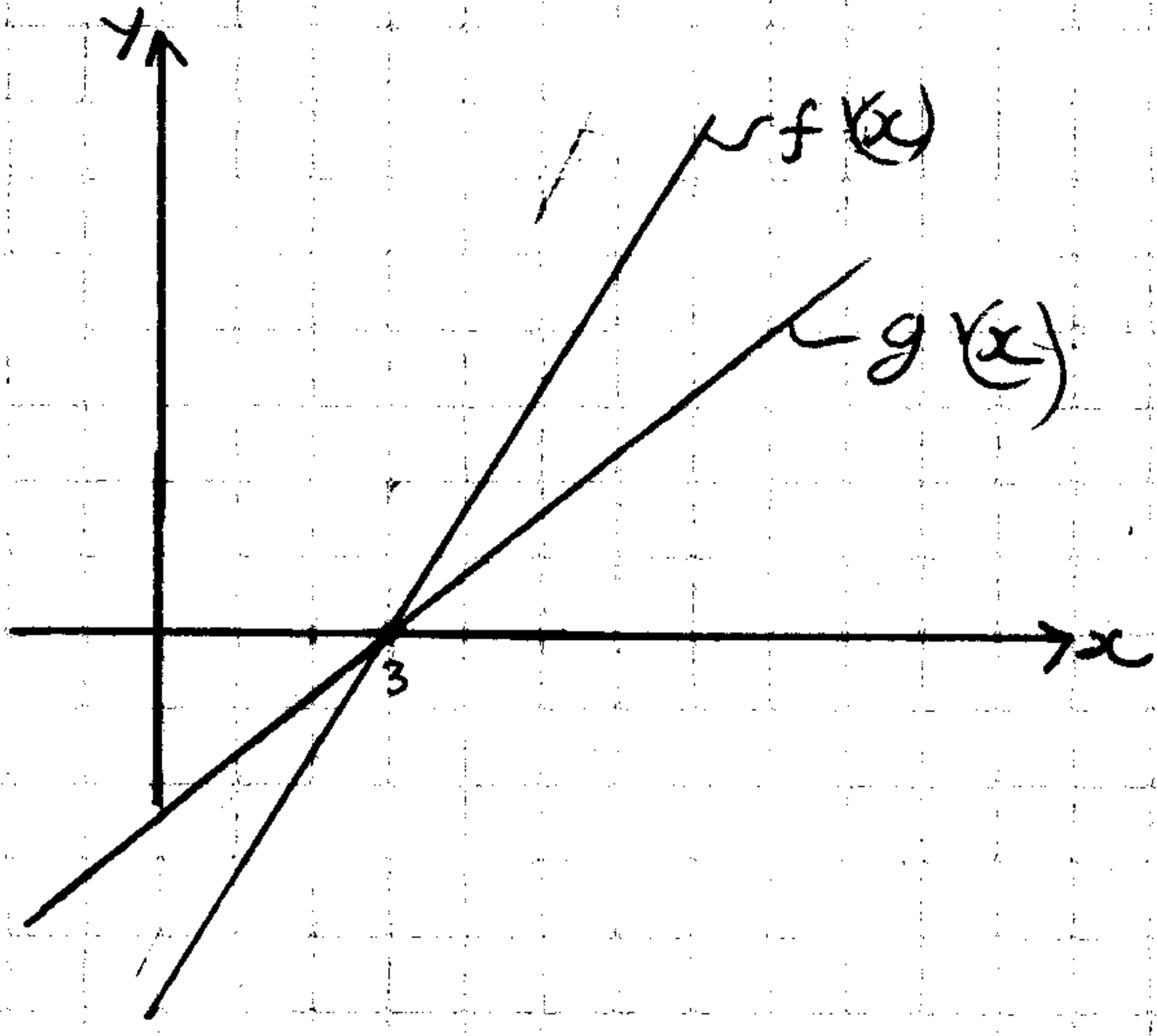
2

F

Solution

Main Grid

9)

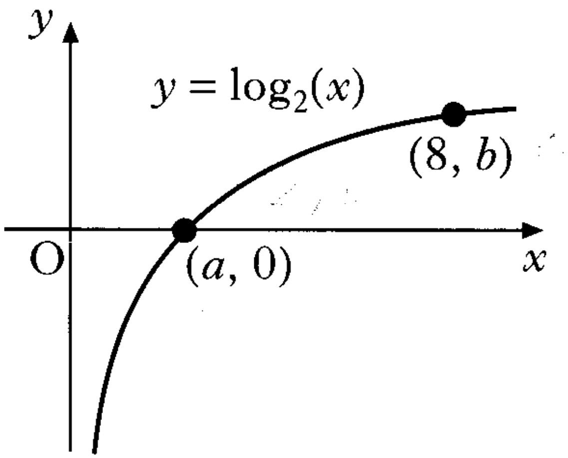


Main Grid

10. The diagram shows a sketch of part of the graph of $y = \log_2(x)$.

(*a*) State the values of a and b. **1**

(*b*) Sketch the graph of $y = \log_2(x + 1) - 3$. **3**



F

Solution

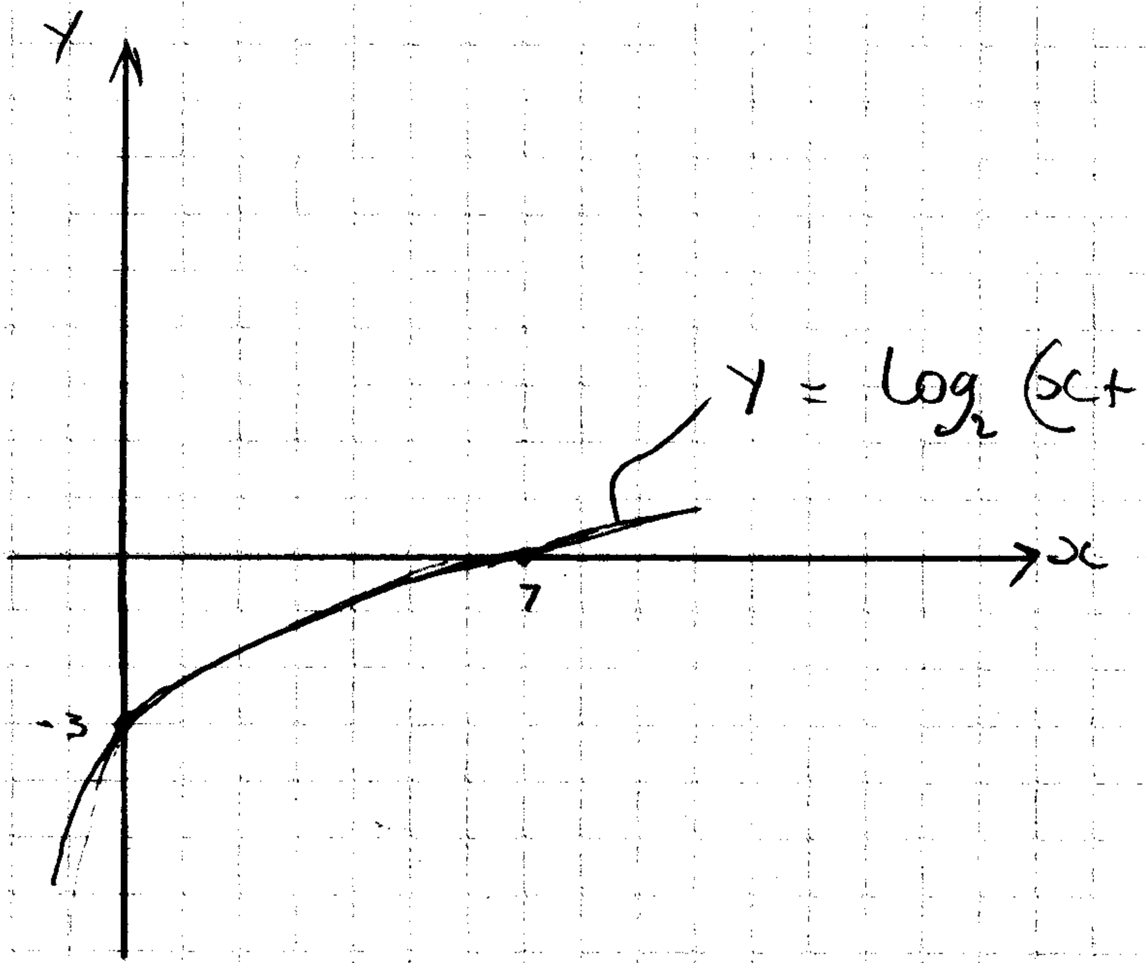
Main Grid

10) a) at a: $\log_2 a = 0$ at b: $\log_2 8 = b$

$\therefore a = 2^0$ $8 = 2^b$

$\underline{\underline{a = 1}}$ $\underline{\underline{b = 3}}$

b)



Main Grid

11. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

(*a*) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch. **4**

(*b*) Find the equation of the tangent to circle Q at the point $(-4, 1)$. **3**

(*c*) The tangent in (*b*) intersects circle P in two points. Find the *x*-coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$. **3**

F

Solution 11a **Solution 11b** **Solution 11c** **Main Grid**

11) $x^2 + y^2 + 2gx + 2fy + c = 0$ centre $= (-g, -f)$

radius $= \sqrt{g^2 + f^2 - c}$

a) i) $2g = -8$ $2f = -10$

$g = -4$ $f = -5$ $c = 9$

$r = \sqrt{16 + 25 - 9}$

$= \sqrt{32} = \sqrt{16}\sqrt{2}$

$= \underline{\underline{4\sqrt{2}}}$

ii) Centre of P = (4, 5)

$\therefore$ distance PQ $= \sqrt{6^2 + 6^2}$

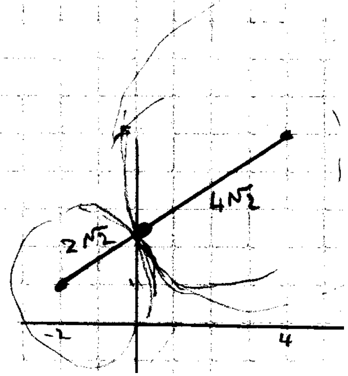
$= \sqrt{72} = \sqrt{36}\sqrt{2}$

$= \underline{\underline{6\sqrt{2}}}$

Now $r_p + r_q = 4\sqrt{2} + 2\sqrt{2}$

$= \underline{\underline{6\sqrt{2}}}$

$\therefore$ The circles must touch



Solution 11b

Solution 11c

Main Grid

b) $M_1 = \frac{1+1}{-4+2} = \frac{2}{-2} = \underline{\underline{-1}}$

$M_1 M_2 = -1 \quad \therefore \underline{\underline{M_2 = 1}}$

$y - b = m(x - a) \qquad m = 1, \quad (a, b) = (-4, 1)$

$y - 1 = 1(x + 4)$

$y - 1 = x + 4$

$\underline{\underline{x - y + 5 = 0}}$

Solution 11c

Main Grid

11) c) $y = x + 5$

Sub: $x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$

$x^2 + x^2 + 10x + 25 - 8x - 10x - 50 + 9 = 0$

$2x^2 - 8x - 16 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{8 \pm \sqrt{192}}{4}$$

$$\frac{8 \pm \sqrt{64}\sqrt{3}}{4}$$

$$\frac{8 \pm 8\sqrt{3}}{4}$$

$$\underline{\underline{2 \pm 2\sqrt{3}}}$$

$a = 2,\ b = -8,\ c = -16$

$b^2 - 4ac$

$64 - (4 \times 2 \times -16)$

$64 + 128$

192

$$\begin{array}{r} 16 \\ {}^{4}8 \\ \hline 128 \end{array}$$

$$\begin{array}{r} 64 \\ 3\overline{)1\,9\,2} \end{array}$$

Main Grid

1. (*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. **3**

(*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. **2**

F

Solution

Main Grid

1) a)

$$\begin{array}{r|rrrr} -2 & 2 & 1 & k & 2 \\ & & -4 & 6 & -2(6+k) \\ \hline & 2 & -3 & (6+k) & -2(6+k)+2 \end{array}$$

$$\therefore \quad -2(6+k)+2 = 0$$

$$-12-2k+2 = 0$$

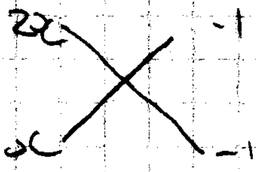
$$-2k-10 = 0$$

$$2k = -10$$

$$\underline{\underline{k = -5}}$$

b) $6+k = 1$

$$\therefore \quad (x+2)(2x^2-3x+1) = 0$$



$$(x+2)(2x-1)(x-1) = 0$$

$$\underline{\underline{x=-2, \quad x=\frac{1}{2}, \quad x=1}}$$

Main Grid

2. A curve has equation $y = x - \dfrac{16}{\sqrt{x}}, \quad x > 0$.

Find the equation of the tangent at the point where $x = 4$. **6**

F

Solution

Main Grid

2) $M = \frac{dy}{dx}$: $y = x - 16x^{-1/2}$

$$\frac{dy}{dx} = 1 + 8x^{-\frac{3}{2}}$$

$$= \underline{\underline{1 + \frac{8}{\sqrt{x^3}}}}$$

at $x = 4$: $M = 1 + \left(8 \div \sqrt{4^3}\right) = 2$

at $x = 4$: $y = 4 - \left(16 \div \sqrt{4}\right) = -4$ $\quad (a, b) = (4, -4)$

$y - b = M(x - a) = y + 4 = 2(x - 4)$

$$y + 4 = 2x - 8$$

$$\underline{\underline{2x - y - 12 = 0}}$$

Main Grid

3. On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1·5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

(*a*) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

Let u_n and u_{n+1} represent the amounts that he owes at the starts of two successive months. Write down a recurrence relation involving u_{n+1} and u_n. **2**

(*b*) Find the date and the amount of the final payment. **4**

F

Solution

Main Grid

3) $U_{n+1} = 1.015 U_n - 300$

Amount owes

April = $1.015 \times 2500 - 300$ = £2237.50

May = $1.015 \times 2237.5 - 300$ = £1971.00

June = $1.015 \times 1971 - 300$ = £1700.60

July = $1.015 \times 1700.6 - 300$ = £1426.10

Aug = $1.015 \times 1426.1 - 300$ = £1147.50

Sept = $1.015 \times 1147.5 - 300$ = £864.74

Oct = $1.015 \times 864.74 - 300$ = £577.71

Nov = $1.015 \times 577.71 - 300$ = £286.37

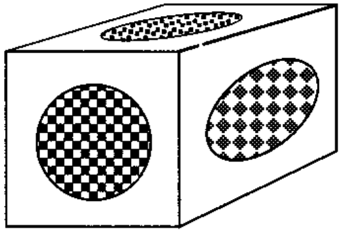
Dec = $1.015 \times 286.37 - 300$ = £−9.32

∴ Last payment in December of

$300 - 9.32$ = £290.68

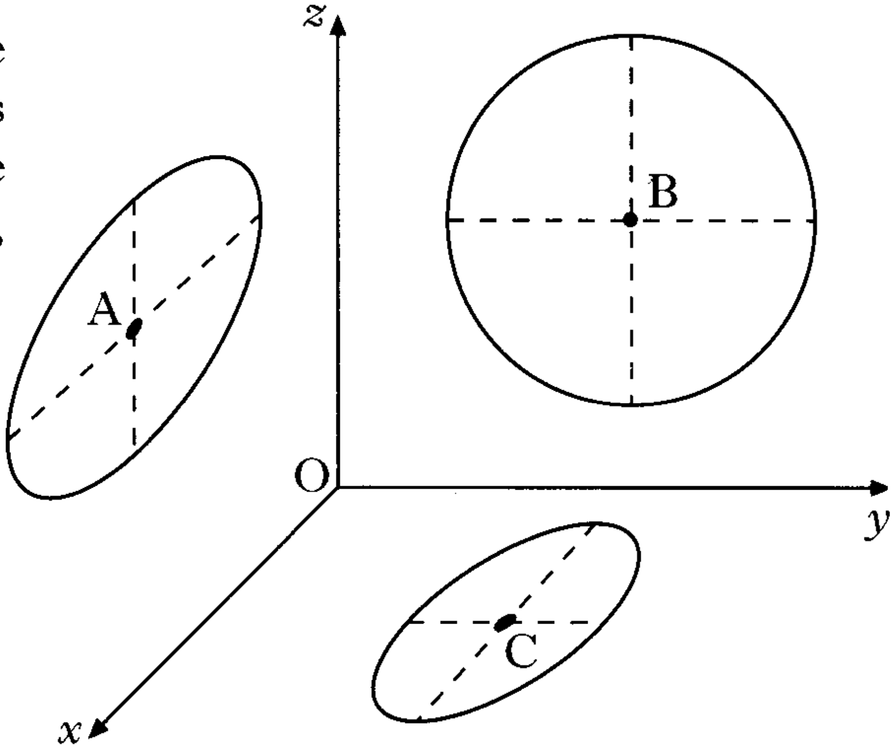
Main Grid

4. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.



The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A(6, 0, 7), B(0, 5, 6) and C(4, 5, 0).

Find the size of angle ABC.



7

F

Solution

Main Grid

4) $\overrightarrow{BA} = \underline{a} - \underline{b} = \begin{pmatrix}6\\0\\7\end{pmatrix} - \begin{pmatrix}0\\5\\6\end{pmatrix} = \underline{\underline{\begin{pmatrix}6\\-5\\1\end{pmatrix}}}$

$\overrightarrow{BC} = \underline{c} - \underline{b} = \begin{pmatrix}4\\5\\0\end{pmatrix} - \begin{pmatrix}0\\5\\6\end{pmatrix} = \begin{pmatrix}4\\0\\-6\end{pmatrix}$

$$\cos\phi = \frac{a.b}{|a||b|}$$

$$a.b = (6\times4) + (-5\times0) + (1\times-6)$$

$$= 24 + 0 - 6$$

$$= \underline{\underline{18}}$$

$$|a| = \sqrt{6^2 + -5^2 + 1^2} = \sqrt{36+25+1} = \underline{\underline{\sqrt{62}}}$$

$$|b| = \sqrt{4^2 + 0^2 + -6^2} = \sqrt{16+0+36} = \sqrt{52}$$

$$\cos\phi = \frac{18}{\sqrt{62}\sqrt{52}} = 0.317$$

$$\phi = \cos^{-1} 0.317 = \underline{\underline{71.5^{\circ}}}$$

Main Grid

5. Express $8\cos x° - 6\sin x°$ in the form $k\cos(x+a)°$ where $k>0$ and $0<a<360$. **4**

F

Solution

Main Grid

5) $8\cos x^\circ - 6\sin x^\circ = k\cos(x+a)^\circ$

$= k(\cos x\cos a - \sin x\sin a)$

$= k\cos x\cos a - k\sin x\sin a$

∴ $8 = k\cos a^\circ$ — ①

$6 = k\sin a^\circ$ — ②

①^2 + ②^2

$8^2 + 6^2 = k^2\cos^2 a + k^2\sin^2 a$

$100 = k^2(\cos^2 a + \sin^2 a)$

$100 = k^2$

$k = \sqrt{100}$

$k = 10$

② ÷ ①

$\frac{\cancel{k}\sin a^\circ}{k\cos a^\circ} = \frac{6}{8}$

$\frac{\sin a}{\cos a} = \frac{3}{4}$

$\tan a^\circ = \frac{3}{4}$

$\sin a^\circ$, $\cos a^\circ$, $\tan a^\circ$ all +ve ∴ 1st quadrant

$a^\circ = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$

∴ $8\cos x^\circ - 6\sin x^\circ = 10\cos(x + 36.87)^\circ$

Main Grid

6. Find $\int \frac{(x^2-2)(x^2+2)}{x^2}dx, \quad x \neq 0$ **4**

F

Solution **Main Grid**

6) $$\int \frac{(x^2-2)(x^2+2)}{x^2}\,dx$$

$$= \int \frac{x^4 - 4}{x^2}\,dx$$

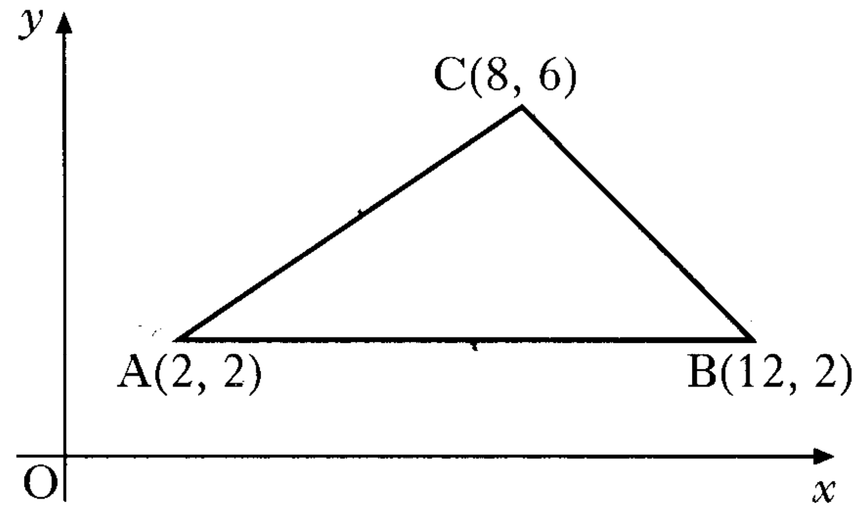
$$= \int x^2 - 4x^{-2}\,dx$$

$$= \frac{x^3}{3} + 4x^{+1} + c$$

$$= \frac{x^3}{3} + \frac{4}{x} + c$$

Main Grid

7. Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6).



(*a*) Write down the equation of l_1, the perpendicular bisector of AB. **1**

(*b*) Find the equation of l_2, the perpendicular bisector of AC. **4**

(*c*) Find the point of intersection of lines l_1 and l_2. **1**

(*d*) Hence find the equation of the circle passing through A, B and C. **2**

F

Solution

Main Grid

7) a) Mid point of AB = $(7, 2)$

Equation of perpendicular bisector is $\underline{\underline{x = 7}}$

b) Mid point of AC = $(5, 4)$; $M_{AC} = \frac{6-2}{8-2} = \frac{4}{6} = \underline{\underline{\frac{2}{3}}}$

$M_1 M_2 = -1 \quad \therefore \quad M_2 = \underline{\underline{\frac{-3}{2}}}$

$Y - b = M(x - a) \qquad M = \frac{-3}{2} \qquad (a, b) = (5, 4)$

$Y - 4 = \frac{-3}{2}(x - 5)$ $(\times 2)$

$2Y - 8 = -3x + 15$

$\underline{\underline{3x + 2Y - 23 = 0}}$

c) Sub $x = 7$ into $3x + 2Y - 23 = 0$

$= 21 + 2Y - 23 = 0$

$2Y = 2$

$Y = 1 \quad \therefore \quad \underline{\underline{(7, 1)}}$

d) Centre $(7, 1)$ radius $= \sqrt{(7-2)^2 + (1-2)^2} = \sqrt{26}$

$\therefore \quad \underline{\underline{(x - 7)^2 + (Y - 1)^2 = 26}}$

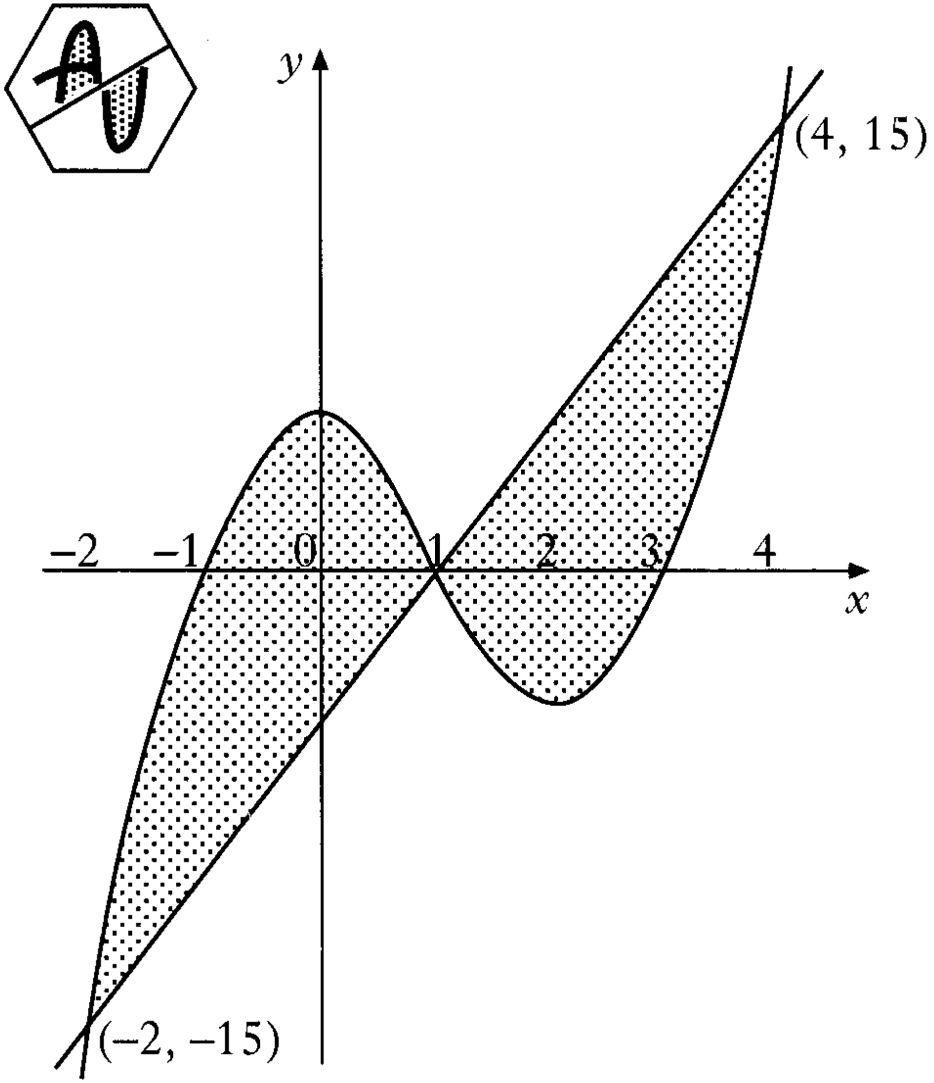
Main Grid

8. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.

A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation $y = (x + 1)(x - 1)(x - 3)$ and the straight line has equation $y = 5x - 5$. The point $(1, 0)$ is the centre of half-turn symmetry.

Calculate the total shaded area.



7

F

Solution

Main Grid

8) $(x^2-1)(x-3)$

x^3-3x^2-x+3

$$\int_1^4 (5x-5)-(x^3-3x^2-x+3)\,dx$$

$$=\int_1^4 -x^3+3x^2+6x-8 \quad dx$$

$$=\left[\frac{-x^4}{4}+x^3+3x^2-8x\right]_1^4$$

$$=\left(\frac{-4^4}{4}+4^3+3\times4^2-8\times4\right)-\left(\frac{-1^4}{4}+1^3+3\times1^2-8\times1\right)$$

$$=(-64+64+48-32)-\left(-\frac{1}{4}+1+3-8\right)$$

$$=(16)-\left(-4\tfrac{1}{4}\right)$$

$$=\underline{\underline{20\tfrac{1}{4}}}$$

$\therefore$ Total Area $= 2\times 20\tfrac{1}{4}$

$$=\underline{\underline{40\tfrac{1}{2}\text{ units}^2}}$$

Main Grid

9. Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and half hours for the area of the forest fire to double, find the value of the constant k. **3**

F

Solution

Main Grid

9)

$$2A_0 = A_0 e^{1.5K}$$

$$2 = e^{1.5K}$$

$$\ln 2 = 1.5K$$

$$0.693 = 1.5K$$

$$\therefore K = 0.693 \div 1.5$$

$$= \underline{\underline{0.462}}$$

Main Grid

10. A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5}{12}\pi,\ \sqrt{3}\right)$.

Find y in terms of x. **4**

F

Solution

Main Grid

10) $\frac{dy}{dx} = 3\sin(2x)$

$$\int 3\sin(2x)\,dx = -\frac{3}{2}\cos 2x + c$$

$$\therefore y = -\frac{3}{2}\cos 2x + c$$

at $\left(\frac{5}{12}\pi, \sqrt{3}\right)$

$$\sqrt{3} = -\frac{3}{2}\cos\left(2\times\frac{5}{12}\pi\right) + c$$

$$\Rightarrow \sqrt{3} = -\frac{3}{2} \times \frac{-\sqrt{3}}{2} + c$$

$$\Rightarrow \sqrt{3} = \frac{3\sqrt{3}}{4} + c \Rightarrow c = \frac{\sqrt{3}}{4}$$

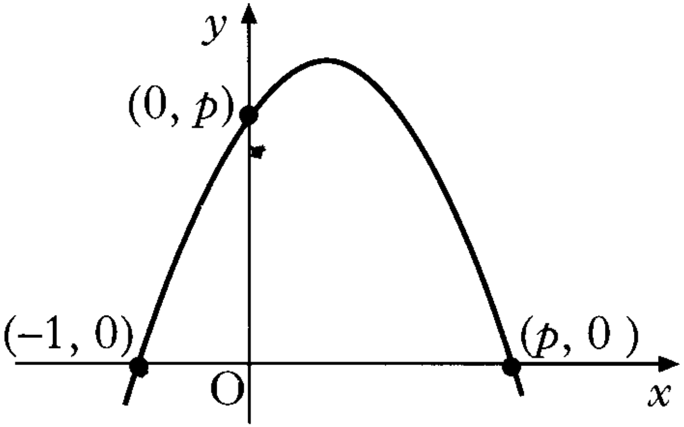
$$\therefore y = -\frac{3}{2}\cos 2x + \frac{\sqrt{3}}{4}$$

Main Grid

11. The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.

(*a*) Show that the equation of the parabola is $y = p + (p - 1)x - x^2$. **3**

(*b*) For what value of p will the line $y = x + p$ be a tangent to this curve? **3**



F

Solution

Main Grid

11) a) $y = (x+1)(x - p)$

$y = k[(x+1)(x-p)]$ where k is a constant

at $(0, p)$: $p = k(1 \times -p)$

$p = -kp$

$\therefore \underline{\underline{k = -1}}$

so: $-1[(x+1)(x - p)]$

$= -1(x^2 - px + x - p)$

$= -x^2 + px - x + p$

$\therefore \underline{\underline{y = p + (p-1)x - x^2}}$

b) For tangency discriminant $= 0$

$p + (p-1)x - x^2 = x + p$

$\cancel{p} + px - x - x^2 - x - \cancel{p} = 0$

$-x^2 \; px - 2x = 0$

$-x^2 + (p-2)x = 0$

$b^2 - 4ac = 0$

$(p-2)^2 - (4 \times -1 \times 0) = 0$

$(p-2)(p-2) = 0$

$\therefore \underline{\underline{p = 2}}$

Main Grid

1. The point P(2, 3) lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P. **4**

F

Solution

Main Grid

1) Centre $= (-1, 1)$

$$M_{radius} = \frac{1-3}{-1-2} = \frac{-2}{-3} = \frac{2}{3}$$

$$M_r \times M_t = -1 \quad \therefore \quad \underline{M_t = \frac{-3}{2}}$$

$y - b = m(x - a)$ $\quad m = \frac{-3}{2}, \quad (a, b) = (2, 3)$

$$\underline{y - 3 = \frac{-3}{2}(x - 2)}$$

Main Grid

2. The point Q divides the line joining P(–1, –1, 0) to R(5, 2, –3) in the ratio 2 : 1. Find the coordinates of Q. **3**

F

Solution

Main Grid

2)

2 : 1

	P			Q			R	
x:	-1	→	-4	3	2	←	5	$\frac{6}{3} = \underline{\underline{2}}$
y:	-1	→	2	1	1	←	2	$\frac{3}{3} = 1$
z:	0	→	2	-2	1	←	-3	$\frac{3}{3} = 1$

$$\therefore \quad \underline{\underline{Q(3, 1, -2)}}$$

Main Grid

3. Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

F

Solution **Main Grid**

3) a) i) $f(g(x)) = f(2x) = \underline{\underline{\sin 2x^\circ}}$

ii) $g(f(x)) = g(\sin x) = \underline{\underline{2\sin x^\circ}}$

b) $2\sin 2x = 2\sin x$

$2\sin 2x - 2\sin x = 0$

$4\sin x \cos x - 2\sin x = 0$

$2\sin x(2\cos x - 1) = 0$

$2\sin x = 0 \qquad \cos x = 1/2$

$x = 0, 180^\circ, 360^\circ \qquad x = 60^\circ, 300^\circ$

$\underline{\underline{x = 0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ}}$

Main Grid

4. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis. **4**

F

Solution

Main Grid

4) $M = \tan 45^\circ = 1.$

$$M = \frac{dy}{dx} = 4x - 7 = 1$$

$$\therefore \underline{x = 8/4} = \underline{\underline{2}}$$

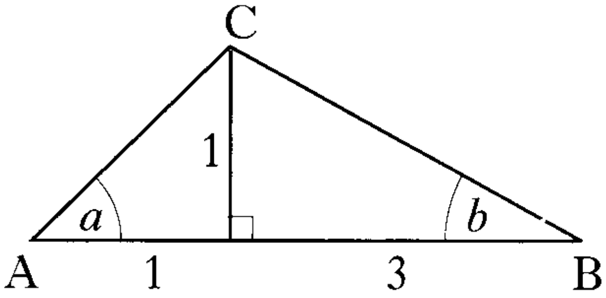
$$y = 2 \times 2^2 - 7 \times 2 + 10$$

$$= 8 - 14 + 10 = 4$$

$$\therefore \underline{\underline{(2, 4)}}$$

Main Grid

5. In triangle ABC, show that the exact value of $\sin(a+b)$ is $\frac{2}{\sqrt{5}}$.



4

F

Solution

Main Grid

5) $AC = \sqrt{2}$ $\qquad CB = \sqrt{10}$

$\sin a = \frac{1}{\sqrt{2}}$, $\cos a = \frac{1}{\sqrt{2}}$, $\sin b = \frac{1}{\sqrt{10}}$, $\cos b = \frac{3}{\sqrt{10}}$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$= \frac{1}{\sqrt{2}} \times \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{10}}$$

$$= \frac{1}{\sqrt{2}}\left(\frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}}\right)$$

$$= \frac{1}{\sqrt{2}} \times \frac{4}{\sqrt{10}} = \frac{4}{\sqrt{2}\sqrt{10}} = \frac{4}{\sqrt{20}}$$

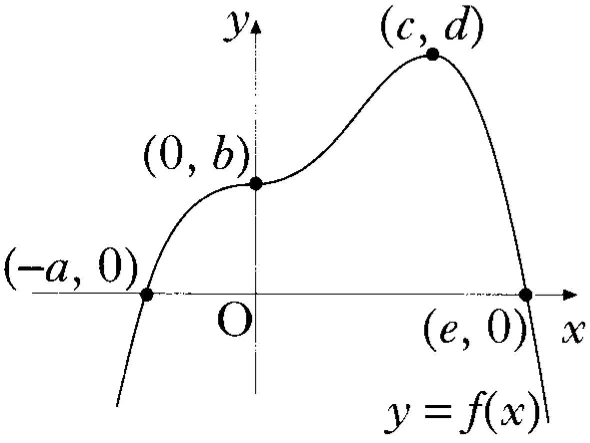
$$= \frac{4}{\sqrt{4}\sqrt{5}} = \frac{4}{2\sqrt{5}} = \underline{\underline{\frac{2}{\sqrt{5}}}}$$

Main Grid

6. The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown.

There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d).

Sketch the graph of the derived function f'.



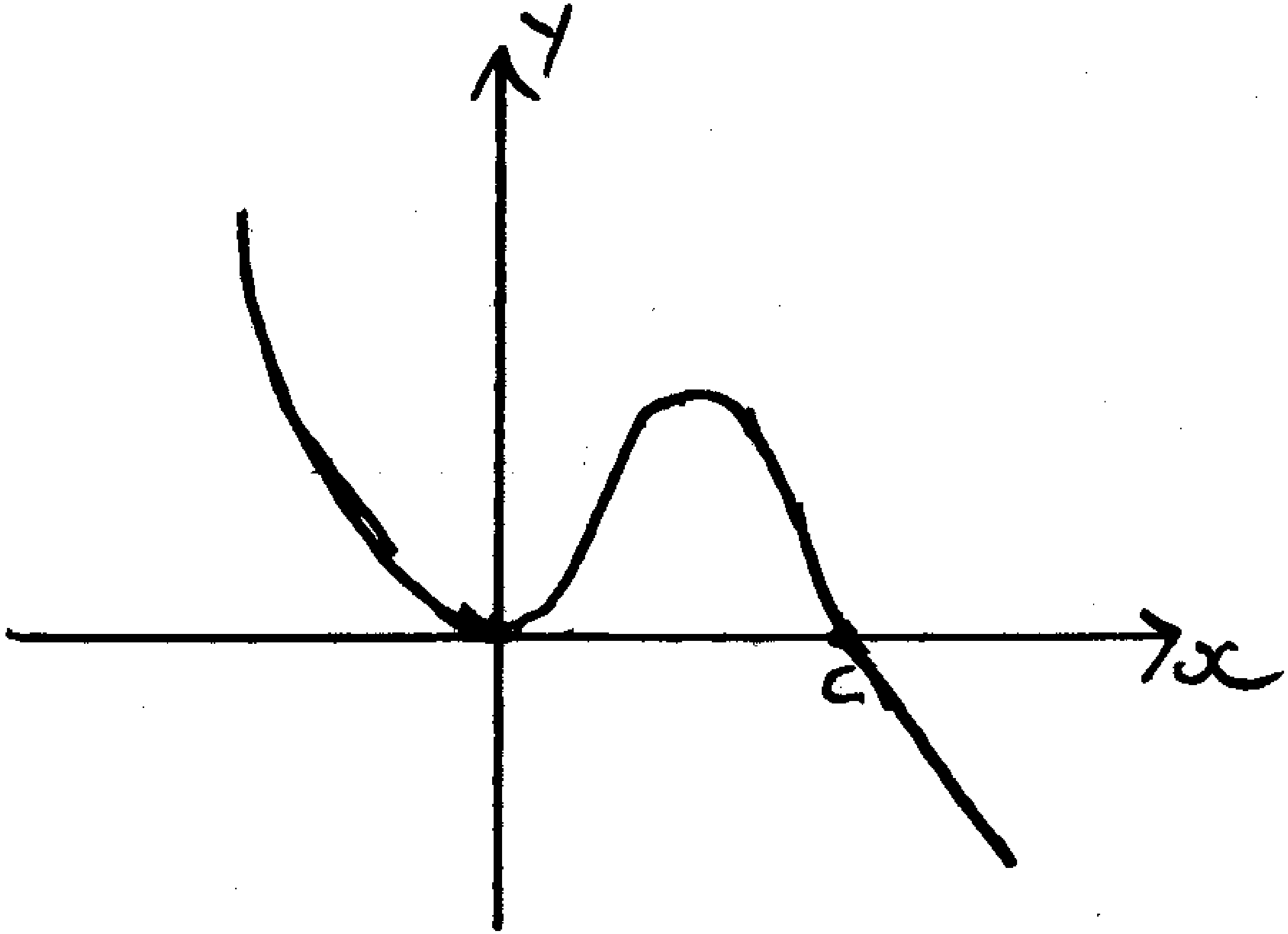
3

F

Solution

Main Grid

6)



Main Grid

7. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

F

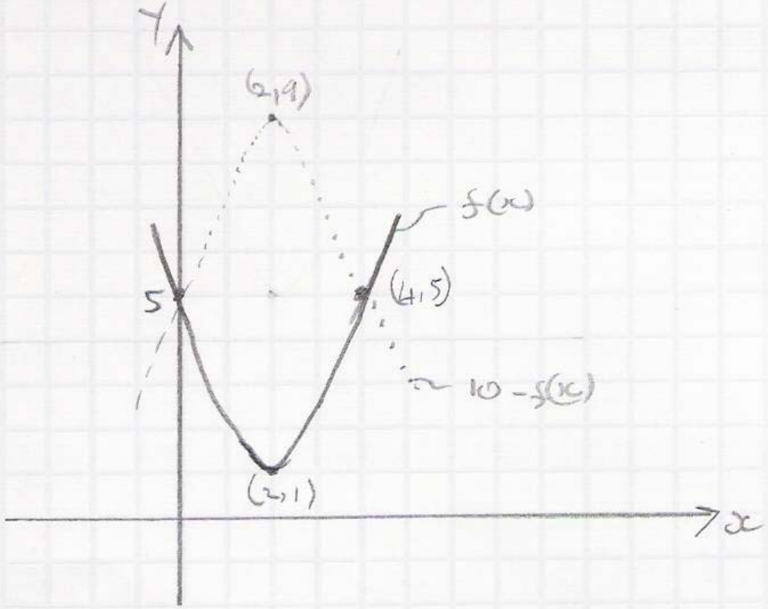
Solution

Main Grid

7) a) $f(x) = x^2 - 4x + 5 \quad = (x^2 - 4x + 4) + 5 - 4$

$$= \underline{(x-2)^2 + 1}$$

b)



c) $10 - (x-2)^2 - 1 = 0$

$(x-2)^2 = 9$

$x - 2 = \pm 3$

$x = -1, 5$

$\therefore \underline{-1 \leq x \leq 5}$

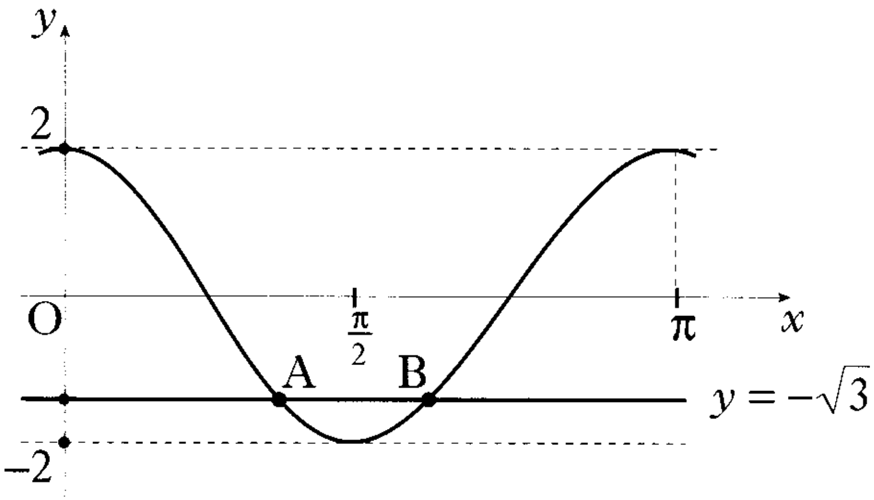
Main Grid

8. The diagram shows the graph of a cosine function from 0 to π.

(*a*) State the equation of the graph. **1**

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at points A and B.

Find the coordinates of B. **3**



F

Solution

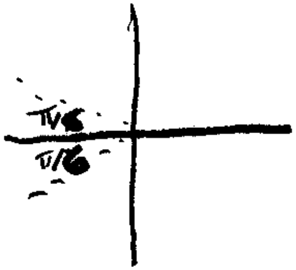
Main Grid

8) a) $Y = 2\cos 2x$

b) $2\cos 2x = -\sqrt{3}$

$\cos 2x = \frac{-\sqrt{3}}{2}$

$R \cdot A = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$



$\therefore \quad 2x = \frac{5\pi}{6} \quad \text{OR} \quad \frac{7\pi}{6}$

$\div 2$

$x = \frac{5\pi}{12} \quad \text{OR} \quad \frac{7\pi}{12}$

$\therefore \quad \underline{\underline{B\left(\frac{7\pi}{12}, -\sqrt{3}\right)}}$

Main Grid

9. (*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \leq a \leq 2\pi$. **4**

(*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \leq x \leq 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis. **3**

F

Solution 9a

Solution 9b

Main Grid

9) a) $\sin x - \cos x = k\sin(x-a)$

$$= K(\sin x\cos a - \cos x\sin a)$$

$$= K\sin x\cos a - K\cos x\sin a$$

$\therefore\ K\cos a = 1 \qquad K\sin a = 1$

Square and add: $K^2(\cos^2 a + \sin^2 a) = 2$

$$\underline{K = \sqrt{2}}$$

$$\frac{K\sin a}{K\cos a} = \frac{1}{1} = \underline{\underline{\tan a = 1}}$$

$\therefore\ \tan a = 1$ sin is +ve , cos is +ve

$\therefore$ 1st Quadrant

$\therefore\ a = \tan^{-1}(1) = \pi/4$

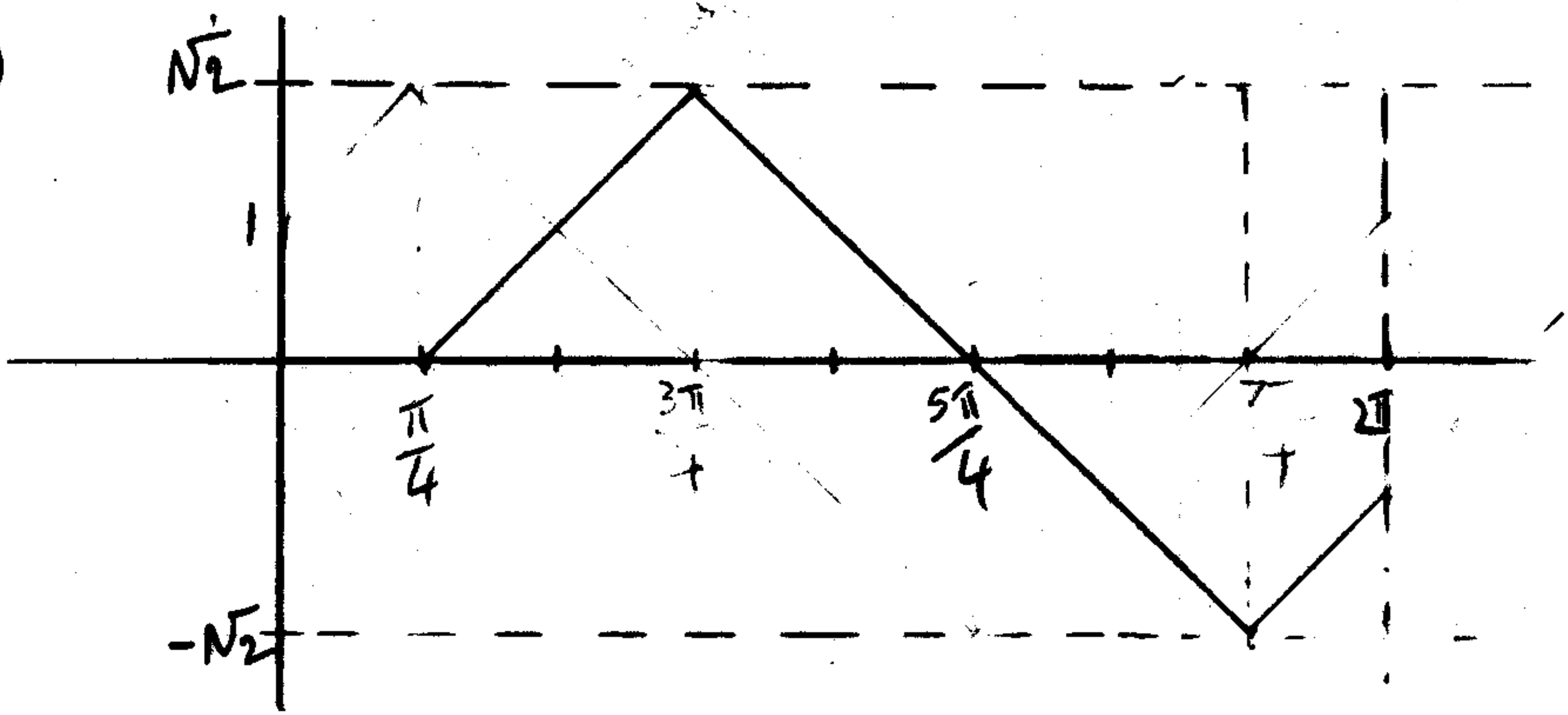
$\therefore\ \sin x - \cos x = \underline{\underline{\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)}}$

$\therefore\ \underline{\underline{K = \sqrt{2} \qquad a = \frac{\pi}{4}}}$

Solution 9b

Main Grid

b)



Main Grid

10. (*a*) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}, \; x < 2.$ **2**

(*b*) Hence write down $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}} dx.$ **1**

F

Solution

Main Grid

10) a) $f(x) = (8 - x^3)^{1/2}$

$$f'(x) = \frac{1}{2}(8-x^3)^{-1/2} \times -3x^2$$

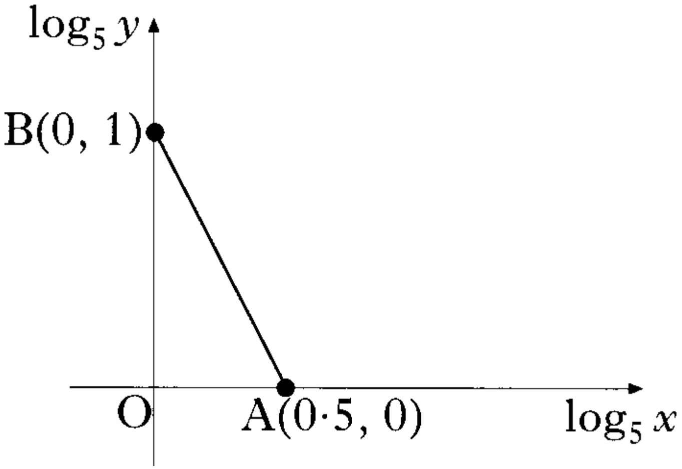
$$= \underline{\underline{\frac{-3x^2}{2\sqrt{(8-x^3)}}}}$$

b) $$\int \frac{x^2}{(8-x^3)^{1/2}} = \underline{\underline{\frac{-2}{3}(8-x^3)^{1/2} + C}}$$

Main Grid

11. The graph illustrates the law $y = kx^n$.

If the straight line passes through A(0·5, 0) and B(0, 1), find the values of k and n.



4

F

Solution

Main Grid

11) If $y = kx^n$ then $n = M$, $c = \log_5 k$

$M = \frac{-1}{0.5} = \underline{\underline{-2}}$

$\therefore \underline{\underline{n = -2}}$

$c = \log_5 k$

$1 = \log_5 k$

$\log_5 5 = \log_5 k$

$\therefore \underline{k = 5}$

$\therefore \underline{\underline{y = 5x^{-2}}}$

Main Grid

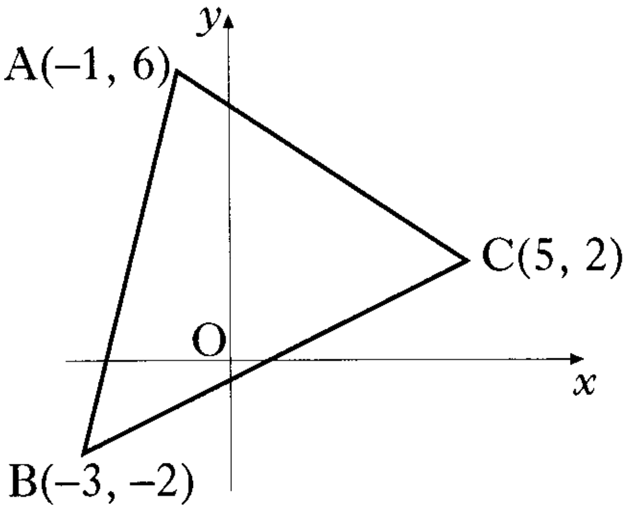
1. Triangle ABC has vertices A(–1, 6), B(–3, –2) and C(5, 2).

Find

(*a*) the equation of the line *p*, the median from C of triangle ABC. **3**

(*b*) the equation of the line *q*, the perpendicular bisector of BC. **4**

(*c*) the coordinates of the point of intersection of the lines *p* and *q*. **1**



F

Solution

Main Grid

1) a) Mid Point AB = $(-2, 2)$

$M_p = \frac{2-2}{5--2} = \frac{0}{7} = 0$

$\therefore$ P is $\underline{\underline{y=2}}$

b) $M_{BC} = \frac{2--2}{5--3} = \frac{4}{8} = \frac{1}{2}$ $\therefore M_1M_2 = -1$ $\therefore M_q = -2$

Mid Point BC = $(1, 0)$

$y - b = M(x - a)$ $M = -2, \ (a, b) = (1, 0)$

$y - 0 = -2(x - 1)$

$\underline{\underline{y = -2x + 2}}$ $\therefore$ $\underline{\underline{y = 2 - 2x}}$

c) $y = 2$ $\therefore$ $2 = 2 - 2x$

$4 \quad -x \quad -1$

$\underline{\underline{x = 0}}$

$\therefore$ $\underline{\underline{(0, 2)}}$

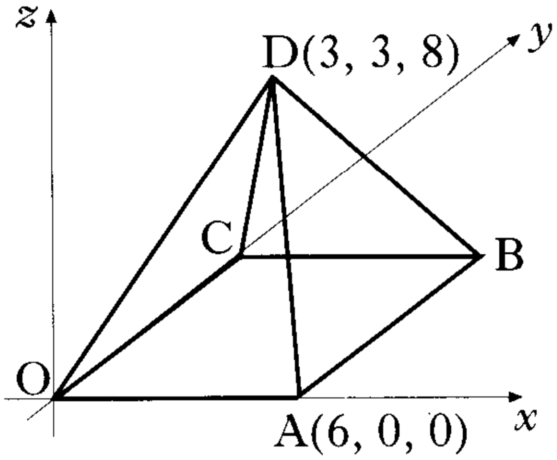
Main Grid

2. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

The coordinates of A and D are (6, 0, 0) and (3, 3, 8).

C lies on the y-axis.



(a) Write down the coordinates of B. **1**

(b) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2**

(c) Calculate the size of angle ADB. **4**

F

Solution

Main Grid

2) a) $B(6,6,0)$

b) $\overrightarrow{DA} = a - d = \begin{pmatrix}6\\0\\0\end{pmatrix} - \begin{pmatrix}3\\3\\8\end{pmatrix} = \underline{\underline{\begin{pmatrix}3\\-3\\-8\end{pmatrix}}}$ $\quad \overrightarrow{DB} = b - d = \begin{pmatrix}6\\6\\0\end{pmatrix} - \begin{pmatrix}3\\3\\8\end{pmatrix} = \underline{\underline{\begin{pmatrix}3\\3\\-8\end{pmatrix}}}$

c) $\cos\phi = \dfrac{a \cdot b}{|a||b|} = \dfrac{\overrightarrow{DA} \cdot \overrightarrow{DB}}{|\overrightarrow{DA}||\overrightarrow{DB}|}$

$\overrightarrow{DA} \cdot \overrightarrow{DB} = (3 \times 3) + (-3 \times 3) + (-8 \times -8) = 9 - 9 + 64 = \underline{\underline{64}}$

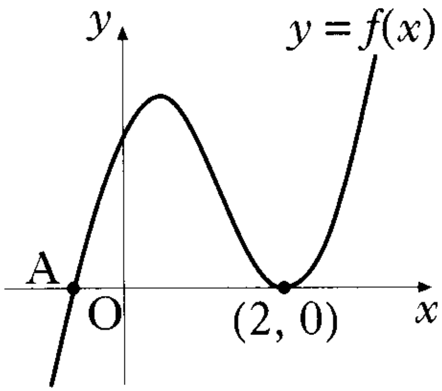
$|\overrightarrow{DA}| = \sqrt{3^2 + -3^2 + -8^2} = \underline{\underline{9.055}}$ $\quad |\overrightarrow{DB}| = \sqrt{3^2 + 3^2 - 8^2} = \underline{\underline{9.055}}$

$\therefore \cos\phi = \dfrac{64}{9.055 \times 9.055} = 0.781$

$\therefore \phi = \cos^{-1}(0.781) = \underline{\underline{38.6^\circ}}$

Main Grid

3. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(*a*) Find the *x*-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(*c*) State the coordinates of the point A and hence find the values of *x* for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

F

Solution

Main Grid

3)

a) Max at $\frac{dy}{dx} = 0$: $\frac{dy}{dx} = 6x^2 - 14x + 4 = 0$

$$2(3x^2 - 7x + 2) = 0$$

$$2(3x - 1)(x - 2) = 0$$

$$\therefore x = \frac{1}{3} \quad \text{OR } x = 2$$

$\therefore$ at Max $\underline{\underline{x = \frac{1}{3}}}$

b)

$$\begin{array}{r|rrrr} 2 & 2 & -7 & 4 & 4 \\ & & 4 & -6 & -4 \\ \hline & 2 & -3 & -2 & 0 \end{array}$$

$\therefore (x-2)(2x^2-3x-2)$

$\underline{\underline{(x-2)(2x+1)(x-2)}}$

c) $A_x = -\frac{1}{2}$, $A_y = 0$

$\therefore A(-1/2, 0)$

$\therefore \underline{\underline{x < -1/2}}$

Main Grid

4. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0·5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.

(*a*) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run? **3**

(*b*) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition? **3**

F

Solution

Main Grid

4) a) $u_{n+1} = au_n + b$

$u_{n+1} = 0.8\,u_n + 0.5$ $\qquad -1 < 0.8 < 1 \therefore$ limit exists.

limit $\quad L = \dfrac{b}{1-a}$

$$L = \frac{0.5}{1-0.8} = \frac{0.5}{0.2} = \underline{2.5m}$$

b) $L = 2$ find a

$$\therefore \quad 2 = \frac{0.5}{(1-a)} = 2(1-a) = 0.5$$

$$2 - 2a = 0.5$$

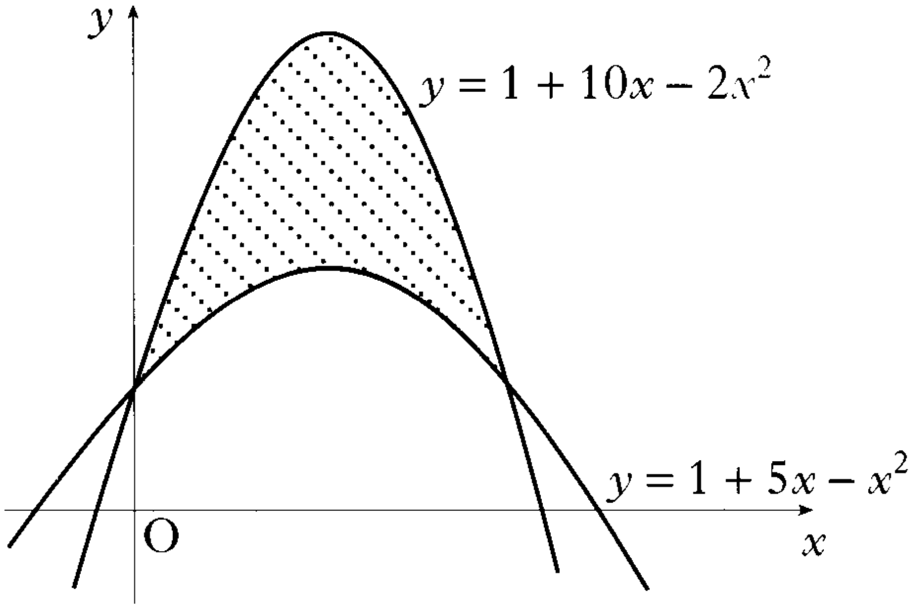
$$-2a = -1.5$$

$$a = 0.75$$

$\therefore$ Trim $\underline{25\%}$

Main Grid

5. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.



6

F

Solution

Main Grid

5) Find limits: $1+10x-2x^2 = 1+5x-x^2$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$\underline{\underline{x=0}} \quad , \underline{\underline{x=5}}$$

$$\therefore \text{Area} = \int_0^5 (1+10x-2x^2)-(1+5x-x^2)\,dx$$

$$= \int_0^5 (5x-x^2)\,dx = \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^5$$

$$= \left(\frac{5\times5^2}{2} - \frac{5^3}{3}\right) - 0 = 62.5 - 41.7$$

$$= \underline{\underline{20.8 \text{ units}^2}}$$

Main Grid

6. Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$. **4**

F

Solution

Main Grid

6)

$$M = \frac{dy}{dx} = 2\cos\left(x - \frac{\pi}{6}\right) \text{ at } \frac{\pi}{3}$$

$$M = 2\cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{6}\right)$$

$$= \frac{2\sqrt{3}}{2} = \underline{\underline{\sqrt{3}}}$$

$$\text{at } x = \frac{\pi}{3} : \; y = 2\sin\left(\frac{\pi}{6}\right) = 2 \times \frac{1}{2} = \underline{\underline{1}}$$

$$y - b = M(x - a) \qquad M = \sqrt{3} \quad (a, b) = \left(\frac{\pi}{3}, 1\right)$$

$$\underline{\underline{y - 1 = \sqrt{3}\left(x - \frac{\pi}{3}\right)}}$$

Main Grid

7. Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3 (x - 2) + 1$ intersects the x-axis. **3**

F

Solution

Main Grid

7) Cuts x-axis at $Y=0$

$\therefore \quad \log_3(x-2)+1=0$

$\log_3(x-2) = -1$

$\log_3(x-2) = \log_3 1/3$

$x-2 = 1/3$

$x = 2\,1/3$

$-1 = \log_3 \frac{1}{3}$

Main Grid

8. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. **4**

F

Solution

Main Grid

8) If $a = \frac{dv}{dt}$ then $v = \int a \, dt$

$$V = \int 2(4-t)^{-1/2} \, dt = \frac{2}{-1 \times \frac{3}{2}} (4-t)^{-1/2} + c$$

$$= -\frac{4}{3} (4-t)^{3/2} + c$$

$$= \frac{-4\sqrt{(4-t)^3}}{3} + c$$

at $t = 0$, $V = 0$

$$\therefore \quad 0 = \frac{-4\sqrt{(4)^3}}{3} + c$$

$$0 = -10.67 + c$$

$$\therefore c = \underline{\underline{10.67}}$$

$$\therefore \quad \underline{\underline{V = \frac{-4\sqrt{(4-t)^3}}{3} + 10.67}}$$

Main Grid

9. Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

F

Solution

Main Grid

9) $b^2 - 4ac \geq 0$

$a = (1-2k)$ $\quad$ $b = -5k$ $\quad$ $c = -2k$

$$= (-5k)^2 - (4 \times (1-2k) \times -2k)$$

$$= 25k^2 - (-8k(1-2k))$$

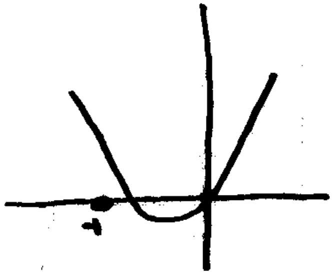
$$= 25k^2 - (-8k + 16k^2)$$

$$= 25k^2 + 8k - 16k^2$$

$$= \underline{\underline{9k^2 + 8k}} \geq 0$$

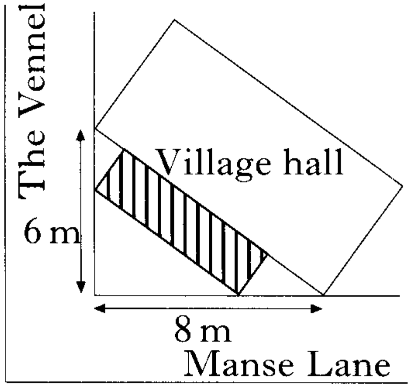
$\therefore$ $k(9k+8) \geq 0$

$k = 0$ or $k = \frac{-8}{9}$

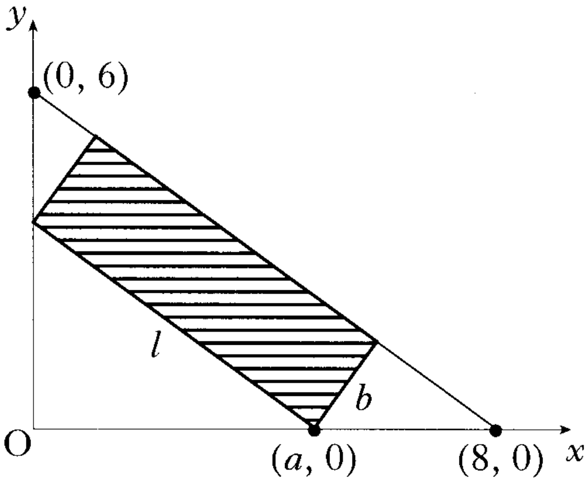


$\therefore$ all <u>integer</u> values are greater than or equal to zero.

10. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



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Solution 10a

(*a*) (i) Show that $l = \frac{5}{4}a$.

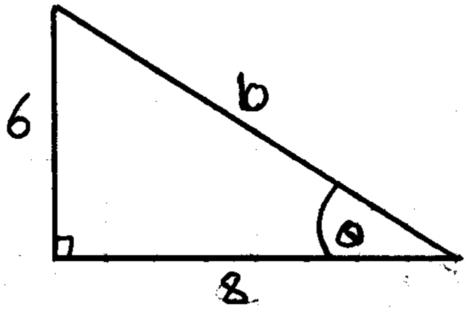
Solution 10b

(ii) Express b in terms of a and hence deduce that the area, $A\text{ m}^2$, of the extension is given by $A = \frac{3}{4}a(8-a)$. 3

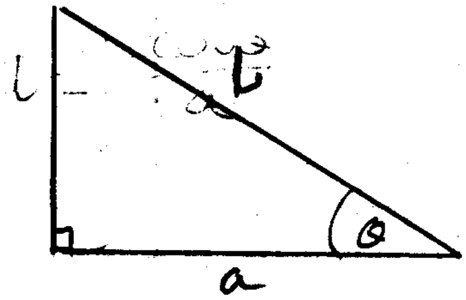
Main Grid

(*b*) Find the value of a which produces the largest area of the extension. 4

10) a) i) Similar triangles.



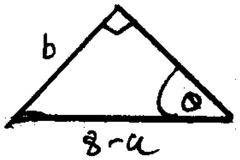
$$\therefore \sin\theta = \frac{6}{10} = \frac{3}{5}, \quad \cos\theta = \frac{8}{10} = \frac{4}{5}, \quad \tan\theta = \frac{6}{8} = \frac{3}{4}$$



$$\cos\theta = \frac{a}{l} \qquad \therefore l = \frac{a}{\cos\theta} = \frac{a}{\frac{4}{5}}$$

$$\therefore \underline{\underline{l = \frac{5a}{4}}}$$

ii)



$$\sin\theta = \frac{b}{8-a} \qquad \therefore b = \frac{3}{5}(8-a).$$

$$\therefore A = \frac{5}{4}a \times \frac{3}{5}(8-a) = \underline{\underline{\frac{3}{4}a(8-a)}}$$

Solution 10b

Main Grid

10) b) Max at $\frac{dA}{da} = 0$

$\therefore \quad A = 6a - \frac{3}{4}a^2$

$$\frac{dA}{da} = 6 - \frac{3}{2}a = 0$$

$\therefore \quad \frac{3}{2}a = 6$

$$a = \frac{2 \times 6}{3} = \underline{\underline{4}}$$

Check:

a	4^-	4	4^+
$\frac{dA}{da}$	$+$	0	$-$
SHAPE	/	—	\

$\therefore$ <u>Max at $a = 4$</u>

Main Grid

1. Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x + y - 1 = 0$. **3**

F

Solution

Main Grid

1) $y = -4x + 1 \quad \therefore \quad M = -4$

$M_1 M_2 = -1 \qquad -4 \times M_2 = -1$

$$\underline{\underline{M_2 = 1/4}}$$

$y - b = M(x - a) \qquad (a, b)\ (-1, 3)$

$y - 3 = \frac{1}{4}(x + 1)$

$4y - 12 = x + 1$

$$\underline{\underline{x - 4y + 13 = 0}}$$

Main Grid

2. (*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. **2**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$. **2**

F

Solution

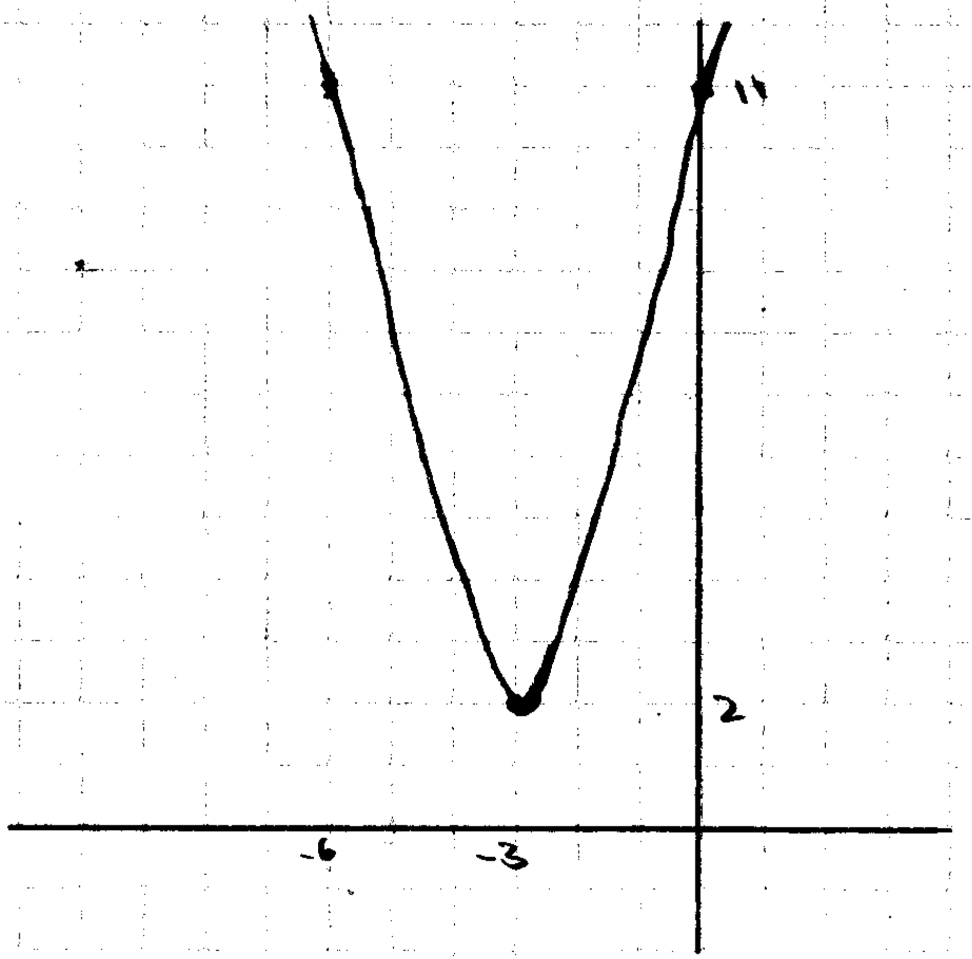
Main Grid

2) a) $x^2 + 6x + 11$

$= x^2 + 6x + 9 - 9 + 11$

$= \underline{\underline{(x+3)^2 + 2}}$

b)



Main Grid

3. Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$.

Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other. **2**

F

Solution

Main Grid

3) If u is perpendicular to v than u.v = 0

$$u \cdot v = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} = (3 \times 2) + (2 \times -3) + (0 \times 4)$$

$$= 6 - 6 = \underline{\underline{0}}$$

∴ u & v are perpendicular

Main Grid

4. A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.

(*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. **2**

(*b*) Find the limit of this recurrence relation as $n \to \infty$. **2**

F

Solution

Main Grid

4) a)

$$15 = 12p + q$$
$$16 = 15p + q$$
$$-1 = -3p$$
$$\underline{\underline{p = 1/3}}$$

$$15 = \frac{12}{3} + q$$
$$15 = 4 + q$$
$$\underline{\underline{q = 11}}$$

b) The sequence will tend to a limit because $-1 < \frac{1}{3} < 1$.

$$U_{n+1} = \frac{1}{3}U_n + 11$$

As $n \to \infty$ $\quad L = \frac{1}{3}L + 11$

$$L - \frac{1}{3}L = 11$$
$$\frac{2}{3}L = 11$$
$$L = \frac{11 \times 3}{2} = \frac{33}{2} = \underline{\underline{16\tfrac{1}{2}}}$$

Main Grid

5. Given that $f(x) = \sqrt{x} + \dfrac{2}{x^2}$, find $f'(4)$.

F

Solution

Main Grid

5)

$$f(x) = x^{1/2} + 2x^{-2}$$

$$f'(x) = \frac{1}{2}x^{-1/2} - 4x^{-3}$$

$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{4}{x^3}$$

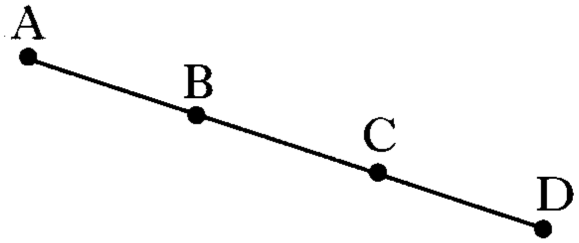
$$f'(4) = \frac{1}{2\sqrt{4}} - \frac{4}{4^3} = \frac{1}{4} - \frac{1}{16} = \underline{\underline{\frac{3}{16}}}$$

Main Grid

6. A and B are the points $(-1, -3, 2)$ and $(2, -1, 1)$ respectively.

B and C are the points of trisection of AD, that is AB = BC = CD.

Find the coordinates of D.



3

F

Solution

Main Grid

6)

$C = (2+3, \ -1+2, \ 1-1)$

$\underline{\underline{C(5, 1, 0)}}$

$D = (5+3, \ 1+2, \ 0-1)$

$\underline{\underline{D(8, 3, -1)}}$

Main Grid

7. Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. **5**

F

Solution

Main Grid

7) sub $y = 2x + 1$ into $y = x^2 + 3x + 4$

$$y = y$$

$$x^2 + 3x + 4 = 2x + 1$$

$$x^2 + x + 3 = 0$$

$a = 1$ $b = 1$ $c = 3$

$$b^2 - 4ac = 1 - (4 \times 1 \times 3)$$

$$= 1 - 12$$

$$= -11 < 0 \therefore \text{no solutions}$$

$\therefore$ the line does not intersect the parabola

Main Grid

8. Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$. **4**

F

Solution **Main Grid**

8)

$$\int_0^1 (3x+1)^{-1/2}\,dx = \left[\frac{1}{3(-1/2+1)}(3x+1)^{1/2}\right]_0^1 = \left[\frac{1}{3/2}(3x+1)^{1/2}\right]_0^1$$

$$= \left[\frac{2\sqrt{(3x+1)}}{3}\right]_0^1 = \frac{2\sqrt{4}}{3} - \frac{2\sqrt{1}}{3} = \frac{4}{3} - \frac{2}{3} = \underline{\underline{\frac{2}{3}}}$$

Main Grid

9. Functions $f(x) = \dfrac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

(*a*) Find an expression for $h(x)$ where $h(x) = f(g(x))$. **2**

(*b*) Write down any restriction on the domain of h. **1**

F

Solution

Main Grid

9) a) $h(x) = \frac{1}{(2x+3)-4} = \underline{\underline{\frac{1}{2x-1}}}$

b) $2x-1 = 0$

$x = \frac{1}{2} \quad \therefore \quad \underline{\underline{x \neq \frac{1}{2}}}$

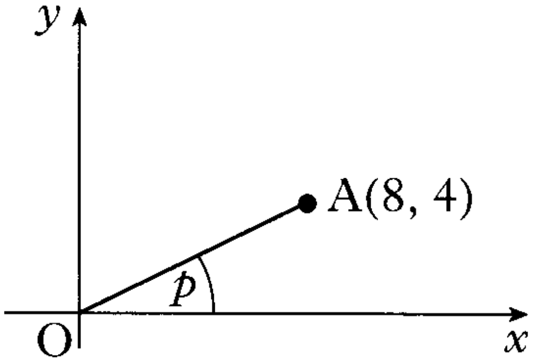
Main Grid

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis.

(*a*) Find the exact values of:

(i) $\sin(2p)$;

(ii) $\cos(2p)$.



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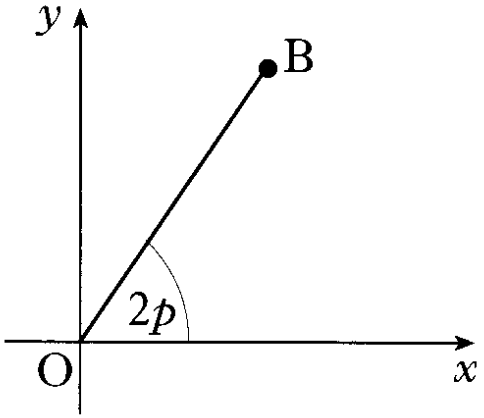
The line OB is inclined at an angle $2p$ radians to the x-axis.

(*b*) Write down the exact value of the gradient of OB.



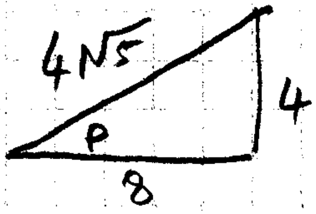
1

F

Solution

Main Grid

10)



$\sin P = \frac{4}{4\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\cos P = \frac{8}{4\sqrt{5}} = \frac{2}{\sqrt{5}}$

a) i) $\sin(2P) = 2 \sin P \cos P$

$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$

ii) $\cos(2P) = 2(\cos P)^2 - 1$

$= 2 \times \left(\frac{2}{\sqrt{5}}\right)^2 - 1$

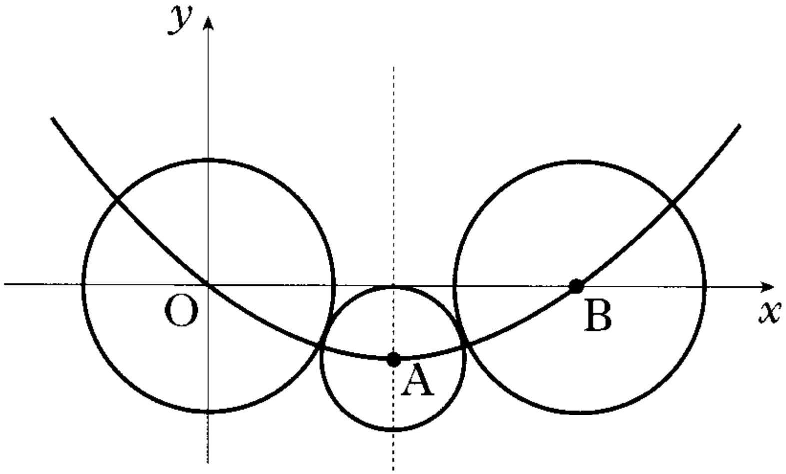
$= 2 \times \frac{4}{5} - 1$

$= \frac{8}{5} - 1 = \frac{3}{5}$

b) $M_{OB} = \tan(2P) = \frac{\sin 2P}{\cos 2P} = \frac{4}{5} \times \frac{5}{3} = \frac{4}{3}$

Main Grid

11.
- O, A and B are the centres of the three circles shown in the diagram below.
- The two outer circles are congruent and each touches the smallest circle.
- Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
- The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(*a*) (i) State the coordinates of A and find the length of the line OA. **2**

(ii) Hence find the equation of the circle with centre B. **3**

(*b*) The equation of the parabola can be written in the form $y = px(x + q)$.

Find the values of p and q. **2**

F

Solution **Main Grid**

11) a) i) $A(12, -5)$ $|OA| = \sqrt{144+25} = \sqrt{169} = \underline{\underline{13}}$

ii) Radius A $= 5$, $|AB| = 13$

$\therefore$ Radius B $= 13 - 5 = \underline{\underline{8}}$

Centre B $= (24, 0)$

$\therefore$ Circle B: $\underline{\underline{(x-24)^2 + (y-0)^2 = 64}}$

b) at $(12, -5)$: $-5 = 12p(12+q) = -5 = 144p + 12pq$

at $(24, 0)$: $0 = 24p(24+q) = 0 = 576p + 24pq$

$\therefore$ $p = \frac{5}{144}$ $pq = -\frac{5}{6}$

$\therefore$ $q = -\frac{5}{6} \times \frac{144}{5} = \underline{\underline{-24}}$ $\therefore$ $\underline{\underline{p = \frac{5}{144}}}$, $q = \underline{\underline{-24}}$

Main Grid

12. Simplify $3\log_e(2e) - 2\log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers. **4**

F

Solution

Main Grid

12)

$$3\log_e(2e) - 2\log_e(3e)$$

$$\log_e(2e)^3 - \log_e(3e)^2$$

$$\log_e 8e^3 - \log_e 9e^2$$

$$\log_e 8 + \log_e e^3 - (\log_e 9 + \log_e e^2)$$

$$\log_e 8 + 3\log_e e - \log_e 9 - 2\log_e e$$

$$\log_e 8 + 3 - \log_e 9 - 2$$

$$\underline{\underline{1 + \log_e 8 - \log_e 9}}$$

Main Grid

1. $f(x) = 6x^3 - 5x^2 - 17x + 6.$

(*a*) Show that $(x - 2)$ is a factor of $f(x)$.

(*b*) Express $f(x)$ in its fully factorised form. **4**

F

Solution **Main Grid**

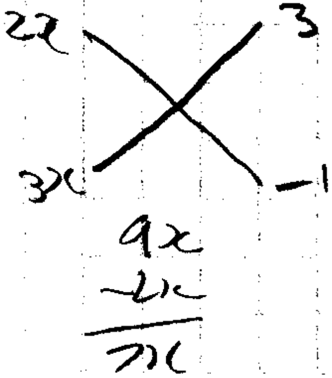
1) a)

$$\begin{array}{r|rrrr} 2 & 6 & -5 & -17 & 6 \\ & & 12 & 14 & -6 \\ \hline & 6 & 7 & -3 & \underline{\underline{0}} \end{array}$$

Remainder $= 0$ ∴ $(x-2)$ is a factor.

b) $(x-2)(6x^2+7x-3)$

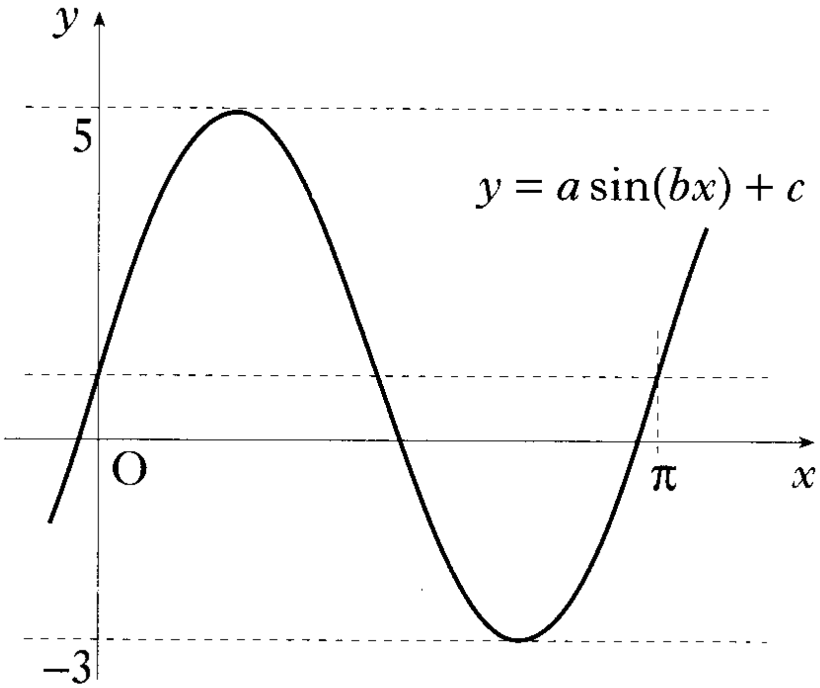
$$\underline{\underline{(x-2)(2x+3)(3x-1)}}$$



Main Grid

2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$.

Determine the values of a, b and c.



3

F

Solution

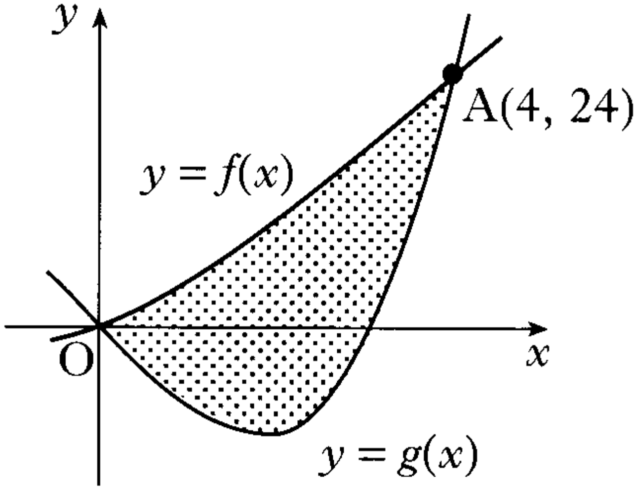
Main Grid

2) | $a = 4$ $b = 2$ $c = 1.$

Main Grid

3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

Find the shaded area enclosed between the curves.



5

F

Solution

Main Grid

3) $$\int_0^4 \left[(x^2+2x)-(x^3-x^2-6x)\right]dx$$

$$= \int_0^4 -x^3+2x^2+8x \; dx$$

$$= \left[\frac{-x^4}{4} + \frac{2x^3}{3} + 4x^2 \right]_0^4$$

$$= \left(-\frac{4^4}{4} + 2\times\frac{4^3}{3} + 4\times 4^2 \right) - (0)$$

$$= -64 + \frac{128}{3} + 64$$

$$= \underline{\underline{\frac{128}{3} \text{ units}^2}}$$

Main Grid

4. (*a*) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. **5**

(*b*) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. **6**

F

Solution 4a

Solution 4b

Main Grid

4) a) at $x=1$: $y = 1^3+2\times1^2-3\times1\ +2$

$= 1+2-3+2$

$= \underline{2}$ $\therefore$ $\underline{\underline{(1, 2)}}$

$$M = \frac{dy}{dx} = 3x^2+4x-3$$

at $x=1$: $M = 3+4-3 = \underline{\underline{4}}$

$$y-b = M(x-a)$$

$$y-2 = 4(x-1)$$

$$y-2 = 4x-4$$

$$\underline{\underline{y = 4x-2}}$$

Solution 4b

Main Grid

b) Sub $y = 4x - 2$ into circle.

$$x^2 + (4x-2)^2 - 12x - 10(4x-2) + 44 = 0$$

$$x^2 + 16x^2 - 16x + 4 - 12x - 40x + 20 + 44 = 0$$

$$17x^2 - 68x + 68 = 0$$

$a = 17 \quad b = -68 \quad c = 68$

$$b^2 - 4ac = (-68^2) - (4 \times 17 \times 68)$$

$$= 4624 - 4624$$

$$= \underline{\underline{0}} \quad \therefore 1 \text{ solution}, 1 \text{ point of contact}$$

$\therefore$ tangent.

$$17x^2 - 68x + 68 = 0$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 \quad \therefore x = 2$$

$$y = 4 \times 2 - 2 = 6$$

$$\underline{\underline{(2, 6)}}$$

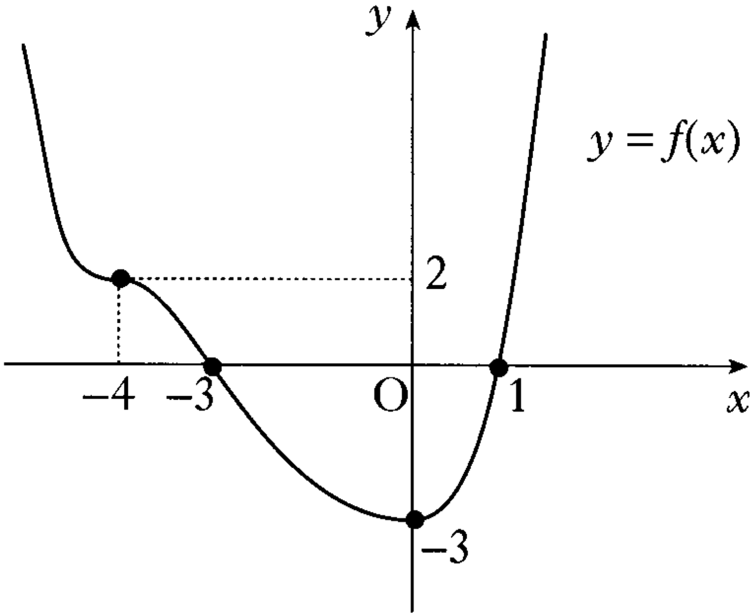
Main Grid

5. The diagram shows the graph of a function f.

f has a minimum turning point at (0, –3) and a point of inflexion at (–4, 2).

(*a*) Sketch the graph of $y = f(-x)$. **2**

(*b*) On the same diagram, sketch the graph of $y = 2f(-x)$. **2**



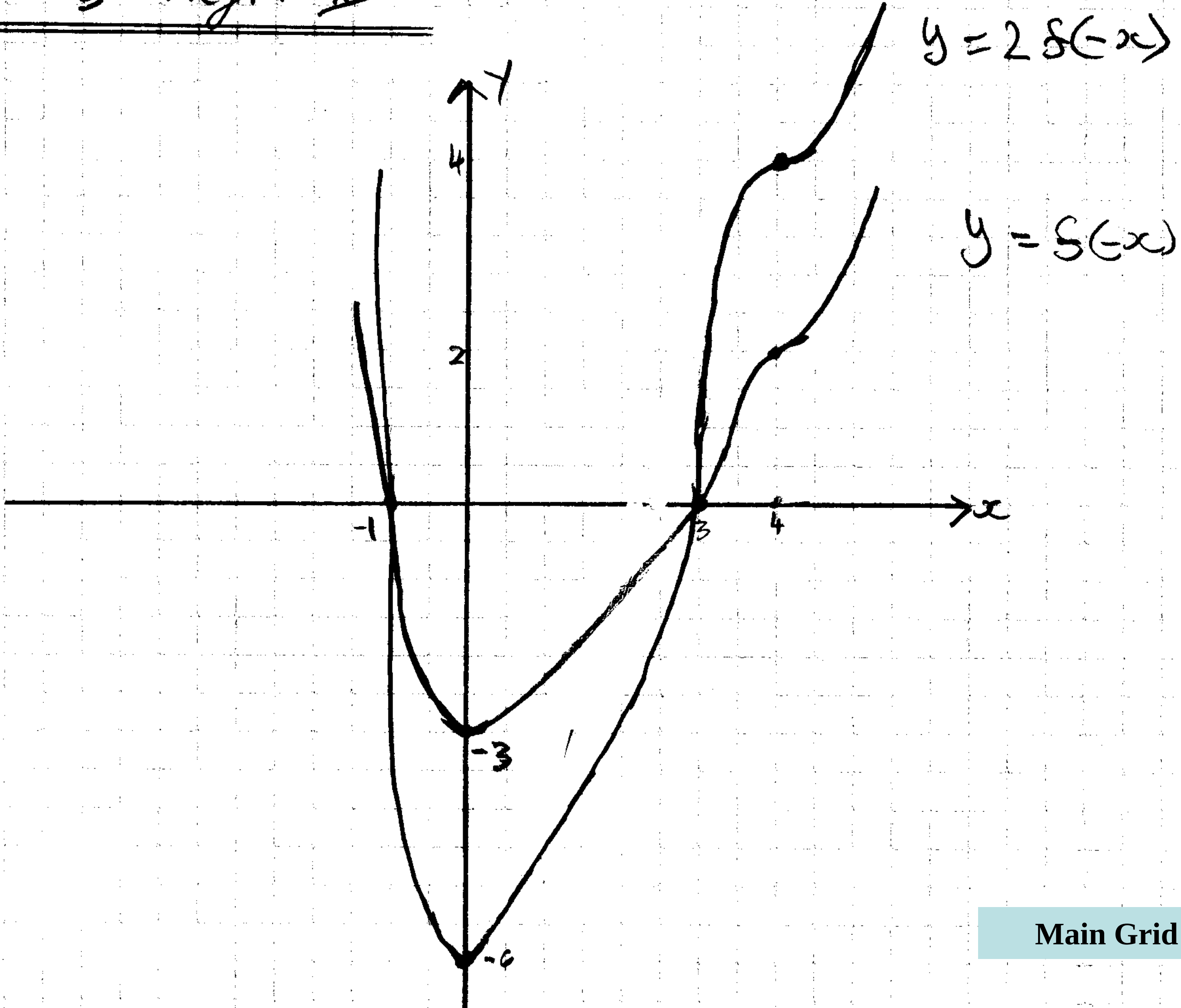
F

Solution

Main Grid

2008 - Higher II

5) a)



Main Grid

6. If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

F

Solution

Main Grid

6)

$f(x) = \cos 2x - 3\sin 4x$

$f'(x) = -2\sin 2x - 12\cos 4x$

$f'\left(\frac{\pi}{6}\right) = -2\sin\frac{\pi}{3} - 12\cos\frac{2\pi}{3}$

$= -2 \times \frac{\sqrt{3}}{2} - 12 \times -\frac{1}{2}$

$= -\sqrt{3} + 6$

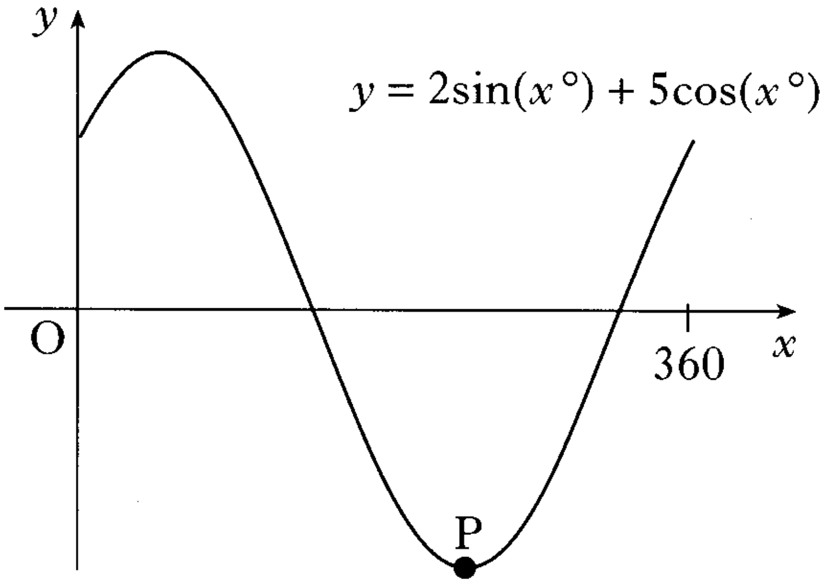
$= \underline{\underline{6 - \sqrt{3}}}$

Main Grid

7. Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.

(*a*) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \leq a < 360$. **4**

(*b*) Find the coordinates of the minimum turning point P. **3**



F

Solution

Main Grid

7) a) Let $2\sin x^\circ + 5\cos x^\circ = K\sin(x+a)^\circ$

$$= K(\sin x\cos a + \cos x\sin a)$$

$$\therefore\ 2\sin x^\circ + 5\cos x^\circ = K\sin x^\circ\cos a^\circ + K\cos x^\circ\sin a^\circ$$

by equating coefficients of $\sin x^\circ$ & $\cos x^\circ$

$$K\cos a^\circ = 2 \qquad K\sin a^\circ = 5$$

by squaring and adding:

$$K^2\cos^2 a^\circ + K^2\sin^2 a^\circ = 4+25$$

$$K^2(\sin^2 a^\circ + \cos^2 a^\circ) = 29$$

$$K^2 \times 1 = 29$$

$$K = \sqrt{29}$$

Also: $\dfrac{K\sin a^\circ}{K\cos a^\circ} = \dfrac{5}{2} = \tan a^\circ$

Sin +ve cos +ve ∴ a is in 1st quadrant.

$$a^\circ = \tan^{-1}\left(5/2\right) = 68.2^\circ$$

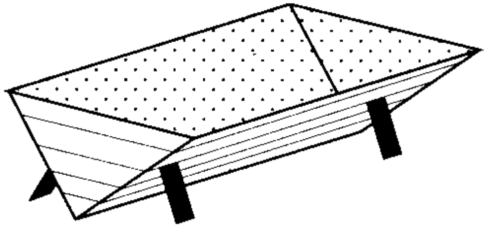
$$\therefore\ 2\sin x^\circ + 5\cos x^\circ = \sqrt{29}\sin(x+68.2)^\circ$$

b) at P: $\sqrt{29}\sin(x+68.2)^\circ = -\sqrt{29} \Rightarrow \sin(x+68.2)^\circ = -1$

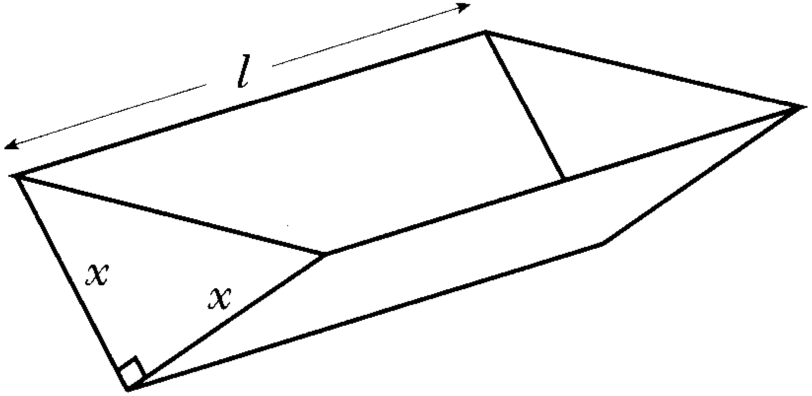
$\Rightarrow x+68.2^\circ = 270^\circ \qquad x = 201.8^\circ \qquad \therefore\ P(201.8, -\sqrt{29})$

Main Grid

8. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.



(*a*) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \dfrac{432000}{x}$. **3**

(*b*) Find the value of x which minimises this surface area. **5**

F

Solution 8a **Solution 8b** **Main Grid**

8) a) Find l: $\frac{1}{2}x^2 \times l = 108000$

$$x^2 l = 216000$$

$$l = \underline{\underline{\frac{216000}{x^2}}}$$

$$A = \left(2 \times \tfrac{1}{2}x^2\right) + 2\left(\frac{216000}{x^2} \times x\right)$$

$$= \underline{\underline{x^2 + \frac{432000}{x}}}$$

Solution 8b

Main Grid

b) Min at $\frac{dy}{dx} = 0$

$$y = x^2 + 432000x^{-1}$$

$$\frac{dy}{dx} = 2x - \frac{432000}{x^2} = 0 \quad \times x^2$$

$$2x^3 - 432000 = 0$$

$$x^3 = 216000$$

$$\underline{\underline{x = 60}}$$

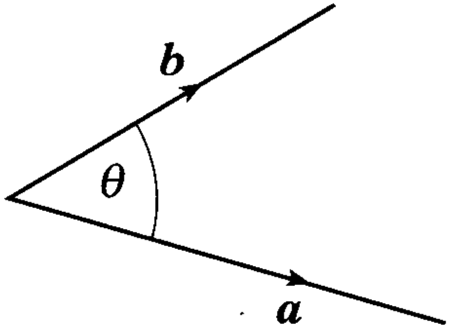
Main Grid

x	60^-	60	60^+
$\frac{dy}{dx}$	-6.1	0	5.9
SHAPE	╲	—	╱

$\therefore$ Min at $\underline{\underline{x = 60}}$

9. The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

If $|\boldsymbol{a}| = 5$, $|\boldsymbol{b}| = 4$ and $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$.



4

F

Solution

Main Grid

9) $a \cdot (a + b) = a \cdot a + a \cdot b = 36$

$$a \cdot b = 36 \div (a \cdot a)$$

$$= 36 \div 5^2$$

$$= \underline{\underline{11}}$$

Now: $\cos\theta = \dfrac{a \cdot b}{|a||b|}$

$$= \frac{11}{5 \times 4} = 0.55$$

$\therefore \theta = \cos^{-1}(0.55) = \underline{\underline{56.6^\circ}}$

Main Grid

10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \leq x \leq \pi$, correct to 2 decimal places. **5**

F

Solution

Main Grid

10) $3(2\cos^2 x-1)+10\cos x-1=0$

$6\cos^2 x-3+10\cos x-1=0$

$6\cos^2 x+10\cos x-4=0$

$3\cos^2 x+5\cos x-2=0$

$a=3,\ b=5,\ c=-2$

$\therefore\ \cos x=0.333$ OR ~~$\cos x=-2$~~

$x=\cos^{-1}(0.333)$

$=\underline{\underline{1.23}}$ Only 1 solution $0\leq x\leq\pi$

Main Grid

11. (*a*) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.

(ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$. **2**

(*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$. **3**

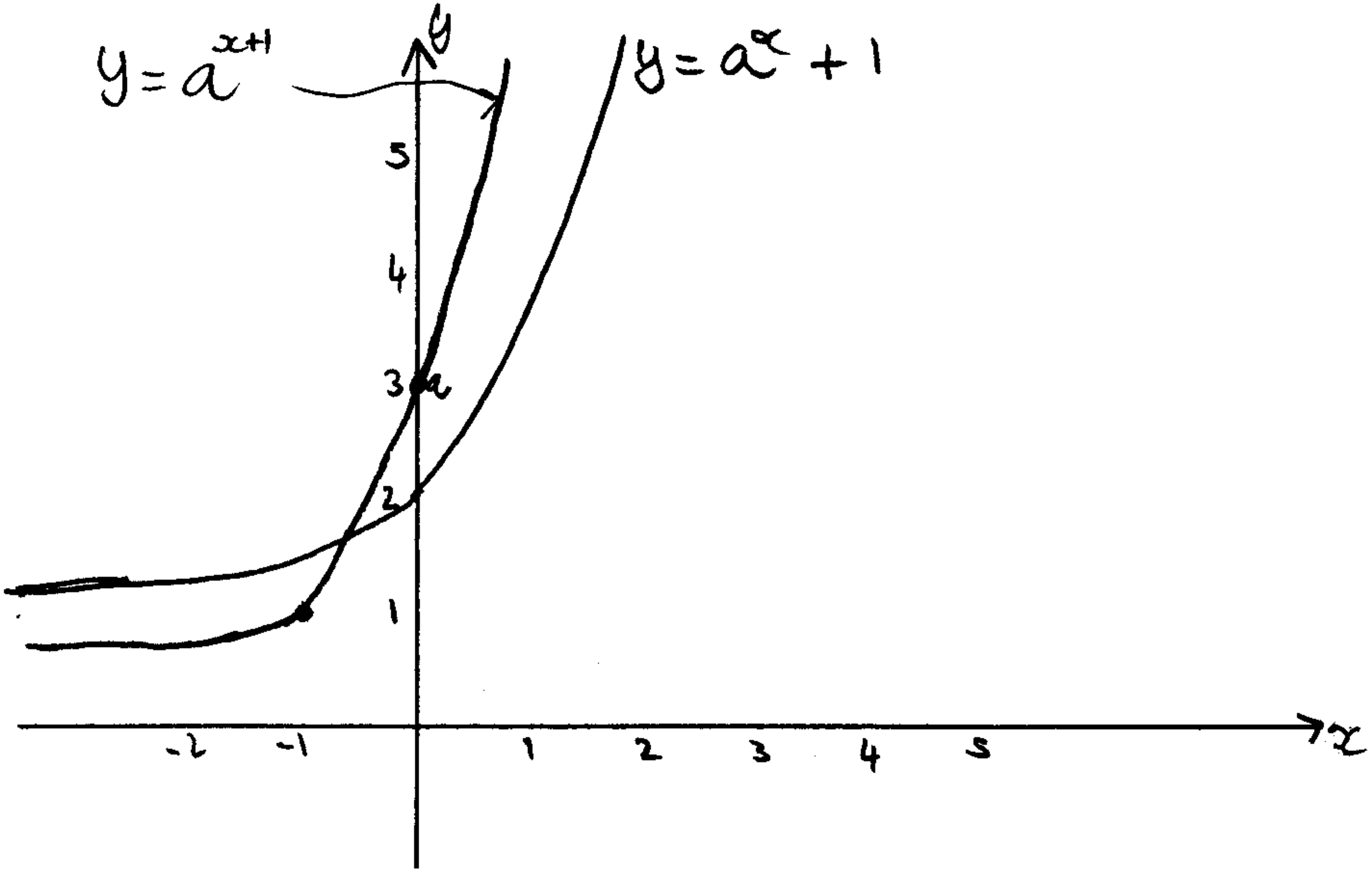
F

Solution 11a

Solution 11b

Main Grid

11)



Solution 11b

Main Grid

b)

$$y = y$$

$$a^{x+1} = a^{x} + 1$$

$$a^{x+1} - a^{x} = 1$$

$$a^{x}(a-1) = 1$$

$$a^{x} = \frac{1}{a-1}$$

$$\log_a a^{x} = \log_a \left(\frac{1}{a-1}\right)$$

$$\underline{\underline{x = \log_a\left(\frac{1}{a-1}\right)}}$$

Main Grid

1. The point A has coordinates (7, 4). The straight lines with equations $x + 3y + 1 = 0$ and $2x + 5y = 0$ intersect at B.

(*a*) Find the gradient of AB. **3**

(*b*) Hence show that AB is perpendicular to only one of these two lines. **5**

F

Solution

Main Grid

1) a)

$$2x + 5y = 0$$
$$x + 3y = -1$$

$$2x + 5y = 0$$
$$2x + 6y = -2$$

$$-y = 2$$
$$\underline{\underline{y = -2}}$$

$$x + (3 \times -2) = -1$$
$$x = -1 + 6$$
$$\underline{\underline{x = 5}}$$

$\therefore$ B(5, -2) A(7, 4)

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4+2}{7-5} = \frac{6}{2} = \underline{\underline{3}}$$

b) $M_1 M_2 = -1$ for perpendicular.

$3 \times M_2 = -1$ $\therefore M_2 = -1/3$

So which line has gradient $= -1/3$

$2x + 5y = 0 \rightarrow y = -\frac{2}{5}x \rightarrow M = -\frac{2}{5}$ $\therefore$ <u>No</u>

$x + 3y = -1 \rightarrow y = -\frac{x}{3} - \frac{1}{3} \rightarrow M = -1/3$ $\therefore$ <u>Yes</u>

Main Grid

2. $f(x) = x^3 - x^2 - 5x - 3$.

(*a*) (i) Show that $(x + 1)$ is a factor of $f(x)$.

(ii) Hence or otherwise factorise $f(x)$ fully. **5**

(*b*) One of the turning points of the graph of $y = f(x)$ lies on the x-axis.

Write down the coordinates of this turning point. **1**

F

Solution

Main Grid

2) a) i)

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -5 & -3 \\ & & -1 & 2 & 3 \\ \hline & 1 & -2 & -3 & \underline{0} \end{array}$$

Remainder zero so $(x+1)$ is a factor.

ii) $(x+1)(x^2-2x-3)$

$\underline{\underline{(x+1)(x-3)(x+1)}}$

b) By inspection double root must be turning point

$\therefore$ T.P. at $\underline{\underline{(-1,0)}}$

Main Grid

3. Find all the values of x in the interval $0 \leq x \leq 2\pi$ for which $\tan^2(x) = 3$. **4**

F

Solution

Main Grid

3)

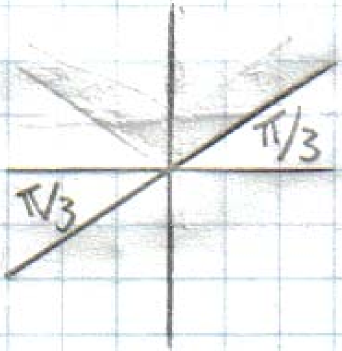
$\tan^2 x = 3$

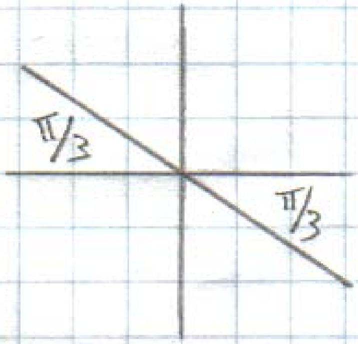
$\tan x = \sqrt{3}$

$\tan x = +\sqrt{3}$ OR $-\sqrt{3}$

$x = \tan^{-1}(\sqrt{3})$ OR $\tan^{-1}(-\sqrt{3})$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$ $x = \frac{2\pi}{3}, \frac{5\pi}{3}$





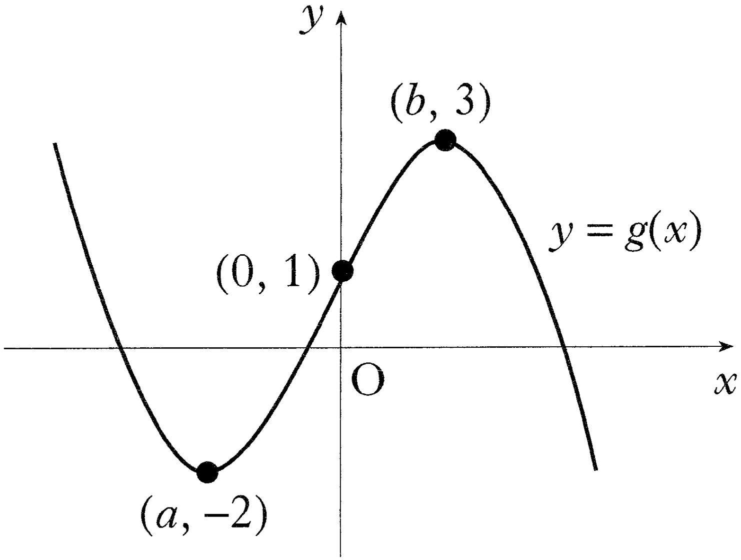
$$\underline{\underline{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}}$$

Main Grid

4. The diagram shows the graph of $y = g(x)$.

(*a*) Sketch the graph of $y = -g(x)$. **2**

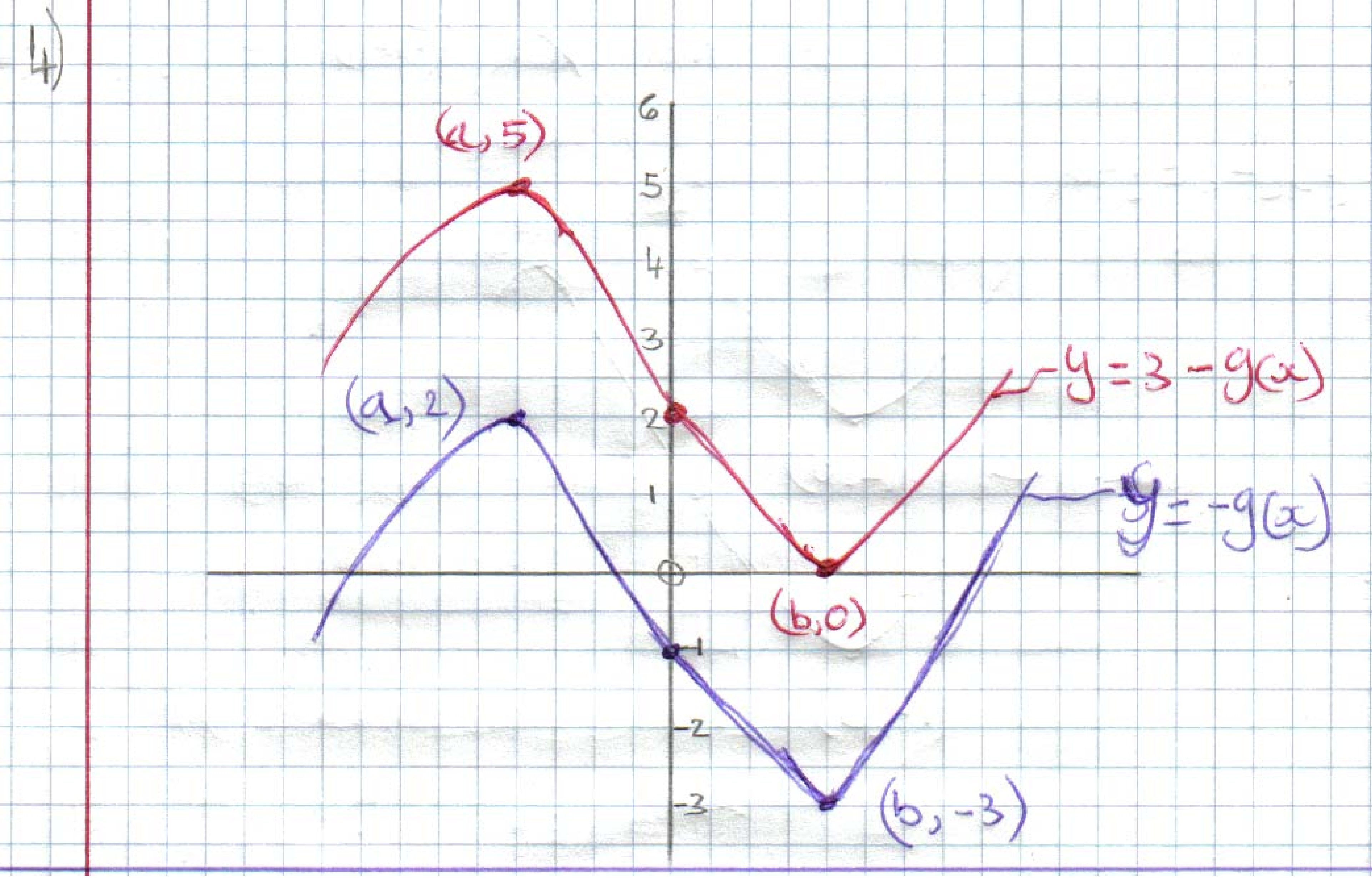
(*b*) On the same diagram, sketch the graph of $y = 3 - g(x)$. **2**



F

Solution

Main Grid



Main Grid

5. A, B and C have coordinates (–3, 4, 7), (–1, 8, 3) and (0, 10, 1) respectively.

(*a*) Show that A, B and C are collinear. **3**

(*b*) Find the coordinates of D such that $\overrightarrow{AD} = 4\overrightarrow{AB}$. **2**

F

Solution

Main Grid

5) $\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix}-1\\8\\3\end{pmatrix} - \begin{pmatrix}-3\\4\\7\end{pmatrix} = \begin{pmatrix}2\\4\\-4\end{pmatrix}$

$\vec{BC} = \underline{c} - \underline{b} = \begin{pmatrix}0\\10\\1\end{pmatrix} - \begin{pmatrix}-1\\8\\3\end{pmatrix} = \begin{pmatrix}1\\2\\-2\end{pmatrix}$

Now $\vec{AB} = 2\vec{BC}$ so are parallel and B is a common poive so A, B, C must be collinear.

b) $4\vec{AB} = 4\begin{pmatrix}2\\4\\-4\end{pmatrix} = \underline{\underline{\begin{pmatrix}8\\16\\-16\end{pmatrix}}}$

$\therefore \vec{AD} = \underline{d} - \underline{a} = \begin{pmatrix}8\\16\\-16\end{pmatrix}$

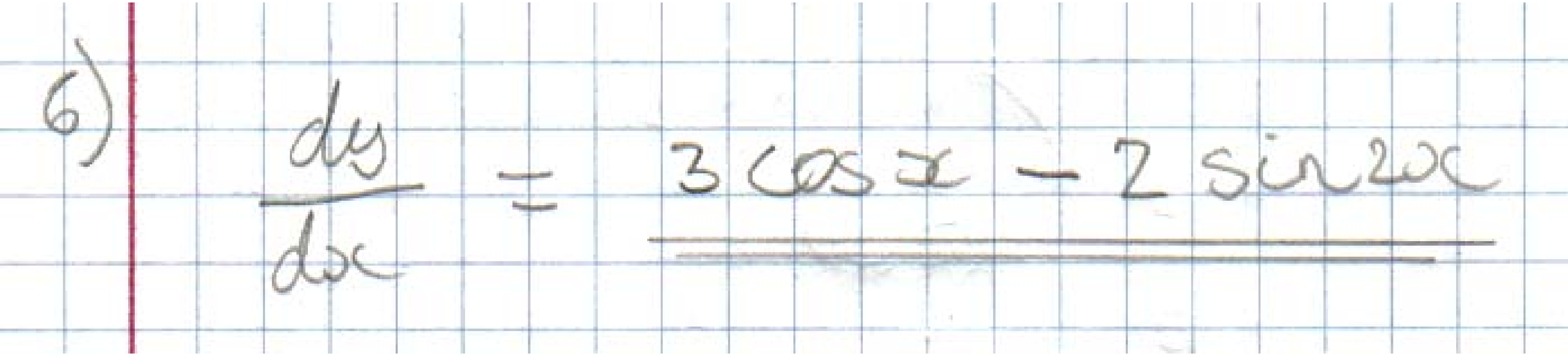
$\begin{pmatrix}x_d\\y_d\\z_d\end{pmatrix} = \begin{pmatrix}8\\16\\-16\end{pmatrix} + \begin{pmatrix}-3\\4\\7\end{pmatrix} = \underline{\underline{\begin{pmatrix}5\\20\\-9\end{pmatrix}}} \quad \therefore \underline{\underline{D(5, 20, -9)}}$

Main Grid

6. Given that $y = 3\sin(x) + \cos(2x)$, find $\dfrac{dy}{dx}$. **3**

F

Solution **Main Grid**



6) $$\frac{dy}{dx} = \underline{\underline{3\cos x - 2\sin 2x}}$$

Main Grid

7. Find $\int_0^2 \sqrt{4x+1}\ dx.$ **5**

F

Solution

Main Grid

7) $$\int_0^2 \sqrt{4x+1}\,dx = \int_0^2 (4x+1)^{1/2}\,dx$$

$$= \left[\frac{(4x+1)^{3/2}}{\frac{3}{2}\times 4}\right]_0^2 = \left[\frac{\sqrt{(4x+1)^3}}{6}\right]_0^2$$

$$= \left(\frac{\sqrt{((4\times 2)+1)^3}}{6}\right) - \left(\frac{\sqrt{((4\times 0)+1)^3}}{6}\right)$$

$$= \frac{\sqrt{729}}{6} - \frac{\sqrt{1}}{6} = \frac{27}{6} - \frac{1}{6} = \frac{26}{6}$$

$$= \frac{13}{3}$$

Main Grid

8. (*a*) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$. **2**

(*b*) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing. **4**

F

Solution

Main Grid

Main Grid

8) a) $(x^2 - 10x + 5^2) + 27 - 5^2$

$(x^2 - 10x + 25) + 2$

$(x-5)(x-5) + 2$

$\underline{\underline{(x-5)^2 + 2}}$

b) If a function is always increasing then gradient is always greater than zero. so $g'(x) > 0$

$g'(x) = x^2 - 10x + 27$

$(x-5)^2 + 2$

for all values of x $g'(x)$ is greater than zero therefore gradient is always positive therefore the function is always increasing

9. Solve the equation $\log_2(x + 1) - 2\log_2(3) = 3$. **4**

F

Solution

Main Grid

9) $\log_2 (x+1) - 2\log_2 3 = 3$

$$\log_2 (x+1) - \log_2 3^2 = \log_2 8$$

$$\log_2 \left(\frac{x+1}{9}\right) = \log_2 8$$

$$\frac{x+1}{9} = 8$$

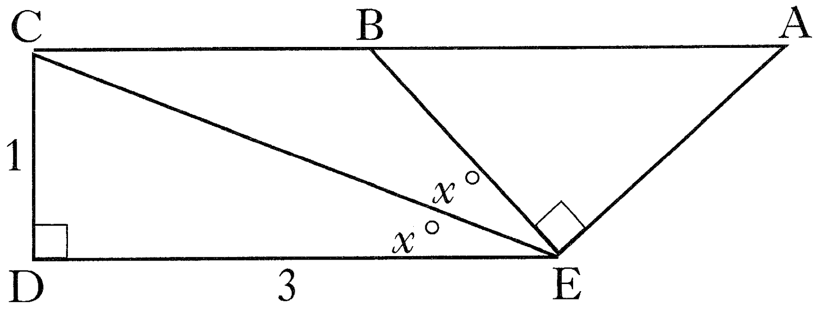
$$x+1 = 72$$

$$\underline{\underline{x = 71}}$$

Main Grid

10. In the diagram
angle DEC = angle CEB = $x°$ and
angle CDE = angle BEA = $90°$.
CD = 1 unit; DE = 3 units.

By writing angle DEA in terms of $x°$, find the exact value of $\cos(\hat{DEA})$.



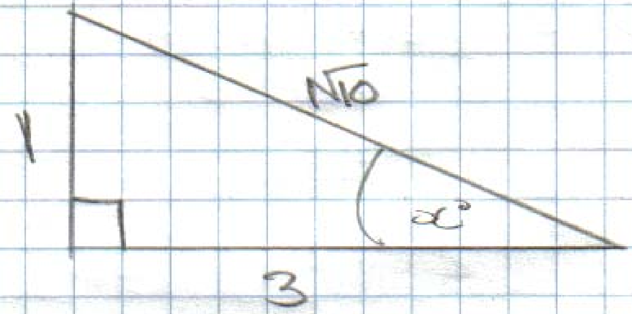
7

F

Solution

Main Grid

10)



$$\sqrt{3^2+1^2} = \sqrt{10}$$

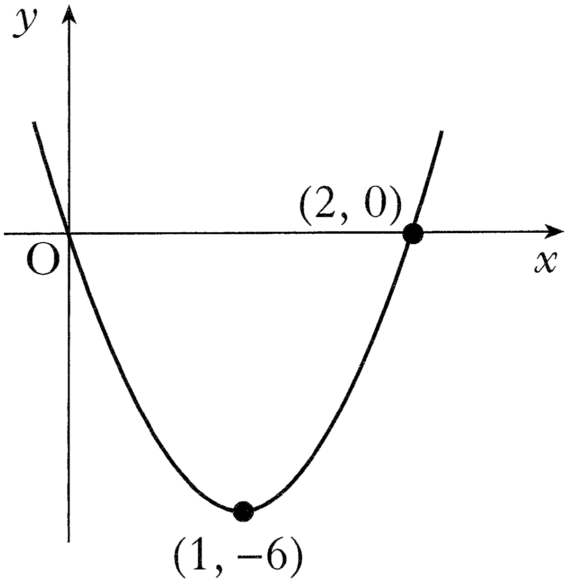
$$\sin x = \frac{1}{\sqrt{10}}$$

$$\cos x = \frac{3}{\sqrt{10}}$$

$$\begin{aligned}
\cos(D\hat{E}A) &= \cos(2x + 90) \\
&= \cos 2x \cos 90 - \sin 2x \sin 90 \\
&= \cos 2x \times 0 \;\; - \sin 2x \times 1 \\
&= - \sin 2x \\
&= -2 \sin x \cos x \\
&= -2 \times \frac{1}{\sqrt{10}} \times \frac{3}{\sqrt{10}} \\
&= \frac{-6}{10} = \underline{\underline{\frac{-3}{5}}}
\end{aligned}$$

Main Grid

11. The diagram shows a parabola passing through the points (0, 0), (1, –6) and (2, 0).



(*a*) The equation of the parabola is of the form $y = ax(x - b)$.

Find the values of a and b. **3**

(*b*) This parabola is the graph of $y = f'(x)$.

Given that $f(1) = 4$, find the formula for $f(x)$. **5**

F

Solution 11a

Solution 11b

Main Grid

11 a) $y = ax(x-b)$

$y = ax(x-2)$

at $(1,-6)$ $-6 = a(1-2)$

$-6 = -a$

$\underline{\underline{a = 6}}$

$\therefore$ $\underline{\underline{y = 6x(x-2)}}$

$\therefore$ $a = 6$

$b = 2$

Solution 11b

Main Grid

b) $f(x) = \int f'(x)\, dx$

$= \int 6x(x-2)dx \quad = \int 6x^2 - 12x\, dx$

$= 2x^3 - 6x^2 + c$

at $f(1)=4$: $2 \times 1^3 - 6 \times 1^2 + c = 4$

$= 2 - 6 + c = 4$

$c = 4 + 6 - 2$

$c = 8$

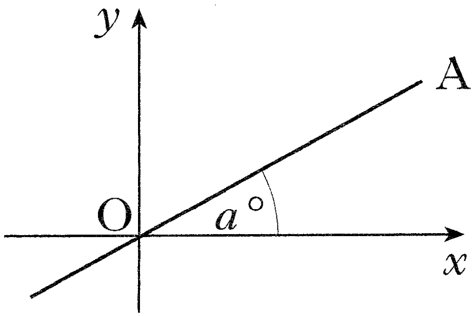
$$\therefore \underline{\underline{f(x) = 2x^3 - 6x^2 + 8}}$$

Main Grid

1. (*a*) The diagram shows line OA with equation $x - 2y = 0$.

The angle between OA and the x-axis is $a°$.

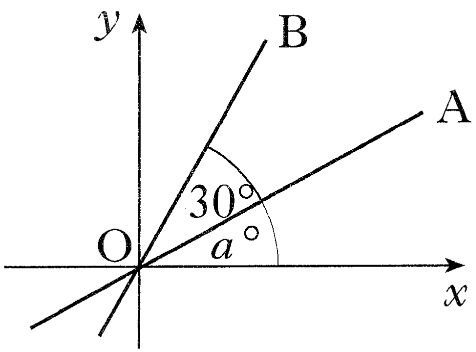
Find the value of a.



3

(*b*) The second diagram shows lines OA and OB. The angle between these two lines is 30°.

Calculate the gradient of line OB correct to 1 decimal place.



1

F

Solution

Main Grid

1) a) $x - 2y = 0$

$y = \frac{1}{2}x \quad \therefore \quad M = \frac{1}{2}$

$M = \tan a^\circ = \frac{1}{2}$

$a^\circ = \tan^{-1}\left(\frac{1}{2}\right) = \underline{\underline{26.6^\circ}}$

b)

$M = \tan\phi \qquad \phi = 26.6 + 30$

$= \underline{\underline{56.6^\circ}}$

$M = \tan 56.6^\circ = 1.517 = \underline{\underline{1.5}}$

Main Grid

2. P, Q and R have coordinates (1, 3, –1), (2, 0, 1) and (–3, 1, 2) respectively.

(*a*) Express the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in component form. **2**

(*b*) Hence or otherwise find the size of angle PQR. **5**

F

Solution

Main Grid

2) a) $\overrightarrow{QP} = \underline{p} - \underline{q} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$

$\overrightarrow{QR} = \underline{r} - \underline{q} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$

b) $\cos Q = \dfrac{a \cdot b}{|a|\,|b|} = \dfrac{\overrightarrow{QP} \cdot \overrightarrow{QR}}{|\overrightarrow{QP}|\,|\overrightarrow{QR}|}$

$|\overrightarrow{QP}| = \sqrt{-1^2 + 3^2 + -2^2} = \sqrt{1+9+4} = \underline{\underline{\sqrt{4}}}$

$|\overrightarrow{QR}| = \sqrt{-5^2 + 1^2 + -1^2} = \sqrt{25+1+1} = \underline{\underline{\sqrt{27}}}$

$\cos PQR = \dfrac{(-1 \times -5) + (3 \times 1) + (-2 \times 1)}{\sqrt{14}\,\sqrt{27}} = \dfrac{5+3-2}{\sqrt{378}} = \underline{\underline{0.309}}$

$\therefore PQR = \cos^{-1}(0.309) = \underline{\underline{72^\circ}}$

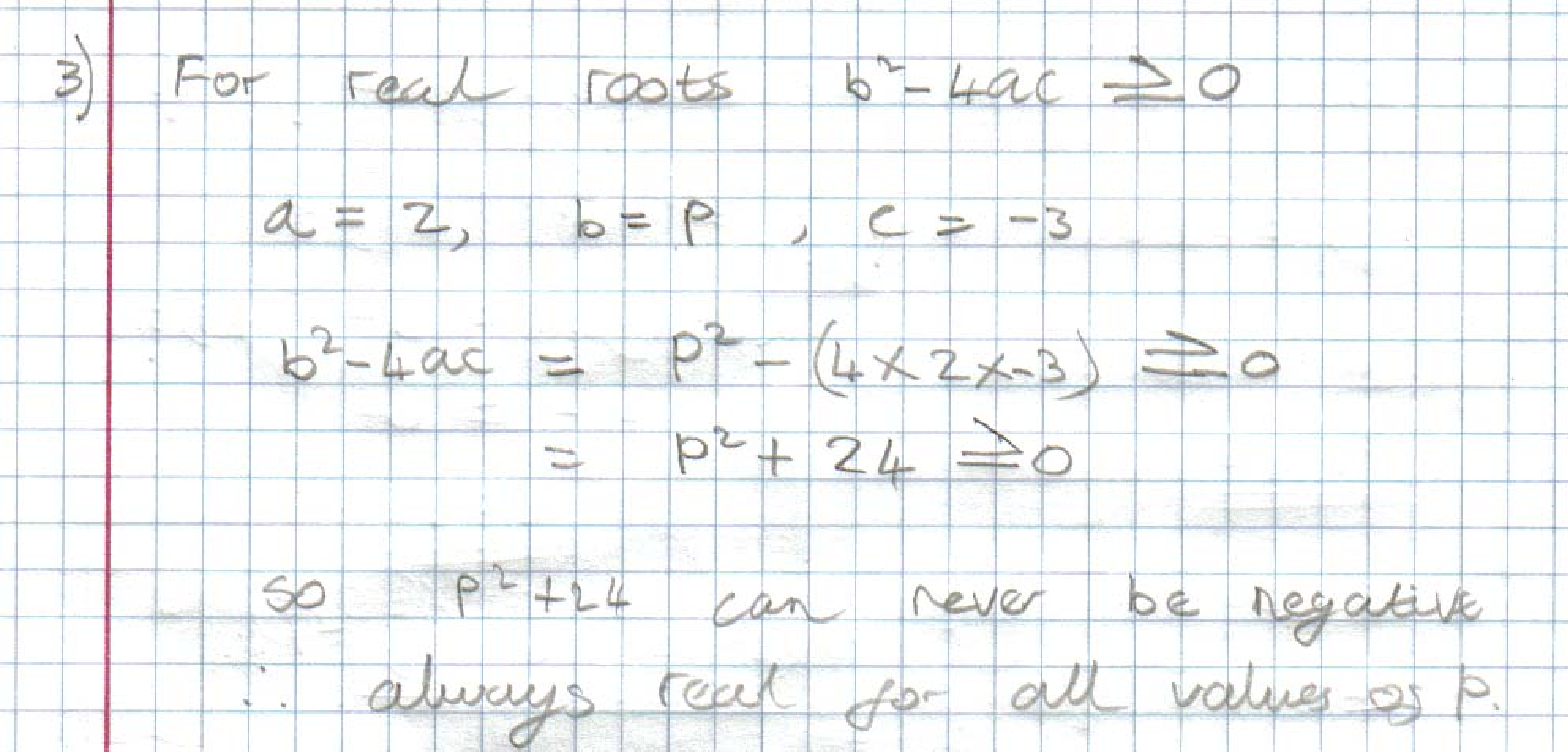
Main Grid

3. Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p. **4**

F

Solution

Main Grid



3) For real roots $b^2 - 4ac \geq 0$

$a = 2, \quad b = p \quad , \quad c = -3$

$$b^2 - 4ac = p^2 - (4 \times 2 \times -3) \geq 0$$
$$= p^2 + 24 \geq 0$$

so $p^2 + 24$ can never be negative

$\therefore$ always real for all values of p.

Main Grid

4. A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$.

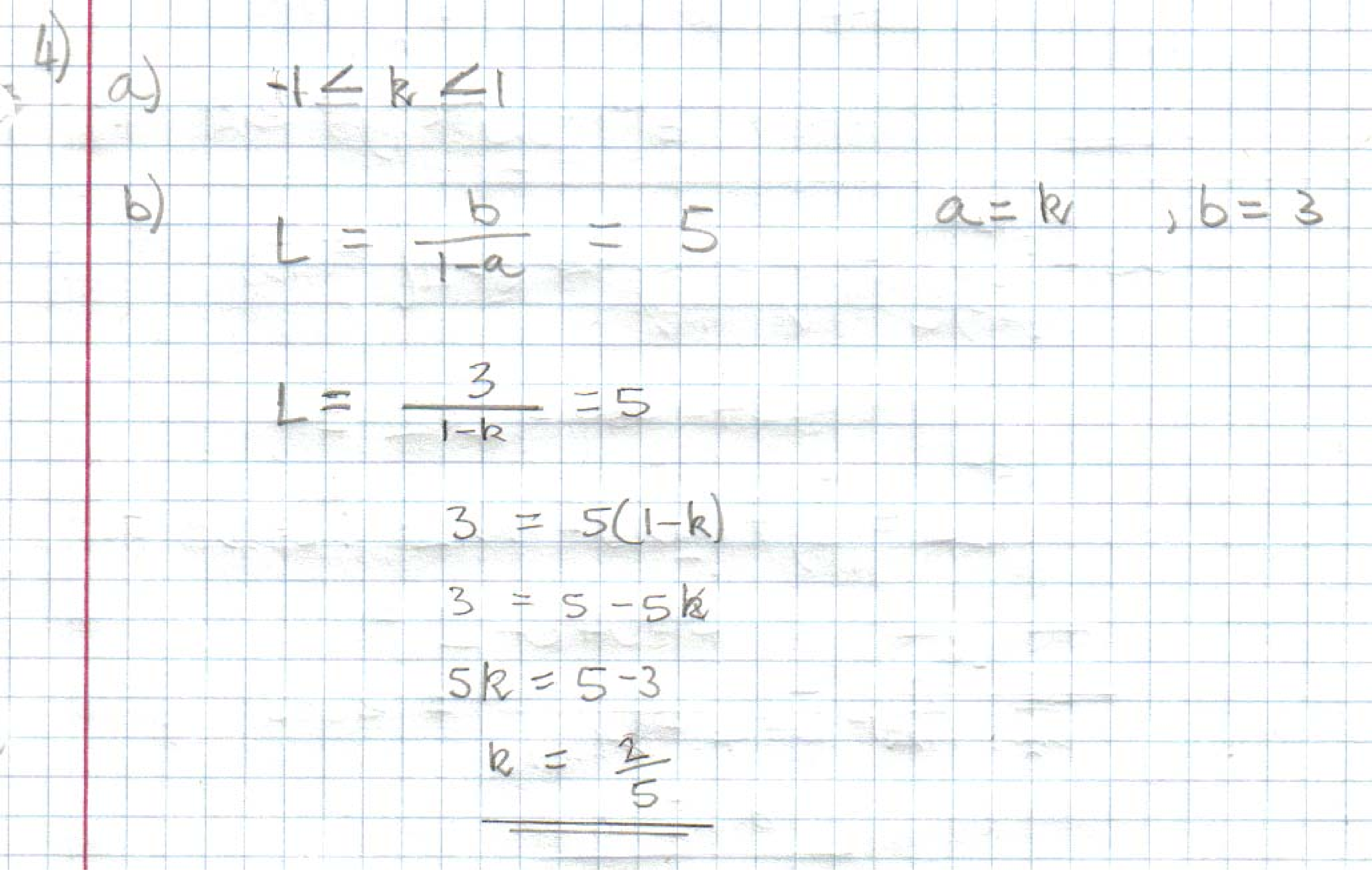
(*a*) Write down the condition on k for this sequence to have a limit. **1**

(*b*) The sequence tends to a limit of 5 as $n \to \infty$. Determine the value of k. **3**

F

Solution

Main Grid



4)

a) $-1 \leq k \leq 1$

b) $L = \frac{b}{1-a} = 5$ $a = k$, $b = 3$

$L = \frac{3}{1-k} = 5$

$3 = 5(1-k)$

$3 = 5 - 5k$

$5k = 5 - 3$

$\underline{\underline{k = \frac{2}{5}}}$

Main Grid

5. The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

(a) Find the value of x for which the gradient of the tangent at P is 12. **5**

(b) Hence find the equation of the tangent at P. **2**

F

Solution

Main Grid

5) a) $M = \frac{dy}{dx} = 12x - 3x^2 = 12$

$$3x^2 - 12x + 12 = 0$$

$$3(x^2 - 4x + 4) = 0$$

$$3(x-2)(x-2) = 0$$

$$\therefore \underline{\underline{x = 2}}$$

b) P: $y = 6 \times 2^2 - 2^3 = 16$ P$(2, 16)$

$y - b = M(x - a)$ $M = 12$, $(a, b) = (2, 16)$

$y - 16 = 12(x - 2)$

$y - 16 = 12x - 24$

$\underline{\underline{y - 12x + 8 = 0}}$

Main Grid

6. (*a*) Express $3\cos(x°) + 5\sin(x°)$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \leq a \leq 90$. **4**

(*b*) Hence solve the equation $3\cos(x°) + 5\sin(x°) = 4$ for $0 \leq x \leq 90$. **3**

F

Solution 6a **Solution 6b** **Main Grid**

6) a) $3\cos x^\circ + 5\sin x^\circ = R\cos(x-a)$

$= R(\cos x\cos a + \sin x \sin a)$

$$3\cos x^\circ + 5\sin x^\circ = R\cos x^\circ \cos a^\circ + R\sin x^\circ \sin a^\circ$$

equate coefficients $R\cos a^\circ = 3$

$R\sin a^\circ = 5$

Square and add : $R^2\cos^2 a^\circ + R^2\sin^2 a^\circ = 3^2 + 5^2$

$R^2(\cos^2 a + \sin^2 a) = 34$

$R^2 = 34$

$\underline{\underline{R = \sqrt{34}}}$

Now: $\dfrac{\cancel{R}\sin a^\circ}{\cancel{R}\cos a^\circ} = \dfrac{5}{3} = \tan a^\circ$

sin is +Ve, cos is +Ve ∴ 1st quadrant.

$\therefore a^\circ = \tan^{-1}\left(\frac{5}{3}\right) = \underline{59.04^\circ}$

$\therefore 3\cos x^\circ + 5\sin x^\circ = \underline{\underline{\sqrt{34}\cos(x^\circ - 59.04^\circ)}}$

Solution 6b

Main Grid

b) $\sqrt{34}\cos(x^\circ - 59.04^\circ) = 4$

$\cos(x^\circ - 59.04^\circ) = 4 \div \sqrt{34} = 0.686$

$x^\circ - 59.04^\circ = 46.7^\circ$ OR -46.7°

$+59.04$

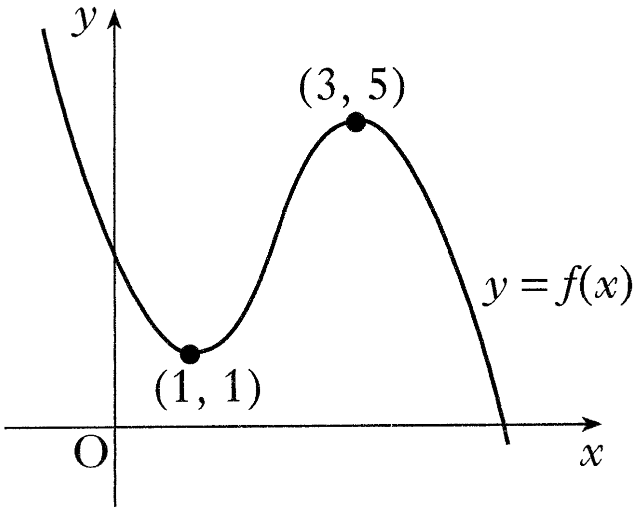
$x^\circ = \cancel{105.74^\circ}$ OR 12.34°

$\underline{\underline{x^\circ = 12.34^\circ}}$

Main Grid

7. The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1, 1) and (3, 5).

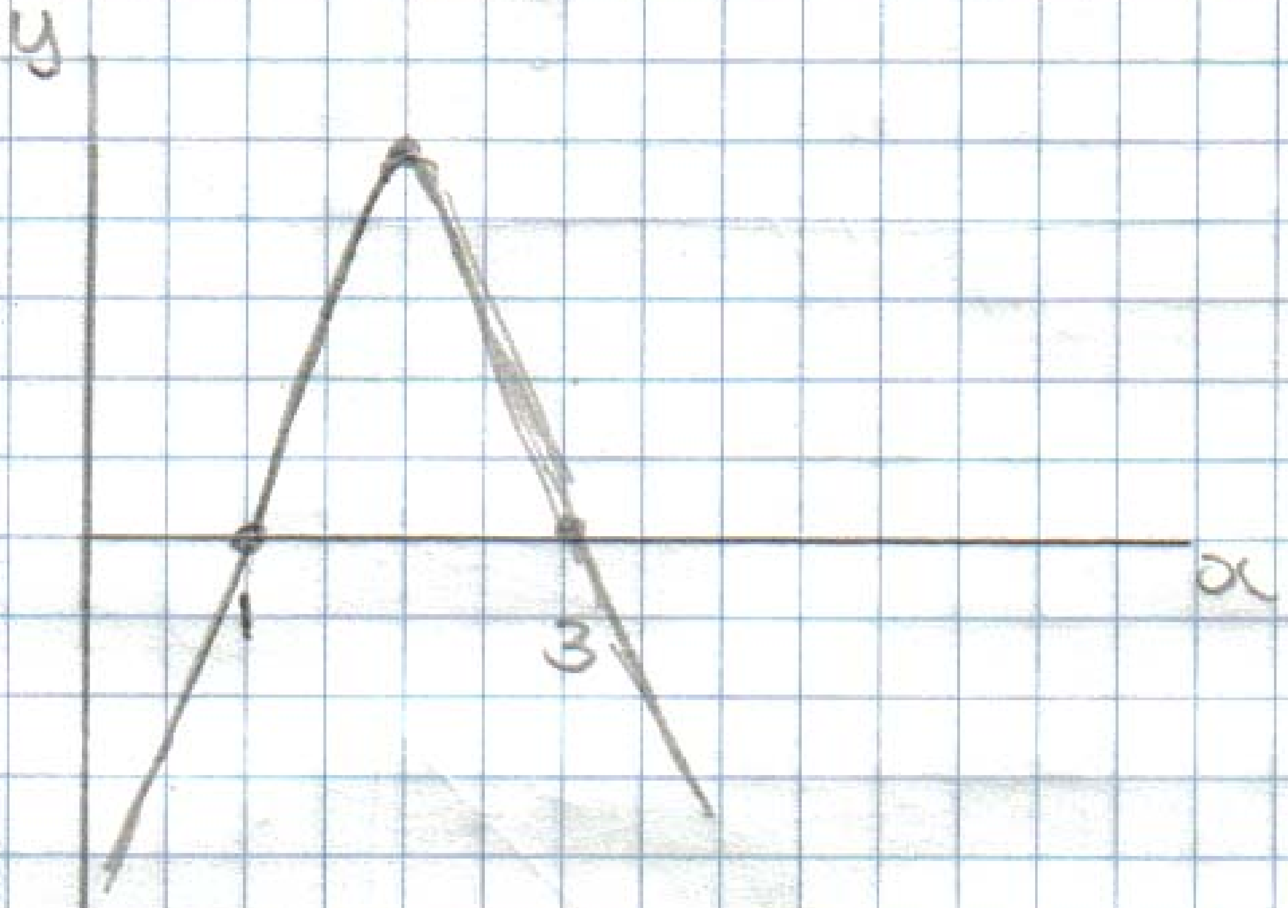
Sketch the graph of $y = f'(x)$.



3

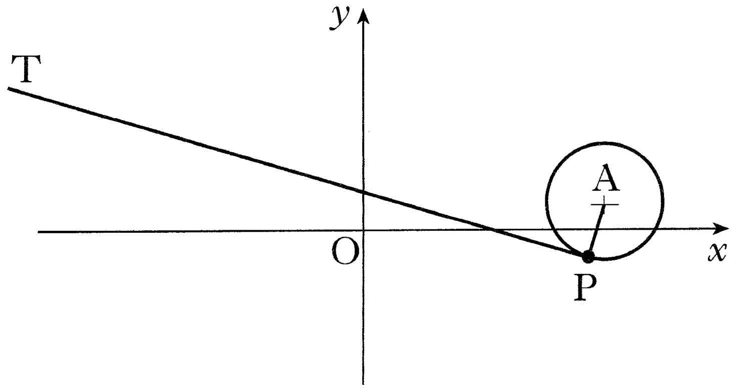
F

7)



Main Grid

8. The circle with centre A has equation $x^2 + y^2 - 12x - 2y + 32 = 0$. The line PT is a tangent to this circle at the point P(5, –1).



(*a*) Show that the equation of this tangent is $x + 2y = 3$. **4**

F

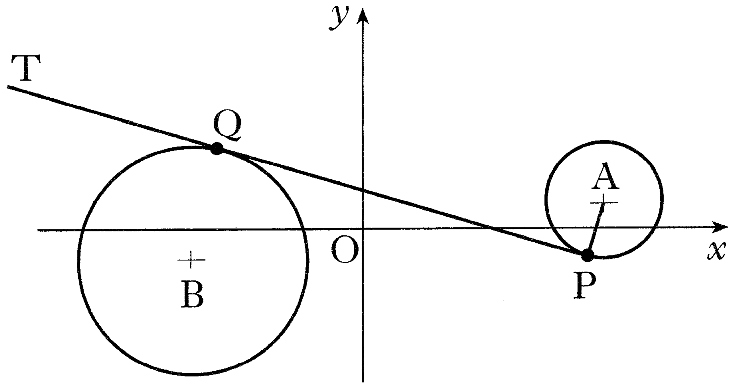
The circle with centre B has equation $x^2 + y^2 + 10x + 2y + 6 = 0$.

Solution 8a

Solution 8b

Solution 8c

Main Grid



(*b*) Show that PT is also a tangent to this circle. **5**

(*c*) Q is the point of contact. Find the length of PQ. **2**

8) Centre $(-g,-f)$ $\quad 2g = -12 \quad 2f = -2$

$\underline{\underline{g = -6}} \quad \underline{\underline{f = -1}}$

Centre $(6,1)$ $\quad M_T M_R = -1$

$$M_R = \frac{-1-1}{5-6} = \frac{-2}{-1} = \underline{\underline{2}} \quad \therefore \quad M_T = \underline{\underline{-1/2}}$$

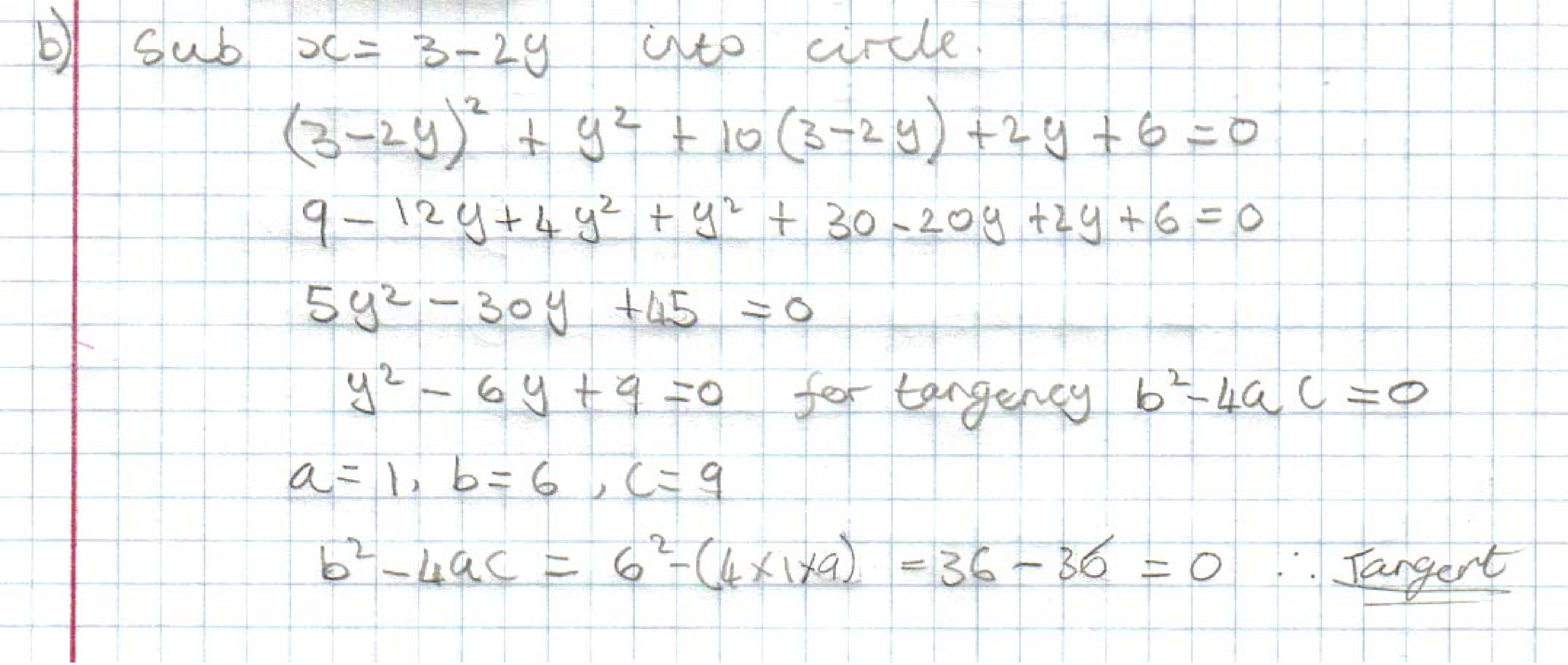
$y - b = M(x-a) \quad M = -1/2 \quad (a,b) = (5,-1)$

$y + 1 = \frac{-1}{2}(x-5)$

$2y + 2 = -x + 5$

$\underline{\underline{x + 2y = 3}}$

Solution 8b Solution 8c Main Grid



b) Sub $x = 3-2y$ into circle.

$$(3-2y)^2 + y^2 + 10(3-2y) + 2y + 6 = 0$$

$$9 - 12y + 4y^2 + y^2 + 30 - 20y + 2y + 6 = 0$$

$$5y^2 - 30y + 45 = 0$$

$y^2 - 6y + 9 = 0$ for tangency $b^2 - 4ac = 0$

$a = 1,\ b = 6,\ c = 9$

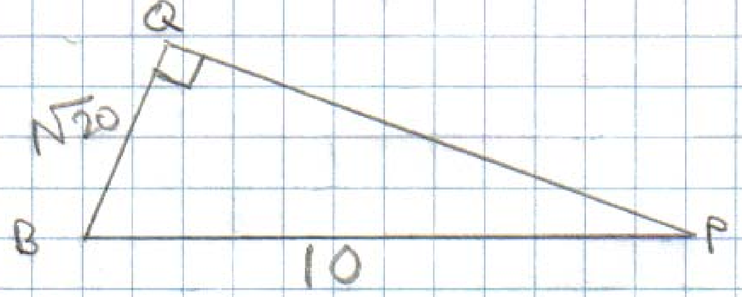
$b^2 - 4ac = 6^2 - (4 \times 1 \times 9) = 36 - 36 = 0$ ∴ Tangent

Solution 8c

Main Grid

8

c)



$B(-g, -f)$ $\quad 2g = 10$ $\quad 2f = 2$

$g = 5$ $\quad f = 1$

$B(-5, -1)$

$BP = \sqrt{10^2 + 0^2} = \underline{\underline{10}}$

$BQ = \text{radius} = \sqrt{g^2 + f^2 - c}$

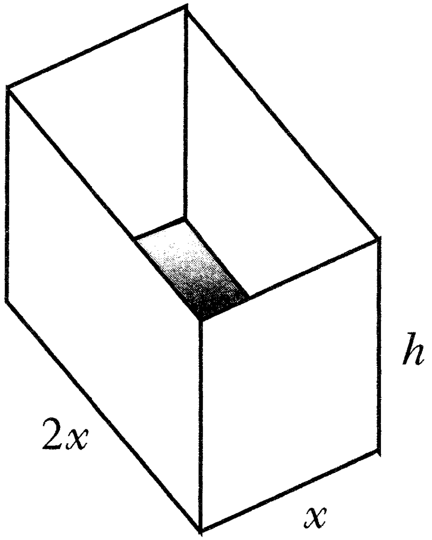
$= \sqrt{5^2 + 1^2 - 6} = \sqrt{20}$

$PQ = \sqrt{10^2 - (\sqrt{20})^2}$

$= \sqrt{100 - 20} = \underline{\underline{\sqrt{80}}} = \underline{8 \cdot 94}$

Marks

9. An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.



(a) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x\left(6 - x^2\right)$. **3**

(b) Find the exact value of x for which this volume is a maximum. **5**

F

Solution 9a

Solution 9b

Main Grid

9) Inner Surface Area $= 2 \times 2xh + 2 \times xh + 2x^2 = 12$

$$= 2x^2 + 6xh = 12$$

$$\therefore h = \underline{\underline{\frac{12 - 2x^2}{6x}}}$$

$$V = lbh = 2x \times x \times \frac{12 - 2x^2}{6x}$$

$$= \frac{24x^2 - 4x^4}{6x}$$

$$= 4x - \frac{2}{3}x^3 .$$

$$= \underline{\underline{\frac{2}{3}x\left(6 - x^2\right)}}$$

Solution 9b

Main Grid

9) b) Max at $V'(x) = 0$

$$V(x) = 4x - \frac{2}{3}x^3$$

$$V'(x) = 4 - 2x^2 = 0$$

$$x^2 = \frac{-4}{-2} = 2$$

$$x = \pm\sqrt{2}$$

$$\underline{\underline{x = \sqrt{2}}}$$

x	$\sqrt{2}^-$	$\sqrt{2}$	$\sqrt{2}^+$
$V'(x)$	+	0	−
SHAPE	/	—	\

∴ Max at $\underline{\underline{x = \sqrt{2}}}$

10. The amount A_t micrograms of a certain radioactive substance remaining after t years decreases according to the formula $A_t = A_0e^{-0 \cdot 002t}$, where A_0 is the amount present initially.

(*a*) If 600 micrograms are left after 1000 years, how many micrograms were present initially? **3**

(*b*) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance? **4**

F

Solution

Main Grid

10) a)

$$600 = A_0 e^{-0.002 \times 1000}$$

$$600 = A_0 e^{-2}$$

$$A_0 = 600 \div e^{-2} = \underline{\underline{4433.4 \text{ mg}}}$$

b) $\frac{A_t}{A_0} = 0.5$ at half life

$$\therefore e^{-0.002t} = 0.5$$

$$-0.002t = \ln 0.5$$

$$t = \ln 0.5 \div -0.002$$

$$\underline{\underline{t = 346.6 \text{ years}}}$$

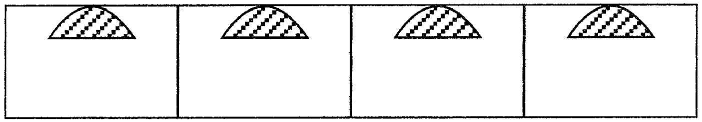
Main Grid

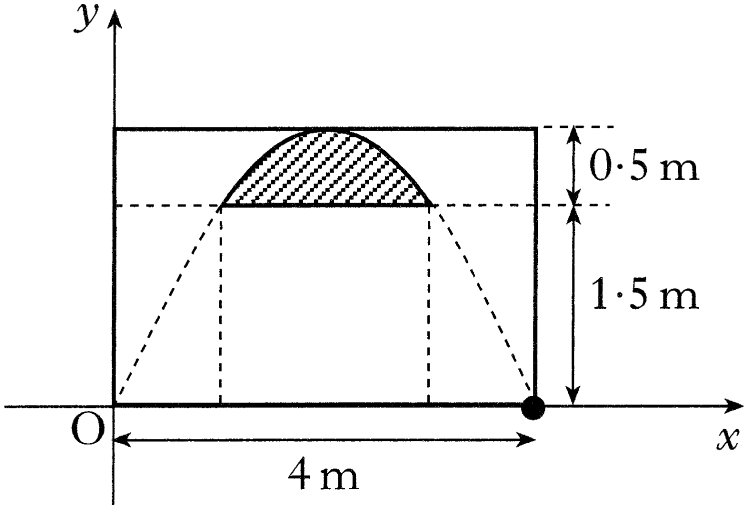
11. An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.

The second diagram shows one such window. The shaded part represents the glass.

The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$.

Find the area in square metres of the glass in one window.





8

F

Solution

Main Grid

11) Area between the curve $y = 2x - \frac{1}{2}x^2$ and the line $y = 1 \cdot 5$

Find the points of intersection:

$$2x - \frac{1}{2}x^2 = \frac{3}{2}$$

$$2x - \frac{1}{2}x^2 - \frac{3}{2} = 0 \quad (\times 2)$$

$$4x - x^2 - 3 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$\underline{x = 1}$ OR $\underline{\underline{x = 3}}$

More Solution

Main Grid

$$A = \int_1^3 2x - \frac{1}{2}x^2 - \frac{3}{2} \; dx$$

$$A = \left[x^2 - \frac{x^3}{6} - \frac{3x}{2} \right]_1^3$$

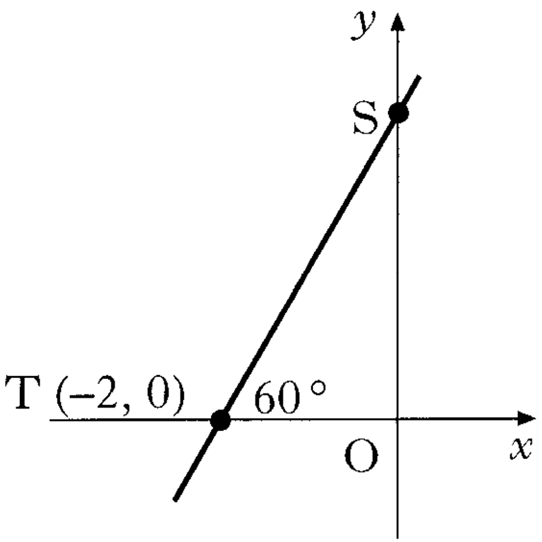
$$A = \left(3^2 - \frac{3^3}{6} - \frac{3 \times 3}{2}\right) - \left(1^2 - \frac{1^3}{6} - \frac{3 \times 1}{2}\right)$$

$$A = \left(9 - \frac{9}{2} - \frac{9}{2}\right) - \left(1 - \frac{1}{6} - \frac{3}{2}\right)$$

$$A = 0 - -\frac{2}{3} \quad = \underline{\underline{\frac{2}{3}\, M^2}}$$

Main Grid

1. Find the equation of the line ST, where T is the point (–2, 0) and angle STO is 60°.



3

F

Solution

Main Grid

1) $M = \tan \theta = \tan 60^\circ = \sqrt{3}$

$y - b = M(x - a)$ $M = \sqrt{3},$ $(a, b) = (-2, 0)$

$y - 0 = \sqrt{3}(x + 2)$

$$\underline{\underline{y = \sqrt{3}x + 2\sqrt{3}}}$$

Main Grid

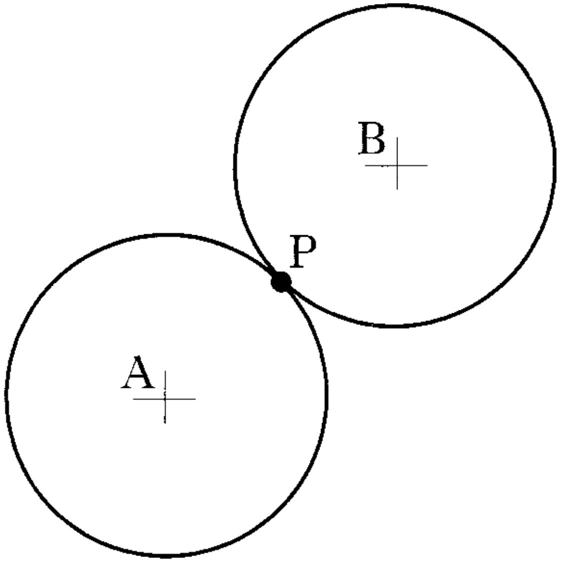
2. Two congruent circles, with centres A and B, touch at P.

Relative to suitable axes, their equations are

$x^2 + y^2 + 6x + 4y - 12 = 0$ and
$x^2 + y^2 - 6x - 12y + 20 = 0.$

(*a*) Find the coordinates of P. **3**

(*b*) Find the length of AB. **2**



F

Solution

Main Grid

2) a) $(-g, -f)$ $\quad A(-3, -2) \quad B(3, 6)$

$$P = \left(\frac{-3+3}{2}, \frac{-2+6}{2}\right) = \underline{\underline{(0, 2)}}$$

b) $$d_{AB} = \sqrt{(-3-3)^2 + (-2-6)^2}$$

$$= \sqrt{36+64}$$

$$= \sqrt{100} = \underline{\underline{10}}$$

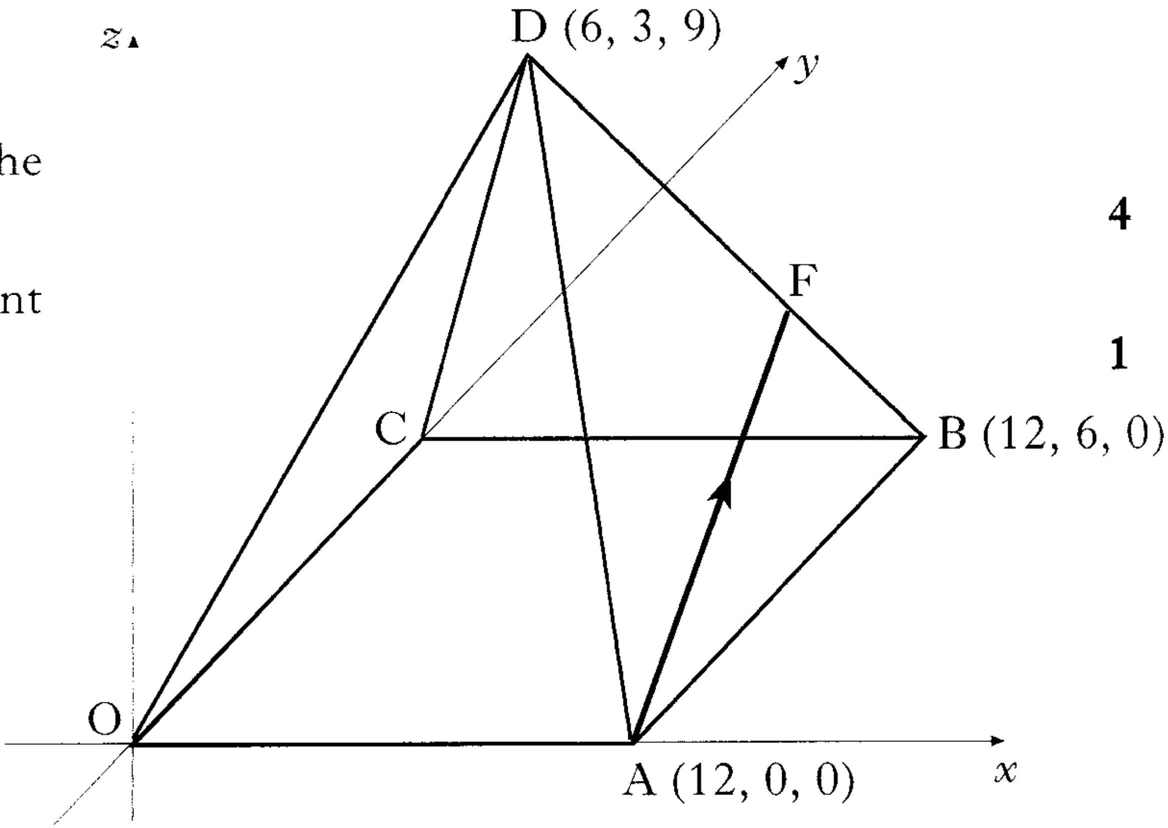
Main Grid

3. D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).

F divides DB in the ratio 2:1.

(*a*) Find the coordinates of the point F. **4**

(*b*) Express $\overrightarrow{AF}$ in component form. **1**

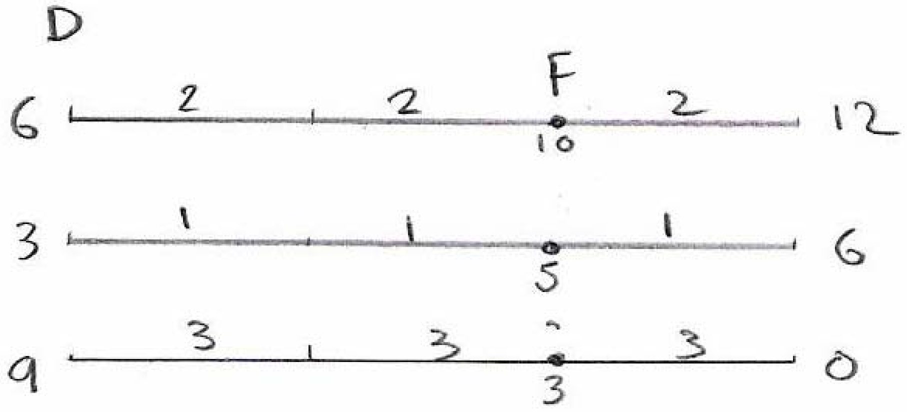


F

Solution

Main Grid

3) a)



F(10, 5, 3)

b) $\overrightarrow{AF} = \underline{f} - \underline{a} = \begin{pmatrix}10\\5\\3\end{pmatrix} - \begin{pmatrix}12\\0\\0\end{pmatrix} = \underline{\underline{\begin{pmatrix}-2\\5\\3\end{pmatrix}}}$

Main Grid

4. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.

(*a*) Find $h(x)$ where $h(x) = g(f(x))$. **2**

(*b*) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.

(ii) Hence state the range of the function h. **2**

F

Solution **Main Grid**

4) a) $h(x) = g(f(x)) = \underline{\underline{(3x-1)^2 + 7}}$

b) i) $\left(\left(\frac{1}{3}\right), 7\right)$.

ii) $h(x) \geq 7$

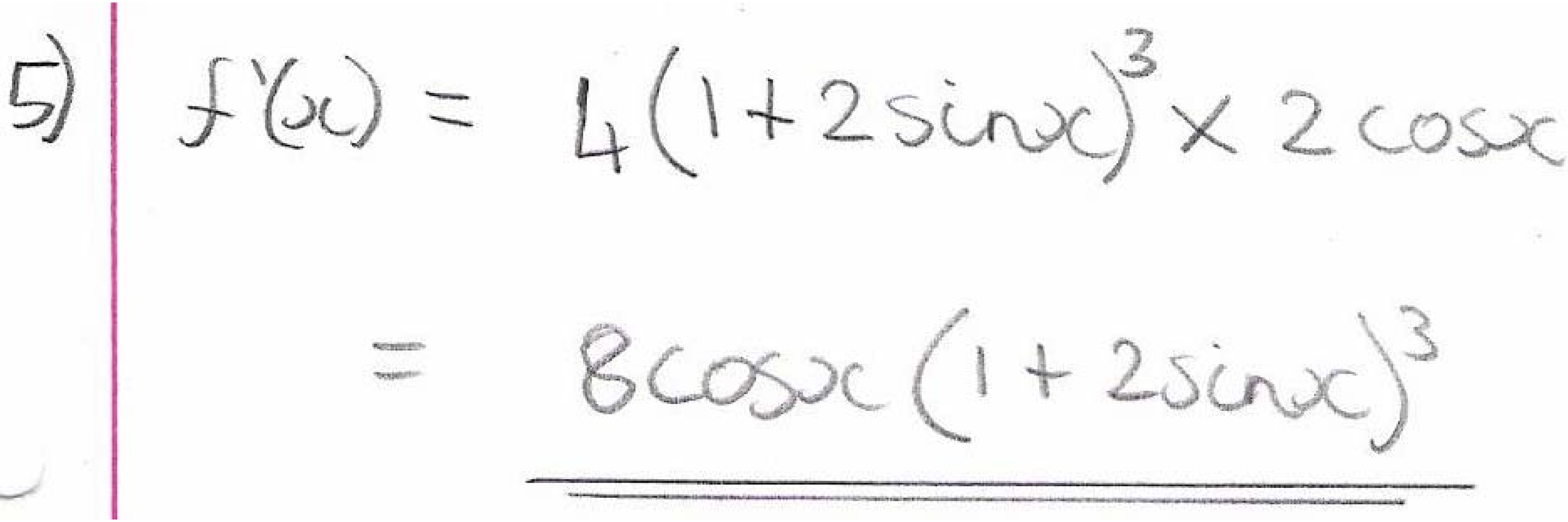
Main Grid

5. Differentiate $(1 + 2\sin x)^4$ with respect to x. **2**

F

Solution

Main Grid



5) $f'(x) = 4(1+2\sin x)^3 \times 2\cos x$

$$= \underline{\underline{8\cos x(1+2\sin x)^3}}$$

Main Grid

6. (*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. **2**

(*b*) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.

(i) Express u_1 and u_2 in terms of m.

(ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. **5**

F

Solution

Main Grid

6) a) $L = \frac{b}{1-a}$ $\quad a = k \quad b = 5$

∴ $\frac{5}{1-k} = 4 \quad = \quad 5 = 4(1-k)$

$5 = 4 - 4k$

$4k = 4 - 5 = -1$

$\underline{\underline{k = -1/4}}$

b) i) $U_1 = 3m + 5.$

$U_2 = m(3m+5) + 5$

$= 3M^2 + 5m + 5.$

ii) $3M^2 + 5M + 5 = 7$

$3M^2 + 5M - 2 = 0$

$(3m - 1)(m + 2) = 0$

$M = 1/3$ OR $M = -2$

for limit $-1 < a < 1$

∴ for no limit $\underline{\underline{M = -2}}$

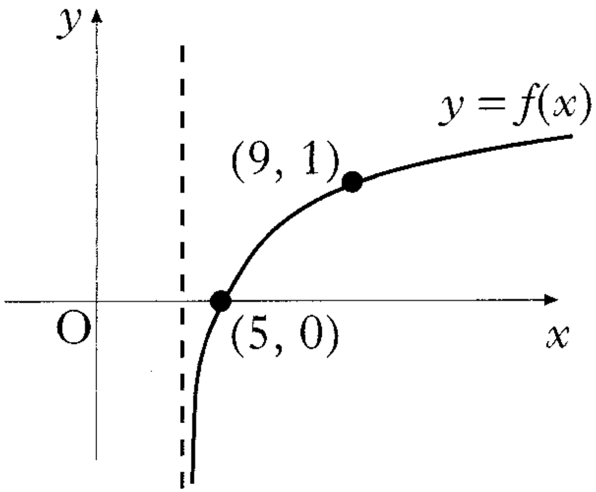
Main Grid

7. The function f is of the form $f(x) = \log_b (x - a)$.

The graph of $y = f(x)$ is shown in the diagram.

(*a*) Write down the values of a and b. **2**

(*b*) State the domain of f. **1**



F

Solution

Main Grid

7) a) $a = 4$ at $(9, 1)$: $1 = \log_b(9-4)$

$1 = \log_b 5$

$\therefore \underline{\underline{b = 5}}$

b) $\underline{\underline{x > 4}}$

Main Grid

8. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **5**

(*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **2**

(*c*) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$. **5**

F

Solution 8ab

Solution 8c

Main Grid

8) a)

$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & 9 \\ & & 6 & -3 & -9 \\ \hline & 2 & -1 & -3 & \underline{\underline{0}} \end{array}$$

Remainder = 0

$\therefore (x-3)$ is a factor

$(x-3)(2x^2-x-3)$

$\underline{\underline{(x-3)(2x-3)(x+1)}}$

b) x-axis: $(-1,0)$ $\left(\frac{3}{2},0\right)$ $(3,0)$

y-axis: $(0-3)(2\times0-3)(0+1) = -3\times-3\times1 = 9$

$\therefore \underline{\underline{(0,9)}}$

Solution 8c

Main Grid

8) c) T.P's at $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 6x^2 - 14x = 0$$

$$2x(3x-7) = 0$$

$$x = 0 \qquad x = 7/3$$

at $x=0$: $y = 2\times0^3 - 7\times0^2 + 9 = \underline{\underline{9}}$

at $x = \frac{7}{3}$: $\frac{7}{3} > 2$ ∴ outside range.

at $x=-2$: $y = 2\times-2^3 - 7\times-2^2 + 9 = -16 - 28 + 9 = \underline{\underline{-35}}$

at $x = 2$: $y = 2\times2^3 - 7\times2^2 + 9 = 16 - 28 + 9 = \underline{\underline{-3}}$

∴ Max Value $= \underline{\underline{9}}$ Min Value $= \underline{\underline{-35}}$

Main Grid

9. If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$. **4**

F

Solution

Main Grid

1) $$2\cos^2 x - 1 = \frac{7}{25}$$

$$2\cos^2 x = \frac{32}{25}$$

$$\cos^2 x = \frac{32}{50} = \frac{16}{25}$$

$$\cos x = \sqrt{\frac{16}{25}} = \underline{\underline{\frac{4}{5}}}$$

OR $\cancel{-\frac{4}{5}}$

$$1 - 2\sin^2 x = \frac{7}{25}$$

$$-2\sin^2 x = \frac{-18}{25}$$

$$\sin^2 x = \frac{9}{25}$$

$$\sin x = \sqrt{\frac{9}{25}} = \underline{\underline{\frac{3}{5}}}$$

OR $\cancel{-\frac{3}{5}}$

Main Grid

10. (*a*) Express $\sin x - \sqrt{3}\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq 2\pi$. **4**

(*b*) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3}\cos x$ in the interval $0 \leq x \leq 2\pi$. **5**

F

Solution 10a

Solution 10b

Main Grid

10) a) $\sin x - \sqrt{3}\cos x = k\sin(x-a)$

$= k(\sin x\cos a - \cos x\sin a)$

$= k\sin x\cos a - k\cos x\sin a$

equating coefficients:

$k\cos a = 1 \qquad k\sin a = \sqrt{3}$

squaring and adding:

$k^2\cos^2 a + k^2\sin^2 a = 1^2 + \sqrt{3}^2$

$k^2(\cos^2 a + \sin^2 a) = 1 + 3$

$k^2 = 4$

$k = \sqrt{4} = \underline{\underline{2}}$

More Solution 10a

Solution 10b

Main Grid

$$\frac{\cancel{k}\sin a}{\cancel{k}\cos a} = \frac{\sqrt{3}}{1} = \tan a = \sqrt{3}$$

sin is +ve, cos is +ve so a is in 1st quad

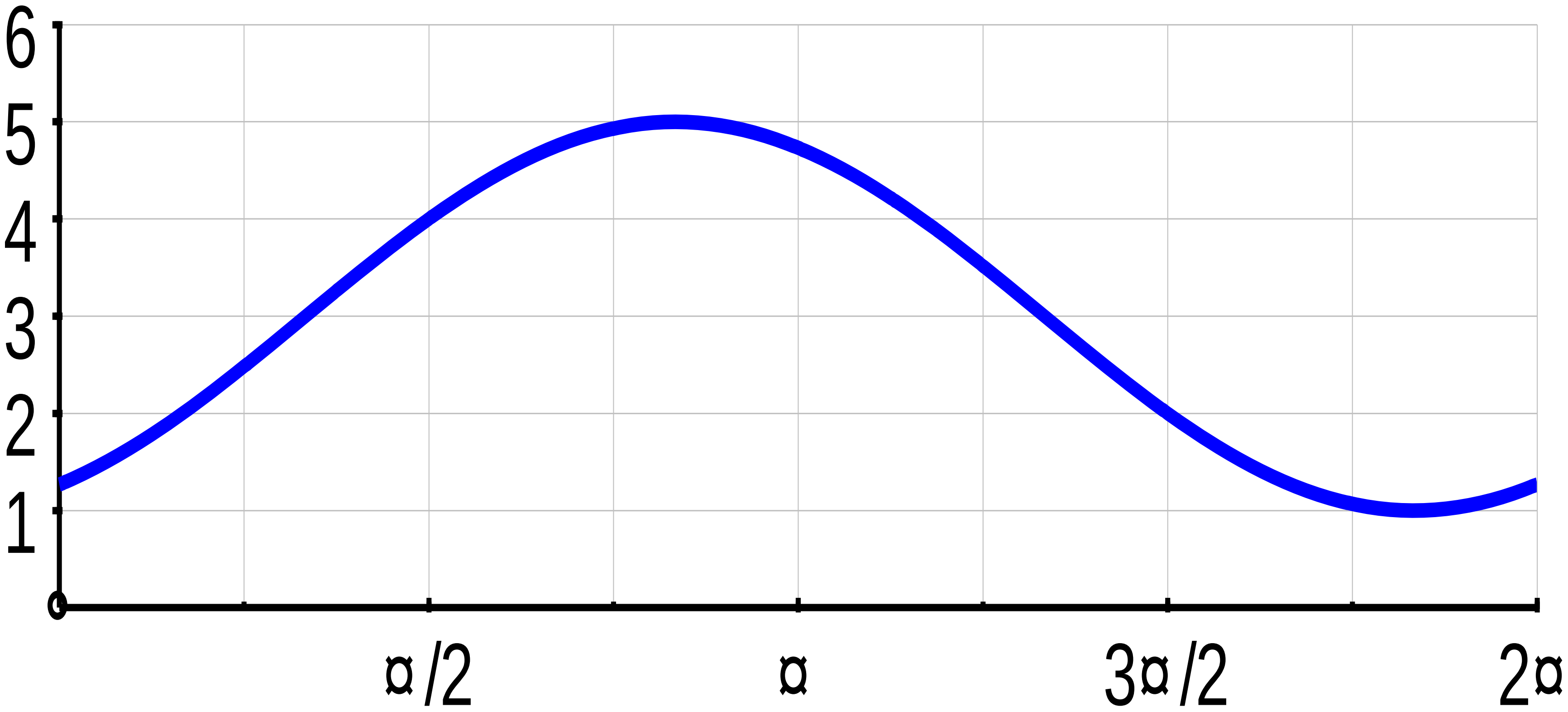
$$a = \tan^{-1}(\sqrt{3}) = \underline{\underline{\pi/3}}$$

$$\therefore \underline{\underline{\sin x - \sqrt{3}\cos x = 2\sin(x - \pi/3)}}$$

Solution 10b

Main Grid

b) $$y = 3 + 2\sin\left(x - \frac{\pi}{3}\right)$$

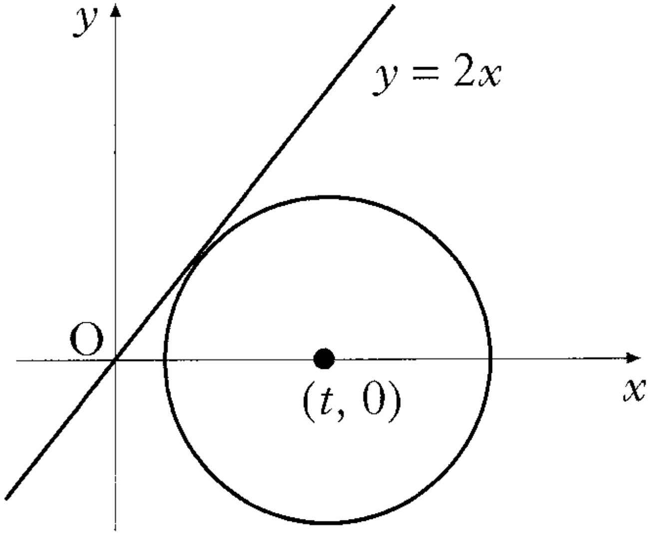


Main Grid

11. (*a*) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units.

Write down the equation of the circle. **1**

(*b*) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle. **5**



F

Solution

Main Grid

11) a) $(x - t)^2 + y^2 = 4$

b) let point of contact have coordinates $(x, 2x)$

$M_r M_t = -1 \quad \therefore \quad M_r = -\frac{1}{2}.$

so: $M_r = \dfrac{0-2x}{t-x} = -\dfrac{1}{2}$

$= \quad -4x = -t + x$

$\underline{\underline{t = 5x}}$

More Solution

Main Grid

to find point of contact substitute line → circle.

$$(x-t)^2+(2x)^2=4$$

at $t=5x$: $(x-5x)^2+4x^2=4$

$$(-4x)^2+4x^2=4$$

$$16x^2+4x^2=4$$

$$20x^2=4$$

$$x^2=\frac{4}{20}=\frac{1}{5}$$

CAN ALSO BE
DONE USING
$b^2-4ac=0$

$$x=\sqrt{\frac{1}{5}}=\frac{1}{\sqrt{5}} \quad \text{OR} \quad \xcancel{-\frac{1}{\sqrt{5}}}$$

$$\therefore \ t=\frac{5}{\sqrt{5}}=\underline{\underline{\sqrt{5}}}$$

Main Grid

1. Find $\int \frac{4x^3 - 1}{x^2}\, dx, \quad x \neq 0.$ **4**

F

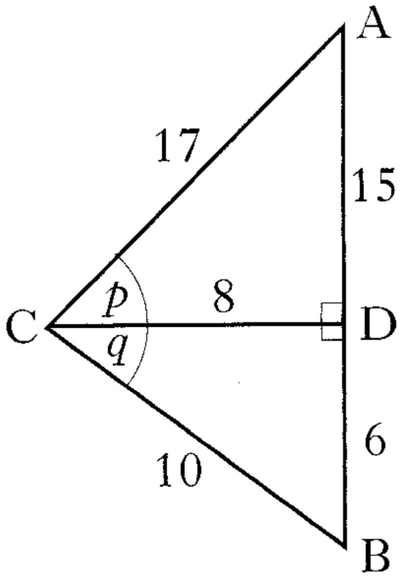
Solution **Main Grid**

1) $$\int \frac{4x^3}{x^2} - \frac{1}{x^2}\,dx = \int 4x - x^{-2}\,dx$$

$$= \frac{4x^2}{2} + \frac{x^{-1}}{1} + c = \underline{\underline{2x^2 + \frac{1}{x} + c}}$$

Main Grid

2. Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.



(*a*) Show that the exact value of $\sin(p+q)$ is $\frac{84}{85}$. **4**

(*b*) Calculate the exact values of:

(i) $\cos(p+q)$;

(ii) $\tan(p+q)$. **3**

F

Solution 2a

Solution 2b

Main Grid

2) a) $\sin P = \frac{15}{17}$ $\cos P = \frac{8}{17}$

$\sin q = \frac{6}{10} = \frac{3}{5}$ $\cos q = \frac{8}{10} = \frac{4}{5}$

$$\sin(P+q) = \sin P \cos q + \cos P \sin q$$

$$= \frac{15}{17} \times \frac{4}{5} + \frac{8}{17} \times \frac{3}{5}$$

$$= \frac{60}{85} + \frac{24}{85} = \underline{\underline{\frac{84}{85}}}$$

Solution 2b

Main Grid

b) i) $\cos(P+q) = \cos P \cos q - \sin P \sin q$

$$= \frac{8}{17} \times \frac{4}{5} - \frac{15}{17} \times \frac{3}{5}$$

$$= \frac{32}{85} - \frac{45}{85} = \underline{\underline{\frac{-13}{85}}}$$

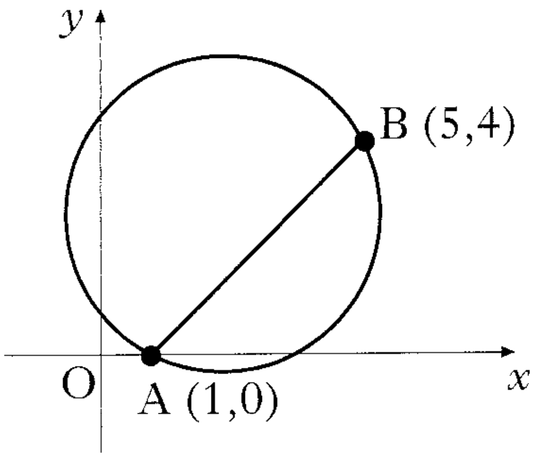
ii) $\tan(P+q) = \dfrac{\sin(P+q)}{\cos(P+q)} = \dfrac{84}{85} \div \dfrac{-13}{85}$

$$= \frac{84}{\cancel{85}} \times \frac{-\cancel{85}}{13} = \underline{\underline{\frac{-84}{13}}}$$

Main Grid

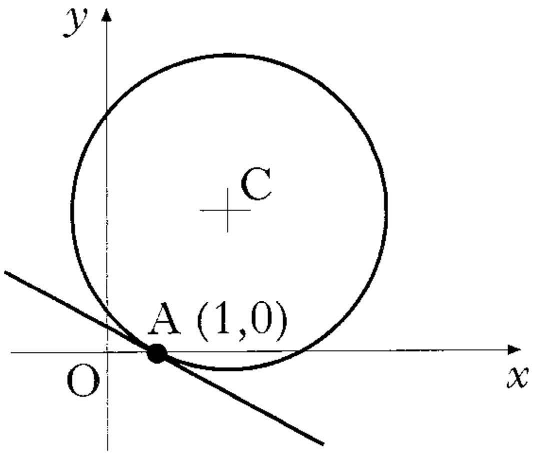
3. (*a*) A chord joins the points A(1,0) and B(5,4) on the circle as shown in the diagram.

Show that the equation of the perpendicular bisector of chord AB is $x + y = 5$. **4**



(*b*) The point C is the centre of this circle. The tangent at the point A on the circle has equation $x + 3y = 1$.

Find the equation of the radius CA. **4**



F

(*c*) (i) Determine the coordinates of the point C.

(ii) Find the equation of the circle. **4**

Solution 3a Solution 3b Solution 3c Main Grid

3) a) Mid-Point of AB $= \left(\frac{1+5}{2}, \frac{0+4}{2}\right) = \underline{\underline{(3,2)}}$

$M_{AB} = \frac{4-0}{5-1} = 1$ $\quad M_1 . M_2 = -1 \quad \therefore \underline{\underline{M = -1}}$

$y - b = M(x-a)$ $\quad M = -1, \; (a,b) = (3,2)$

$y - 2 = -1(x-3)$

$y - 2 = -x + 3$

$\underline{\underline{x + y = 5}}$

Solution 3b

Solution 3c

Main Grid

b) $3y = 1 - x$

$y = \frac{1}{3} - \frac{x}{3} \quad \therefore M_t = -\frac{1}{3} \quad M_t M_r = -1 \quad \underline{\underline{M_r = 3}}$

$y - b = M(x - a) \quad M = 3 \quad , (a, b) = (1, 0)$

$y - 0 = 3(x - 1)$

$\underline{\underline{y = 3x - 3}}$

Solution 3c

Main Grid

c) i) Perpendicular bisectors of chords pass through the centre.

$\therefore$ Sub $y = 3x - 3$ into $x + y = 5$

$= x + 3x - 3 = 5$ $\qquad y = 5 - 2 = 3$

$4x = 8$

$\underline{\underline{x = 2}}$

$\therefore \underline{\underline{C(2,3)}}$

ii) $r = \sqrt{(2-1)^2 + (3-0)^2} = \sqrt{1+9} = \sqrt{10}$

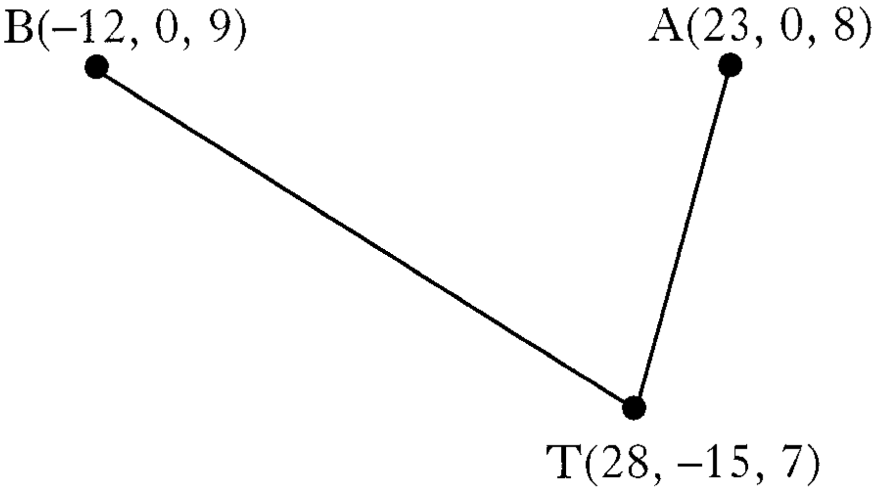
$\therefore \underline{\underline{(x-2)^2 + (y-3)^2 = 10}}$

Main Grid

4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(–12, 0, 9) and T(28, –15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



(*a*) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form. **2**

(*b*) Calculate the angle between these two beams. **5**

F

Solution

Main Grid

4) a) $\overrightarrow{TA} = \underline{a} - \underline{t} = \begin{pmatrix} 23 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}}}$

b) $\overrightarrow{TB} = \underline{b} - \underline{t} = \begin{pmatrix} -12 \\ 0 \\ 9 \end{pmatrix} - \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}}}$

More Solution

Main Grid

b) $\cos\theta = \frac{a.b}{|a||b|} = \frac{\overrightarrow{TA}\cdot\overrightarrow{TB}}{|\overrightarrow{TA}|\,|\overrightarrow{TB}|}$

$\overrightarrow{TA}.\overrightarrow{TB} = (-5\times-40)+(15\times15)+(1\times2) = \underline{\underline{427}}$

$|\overrightarrow{TA}| = \sqrt{-5^2+15^2+1^2} = \underline{\underline{15.84}}$

$|\overrightarrow{TB}| = \sqrt{-40^2+15^2+2^2} = \underline{\underline{42.77}}$

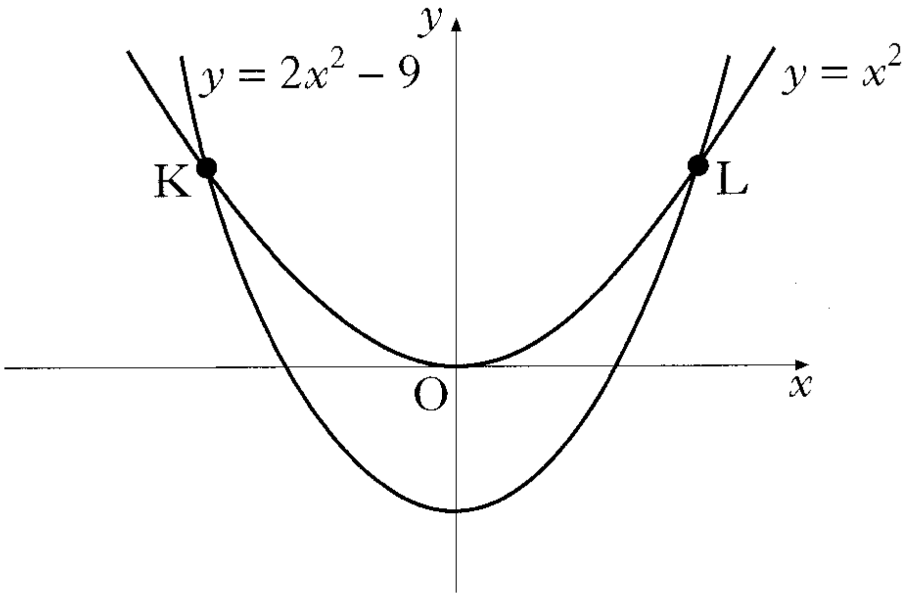
$\cos\theta = \frac{427}{15.84\times42.77} = \underline{\underline{0.63}}$

$\therefore \theta = \cos^{-1}(0.63) = \underline{\underline{50.95^\circ}}$

Main Grid

5. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves.



8

F

Solution

Main Grid

5) Find Lx: $2x^2 - 9 = x^2$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3 \qquad \therefore \underline{\underline{Lx = 3}}$$

$$x^2 - (2x^2 - 9) = -x^2 + 9 = \underline{\underline{9 - x^2}}$$

Because of symmetry integrate from 0→3 and double.

$$A = 2\int_0^3 9 - x^2 \, dx = 2\left[9x - \frac{x^3}{3}\right]_0^3$$

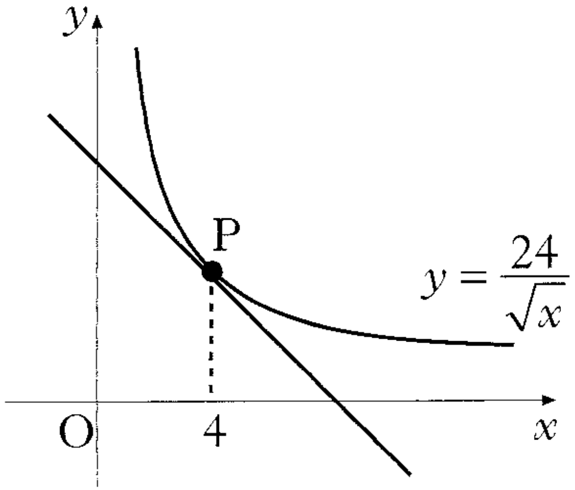
$$= 2\left[(9 \times 3) - \frac{3^3}{3}\right] - 0 = 2 \times 18$$

$$= \underline{\underline{36 \text{ units}^2}}$$

Main Grid

6. The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$.



6

F

Solution

Main Grid

6) $M = \frac{dy}{dx}$: $y = 24x^{-1/2}$ $\frac{dy}{dx} = -12x^{-3/2} = \frac{-12}{\sqrt{x^3}}$

at $x=4$: $M = \frac{-12}{\sqrt{4^3}} = \frac{-12}{8} = \underline{\underline{\frac{-3}{2}}}$

at $x=4$: $y = \frac{24}{\sqrt{4}} = \underline{\underline{12}}$ $\therefore \underline{\underline{(4,12)}}$

$y - b = M(x-a) = y - 12 = \frac{-3}{2}(x-4)$

$2y - 24 = -3x + 12$

$\underline{\underline{2y + 3x - 36 = 0}}$

Main Grid

7. Solve the equation $\log_4(5-x) - \log_4(3-x) = 2$, $x < 3$. **4**

F

Solution

Main Grid

7) $\log_4(5-x) - \log_4(3-x) = \log_4 16$

$$\log_4 \frac{5-x}{3-x} = \log_4 16$$

$$\frac{5-x}{3-x} = 16$$

$$5-x = 16(3-x)$$

$$5-x = 48-16x$$

$$-x+16x = 48-5$$

$$15x = 43$$

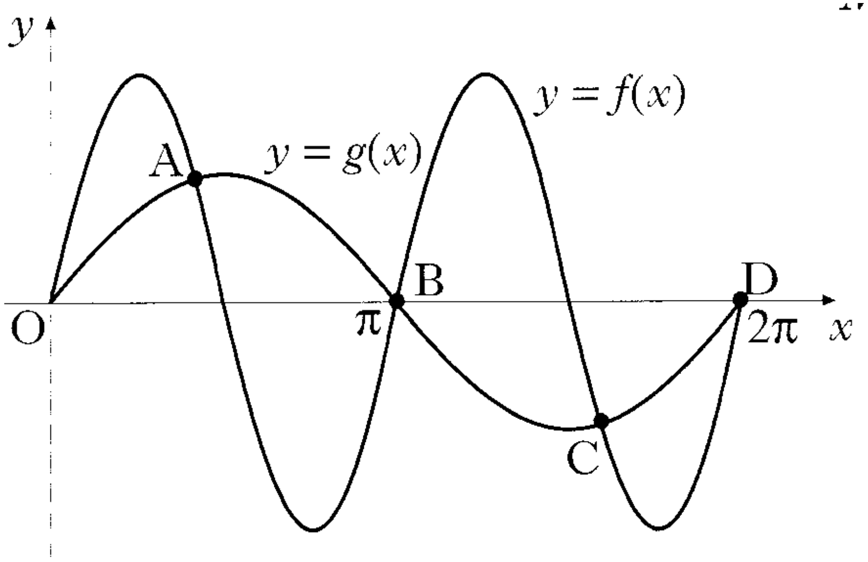
$$\underline{x = 43/15}$$

Main Grid

8. Two functions, f and g, are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$.

The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D.

Show that, at A and C, $\cos x = \dfrac{1}{2k}$.



5

F

Solution

Main Grid

8) at A and C :

$$k\sin 2x = \sin x$$

$$k\sin 2x - \sin x = 0$$

$$k2\sin x\cos x - \sin x = 0$$

$$\sin x(2k\cos x - 1) = 0$$

$\sin x = 0$ $\quad$ $2k\cos x - 1 = 0$

$$\therefore \cos x = 1/2k$$

Main Grid

9. The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0 \cdot 06335t}$.

(*a*) What was its value when launched? **1**

(*b*) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold? **4**

F

Solution

Main Grid

9) a) $t=0$: $V = 252e^0 = \underline{\underline{£252 \text{ million}}}$

b) $20 = 252e^{-0.06335t}$

$$\frac{20}{252} = e^{-0.06335t} = 0.0794$$

$$-0.06335t = \ln 0.0794 = -2.53326$$

$$\therefore t = \frac{-2.53326}{-0.06335} = 39.99$$

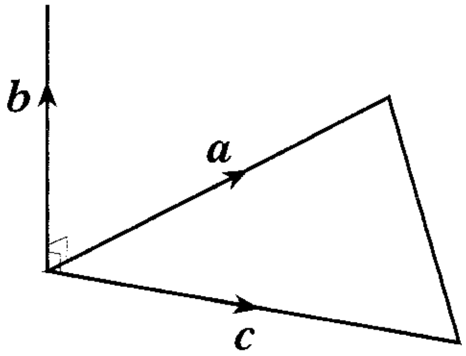
$\therefore$ $\underline{\text{approx } 40 \text{ years.}}$

Main Grid

10. Vectors ***a*** and ***c*** are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram.

Vector ***b*** is 2 units long and ***b*** is perpendicular to both ***a*** and ***c***.

Evaluate the scalar product $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$.

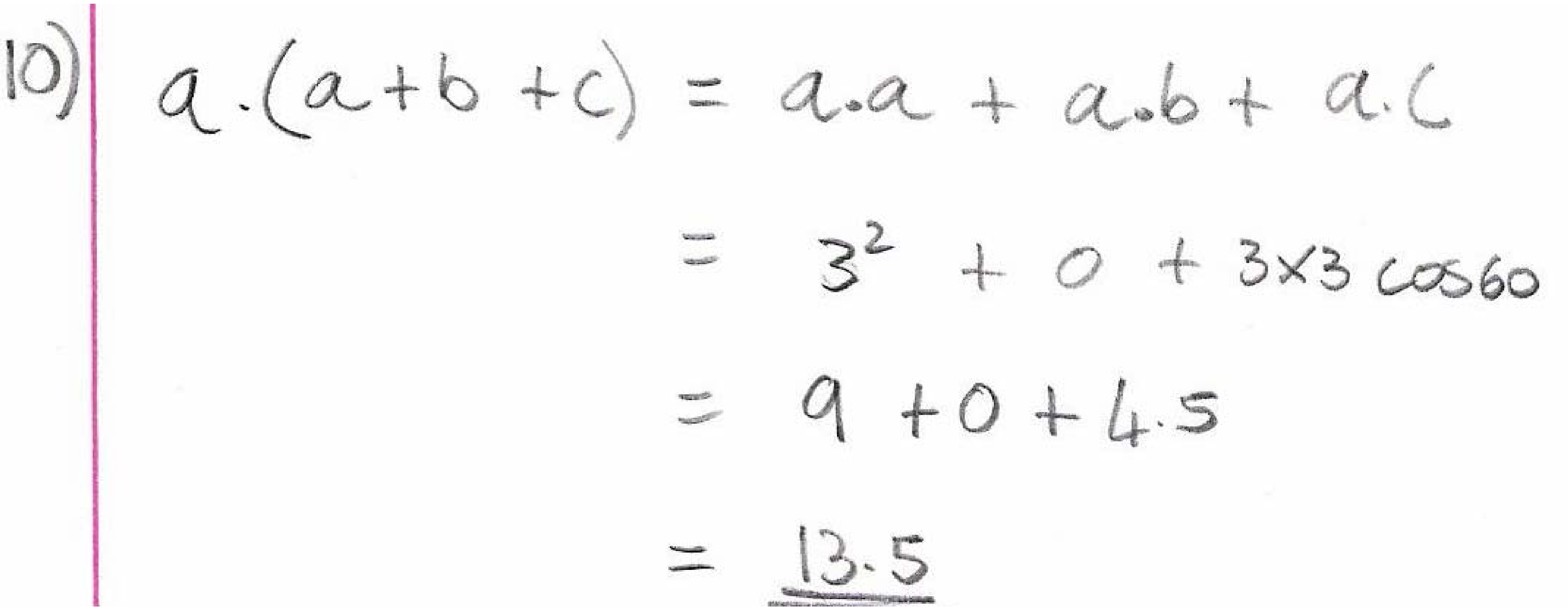


4

F

Solution

Main Grid



10) $$a\cdot(a+b+c) = a\cdot a + a\cdot b + a\cdot c$$

$$= 3^2 + 0 + 3\times 3\cos 60$$

$$= 9 + 0 + 4.5$$

$$= \underline{13.5}$$

Main Grid

11. (*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. **1**

(*b*) Hence find the range of values of p for which all the roots of the cubic equation are real. **7**

F

Solution

Main Grid

11) a) $-1^3 + P \times -1^2 + P \times -1 + 1$

$= -1 + P - P + 1 = 0 \therefore$ Solution.

b)

$$\begin{array}{r|rrrr} -1 & 1 & P & P & 1 \\ & & -1 & -P+1 & -1 \\ \hline & 1 & P-1 & 1 & \underline{\underline{0}} \end{array}$$

$(x+1)(x^2+(P-1)x+1)$ for real roots $b^2-4ac \geq 0$

$a = 1 \quad b = P-1 \quad c = 1$

$(P-1)^2 - (4 \times 1 \times 1)$

$P^2 - 2P + 1 - 4$

$P^2 - 2P - 3$

$(P-3)\,(P+1)$

$\therefore \quad P \geq 3 \qquad P \leq -1$

CHECK VALUES OR

* SKETCH CURVE BEFORE *

STATING RANGE

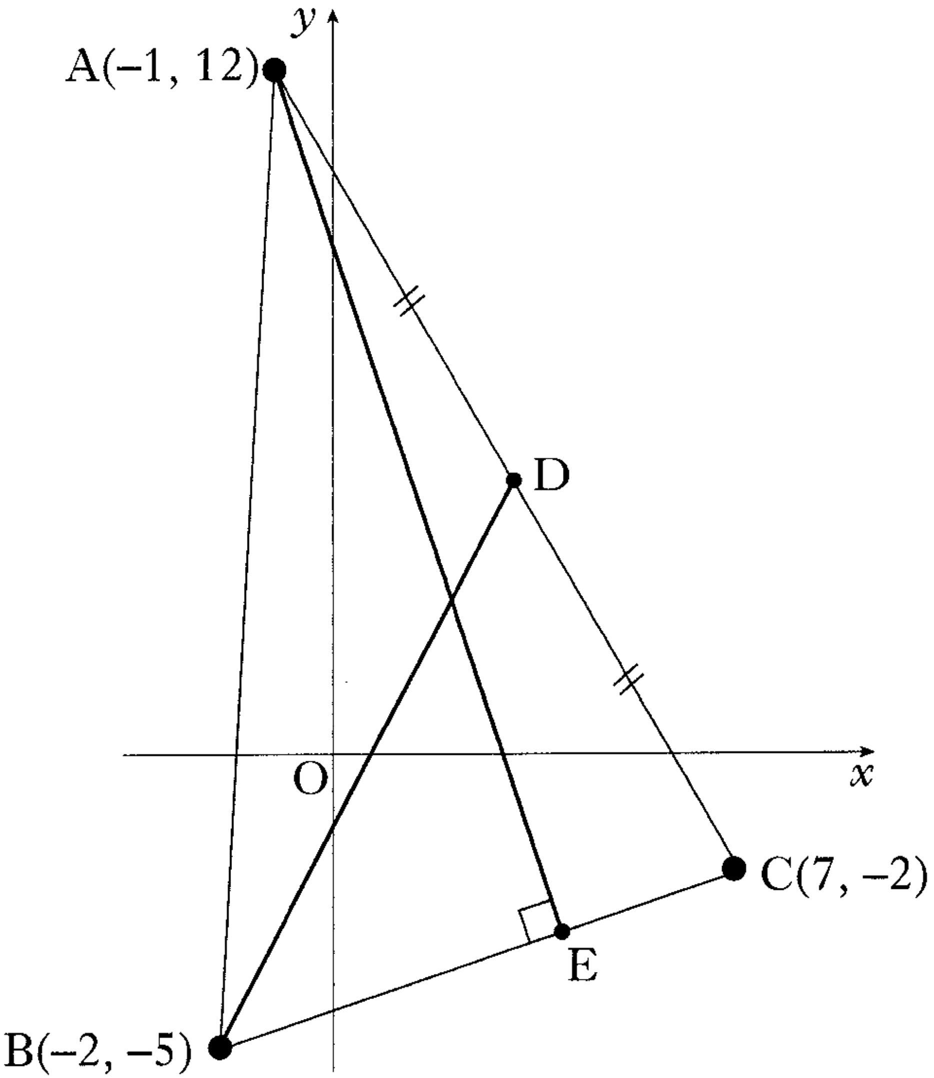
Main Grid

1. Triangle ABC has vertices A(–1, 12), B(–2, –5) and C(7, –2).

(*a*) Find the equation of the median BD. **3**

(*b*) Find the equation of the altitude AE. **3**

(*c*) Find the coordinates of the point of intersection of BD and AE. **3**



F

Solution

Main Grid

1) a) $D = \text{Mid Point } AC = \left(\frac{-1+7}{2}, \frac{12+-2}{2}\right) = \underline{\underline{(3, 5)}}$

$$M_{BD} = \frac{5--5}{3--2} = \frac{10}{5} \; \underline{\underline{= 2}}$$

$y - b = M(x - a)$ $\quad M = 2, \; (a, b) = (3, 5)$

$y - 5 = 2(x - 3)$

$y - 5 = 2x - 6$

$\underline{\underline{y = 2x - 1}}$ OR $\underline{\underline{2x - y = 1}}$

More Solution

Main Grid

b) $M_{BC} = \dfrac{-2--5}{7--2} = \dfrac{3}{9} = \dfrac{1}{3}$

$M_1 M_2 = -1 \quad \therefore \quad M_{AE} = \underline{\underline{-3}}$

$y-b = M(x-a) \qquad M=-3, \quad (a,b) = (-1, 12)$

$y-12 = -3(x+1)$

$y-12 = -3x-3$

$\underline{\underline{y = -3x+9}} \quad \text{OR} \quad \underline{\underline{3x+y=9}}$

More Solution

Main Grid

c)

$$\begin{aligned} 2x - y &= 1 \\ 3x + y &= 9 \\ \hline 5x &= 10 \end{aligned}$$

$$\underline{\underline{x = 2}}$$

$$(2 \times 2) - y = 1$$

$$4 - y = 1$$

$$\underline{\underline{y = 3}}$$

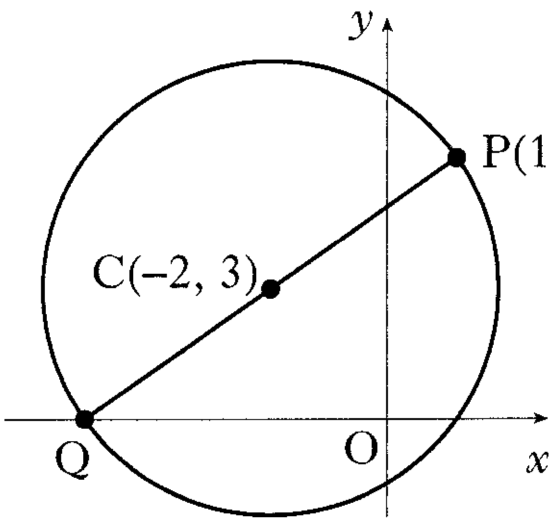
$$\therefore \quad \underline{\underline{(2, 3)}}$$

Main Grid

2. A circle has centre C(–2, 3) and passes through P(1, 6).

(*a*) Find the equation of the circle. **2**

(*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q. **4**



F

Solution

Main Grid

2)a) $r = \sqrt{(1--2)^2 + (6-3)^2}$

$= \underline{\underline{\sqrt{18}}}$

$\underline{\underline{(x+2)^2 + (y-3)^2 = 18}}$

b) $M_r \times M_t = -1$

$M_r = \frac{6-3}{1--2} = \frac{3}{3} = \underline{\underline{1}}$

$\therefore \underline{\underline{M_t = -1}}$

Find Q: $Q = (-2-3, 3-3) = (-5, 0)$

$y - b = M(x-a)$ $M = -1, (a,b) = (-5,0)$

$y - 0 = -1(x+5)$

$y = -x - 5$

$\underline{\underline{x + y + 5 = 0}}$

Main Grid

3. Two functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **3**

(*b*) Determine the least possible value of the product $f(g(x)) \times g(f(x))$. **2**

F

Solution

Main Grid

3)a) $f(g(x)) = 2(2x-3)+3$

$= 4x - 6 + 3$

$= \underline{\underline{4x - 3}}$

$g(f(x)) = 2(2x+3)-3$

$= 4x + 6 - 3$

$= \underline{\underline{4x + 3}}$

b) $(4x-3)(4x+3)$

Main Grid

$\underline{\underline{16x^2 - 9}}$ $\therefore$ $\underline{\underline{\text{least value} = -9}}$

4. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}8u_n + 12$, $u_0 = 4$.

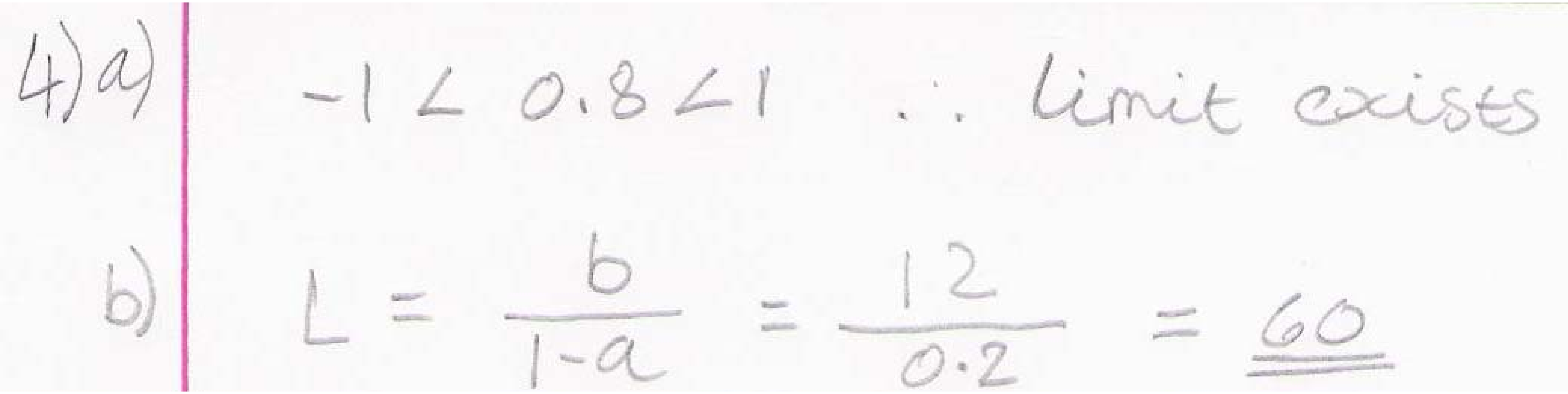
(*a*) State why this sequence has a limit. **1**

(*b*) Find this limit. **2**

F

Solution

Main Grid



4) a) $-1 < 0.8 < 1$ ∴ limit exists

b) $L = \frac{b}{1-a} = \frac{12}{0.2} = \underline{\underline{60}}$

Main Grid

5. A function f is defined by $f(x) = (2x - 1)^5$.

Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. **7**

F

Solution

Main Grid

5) Stationary Points at $f'(x)=0$ ∴

$f'(x) = 5(2x-1)^4 \times 2$

$= 10(2x-1)^4 = 0$

$\underline{\underline{x = \frac{1}{2}}}$

at $x=\frac{1}{2}$: $y = (2\times\frac{1}{2}-1)^5 = 0^5 = \underline{\underline{0}}$ ∴ $\underline{\underline{(\frac{1}{2}, 0)}}$

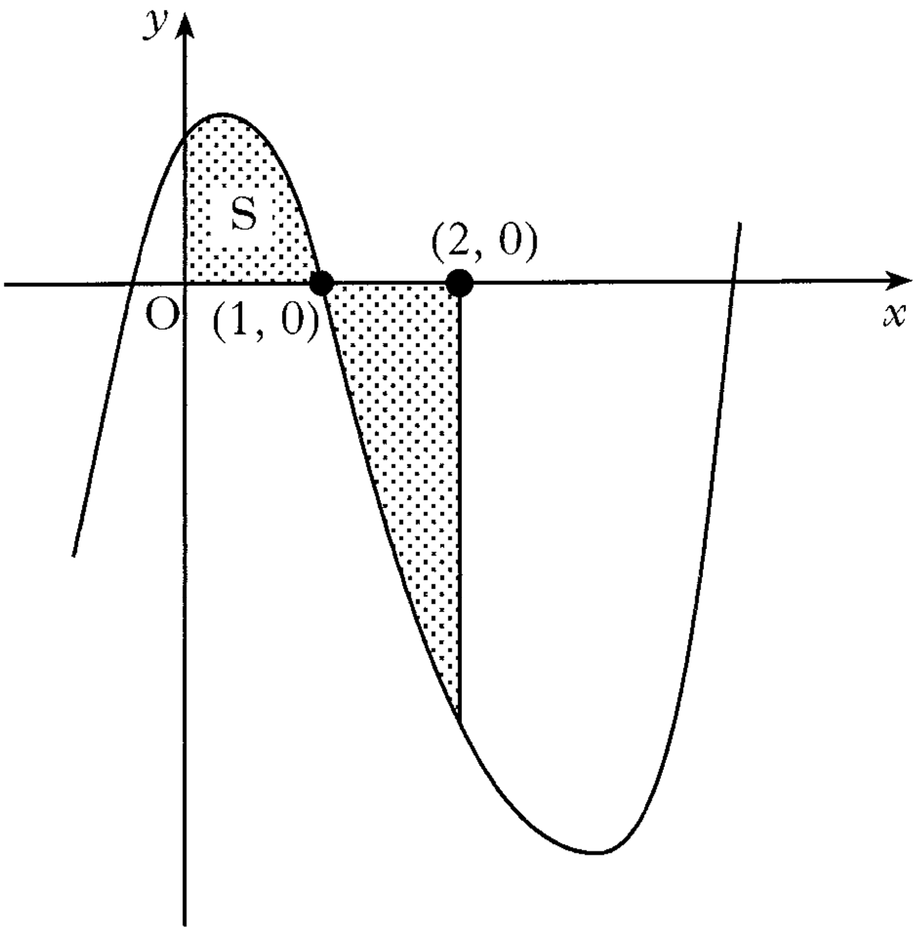
x	$\frac{1}{2}^-$	$\frac{1}{2}$	$\frac{1}{2}^+$
$f'(x)$	+	0	+
SHAPE	/	—	/

∴ Rising Pt. of Inflexion at $(\frac{1}{2}, 0)$

Main Grid

6. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.



(*a*) Calculate the shaded area labelled S. **4**

(*b*) Hence find the total shaded area. **3**

F

Solution

Main Grid

6) a)

$$\int_0^1 x^3 - 6x^2 + 4x + 1 \, dx$$

$$= \left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{4x^2}{2} + x\right]_0^1$$

$$= \left[\frac{x^4}{4} - 2x^3 + 2x^2 + x\right]_0^1$$

$$= \left(\frac{1^4}{4} - 2 \times 1^3 + 2 \times 1^2 + 1\right) - (0)$$

$$= \frac{1}{4} - 2 + 2 + 1 \quad = \underline{\underline{1\tfrac{1}{4} \text{ units}^2}}$$

More Solution

Main Grid

b) $\int_1^2 x^3 - 6x^2 + 4x + 1 \; dx$

$= \left[\frac{x^4}{4} - 2x^3 + 2x^2 + x\right]_1^2$

$= \left(\frac{2^4}{4} - 2\times 2^3 + 2\times 2^2 + 2\right) - \left(\frac{5}{4}\right)$

Main Grid

$= (4 - 16 + 8 + 2) - \left(\frac{5}{4}\right) = -2 - \frac{5}{4} = -3\frac{1}{4}$

$\therefore A = \underline{\underline{3\frac{1}{4} \text{ units}^2}}$

Total Area $= 1\frac{1}{4} + 3\frac{1}{4} = \underline{\underline{4\frac{1}{2} \text{ units}^2}}$

7. Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \le x \le 360$. **4**

F

Solution

Main Grid

7) $\sin x - \sin 2x = 0$

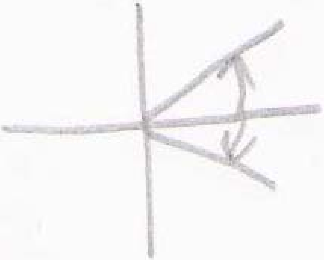
$\sin x - 2\sin x \cos x = 0$

$\sin x(1 - 2\cos x) = 0$

$\sin x = 0$ $\qquad$ $\cos x = \frac{1}{2}$

$x = 0°, 180°$ $\qquad$ $x = 60°$

$360°$ $\qquad$ $x = 300°$



$x = 0°, 60°, 180°, 300°, 360°$

Main Grid

8. (*a*) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$. **3**

(*b*) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$. **1**

F

Solution

Main Grid

8) a) $2x^2 + 4x - 3 = a(x+b)^2 + c$

$= a(x^2 + 2bx + b^2) + c$

$= ax^2 + 2abx + ab^2 + c$

$\therefore \underline{\underline{a = 2}}$

$2ab = 4$

$4b = 4$

$\underline{\underline{b = 1}}$

$ab^2 + c = -3$

$2 + c = -3$

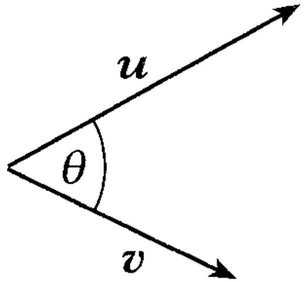
$\underline{\underline{c = -5}}$

$\therefore \underline{\underline{2(x+1)^2 - 5}}$

b) $\underline{\underline{(-1, -5)}}$

Main Grid

9. $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.



(*a*) If $\boldsymbol{u}.\boldsymbol{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. **2**

(*b*) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. **5**

(*c*) Deduce the only possible value of k. **1**

(*d*) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$. **3**

F

Solution

Main Grid

9) a) $(K^3 \times 1) + (1 \times 3K^2) + (K+2) \times -1 = 1$

$K^3 + 3K^2 - K - 2 = 1$

$\underline{\underline{K^3 + 3K^2 - K - 3 = 0}}$

b)

-3	1	3	-1	-3
		-3	0	3
	1	0	-1	$\underline{\underline{0}}$

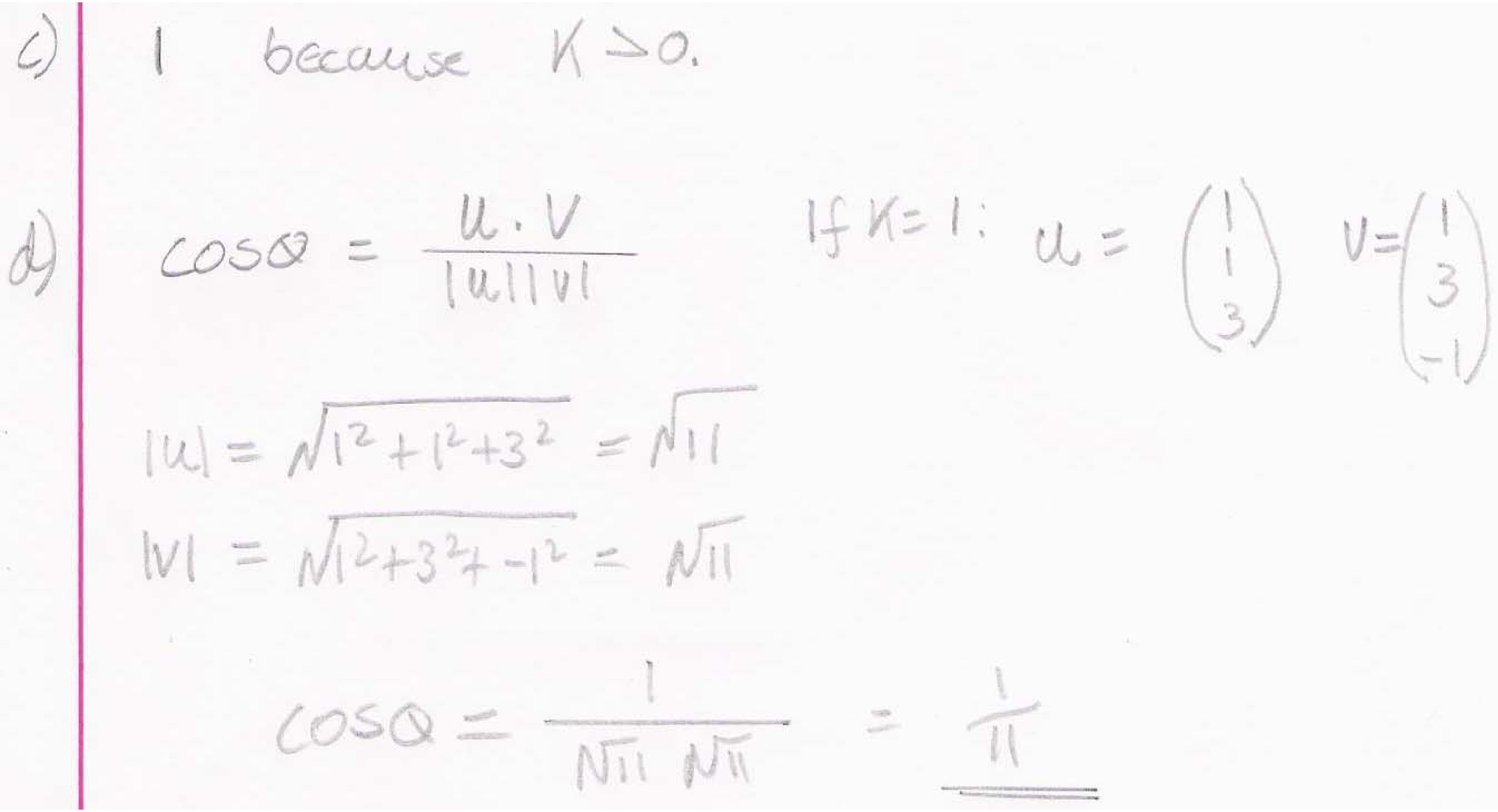
Remainder of zero
so (x+3) is a factor

$(K+3)(K^2-1)$

$\underline{\underline{(K+3)(K+1)(K-1)}}$

More Solution

Main Grid



c) 1 because $K > 0$.

d) $\cos Q = \frac{u \cdot V}{|u||V|}$ If $K = 1$: $u = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ $V = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

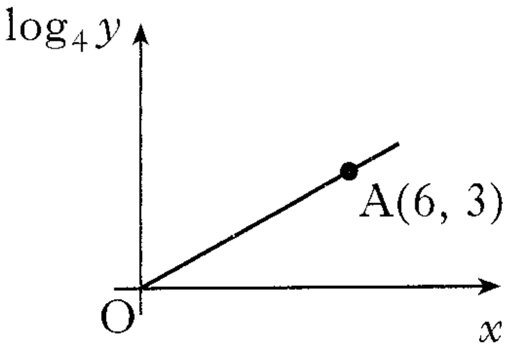
$|u| = \sqrt{1^2 + 1^2 + 3^2} = \sqrt{11}$

$|V| = \sqrt{1^2 + 3^2 + -1^2} = \sqrt{11}$

$\cos Q = \frac{1}{\sqrt{11}\sqrt{11}} = \underline{\underline{\frac{1}{11}}}$

Main Grid

10. Two variables, x and y, are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a.



4

F

Solution

Main Grid

10)

$$y = a^x$$

$$\log_4 y = \log_4 a^x$$

$$\log_4 y = x \log_4 a$$

$$y = Mx + c$$

$$M = \log_4 a$$

$$M = \frac{3}{6} = \frac{1}{2}$$

$$\log_4 a = \frac{1}{2}$$

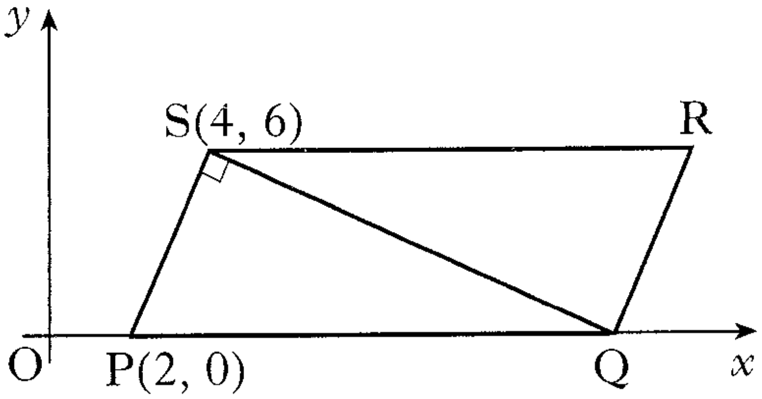
$$\log_4 a = \log_4 2$$

$$\underline{\underline{a = 2}}$$

Main Grid

1. PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x-axis, as shown.

The diagonal QS is perpendicular to the side PS.



(a) Show that the equation of QS is $x + 3y = 22$. **4**

(b) Hence find the coordinates of Q and R. **2**

F

Solution

Main Grid

1) a) $M_{PS} = \frac{6-0}{4-2} = \frac{6}{2} = \underline{\underline{3}}$

$M_{PS} \times M_{QS} = -1 \quad \therefore \quad M_{QS} = \underline{\underline{-\frac{1}{3}}}$

$y - b = M(x-a) \quad M = -\frac{1}{3}, (a,b) = (4,6)$

$y - 6 = -\frac{1}{3}(x-4) \times 3$

$3y - 18 = -1(x-4)$

$3y - 18 = -x + 4$

$\underline{\underline{x + 3y = 22}}$

b) at $y = 0$: $x + 0 = 22 \quad \therefore \quad \underline{\underline{Q(22,0)}}$

$R(22+2, 6) = \underline{\underline{R(24,6)}}$

Main Grid

2. Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. **4**

F

Solution

Main Grid

2) For equal roots $b^2 - 4ac = 0$

$a = K \quad , b = K, \quad c = 6$

$$K^2 - (4 \times K \times 6) = 0$$

$$K^2 - 24K = 0$$

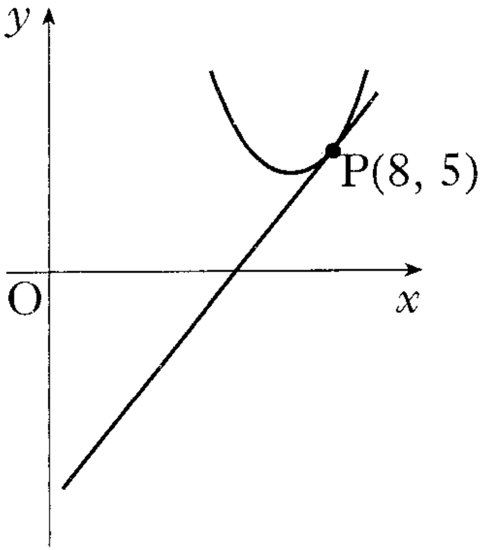
$$K(K - 24) = 0$$

$$\cancel{K = 0} \quad \text{OR} \quad \underline{\underline{K = 24}}$$

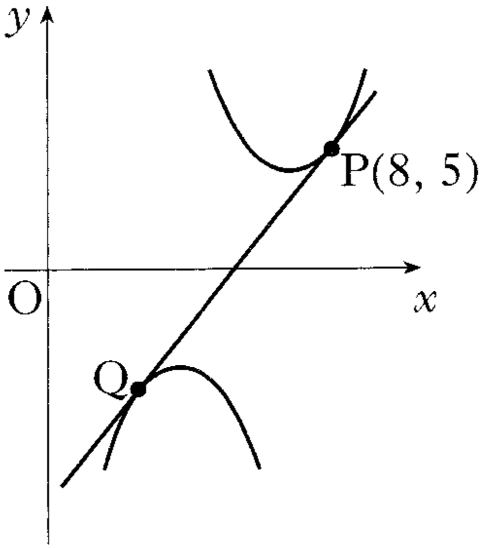
Main Grid

3. The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5).

(*a*) Find the equation of this tangent. **4**



(*b*) Show that the tangent found in (*a*) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q. **5**



F

Solution

Main Grid

3)a) $M = \frac{dy}{dx} = 2x - 14$

at $x = 8$: $M = 2 \times 8 - 14 = \underline{\underline{2}}$

$y - b = M(x - a)$ $M = 2,$ $(a, b) = (8, 5)$

$y - 5 = 2(x - 8)$

$y - 5 = 2x - 16$

$\underline{\underline{y = 2x - 11}}$

More Solution

Main Grid

b) $2x - 11 = -x^2 + 10x - 27$

$x^2 - 8x + 16 = 0$

$a = 1, \; b = -8, \; c = 16$

$b^2 - 4ac = -8^2 - (4 \times 1 \times 16)$

$= 64 - 64 = 0 \; \therefore \; \underline{\underline{\text{tangent}}}.$

$x^2 - 8x + 16 = 0$

$(x - 4)(x - 4) = 0$

$x = 4$

at $x = 4$: $y = 2 \times 4 - 11 = -3$

$\therefore Q\underline{\underline{(4, -3)}}$

Main Grid

4. The circles with equations $(x-3)^2+(y-4)^2=25$ and $x^2+y^2-kx-8y-2k=0$ have the same centre.

Determine the radius of the larger circle. **5**

F

Solution

Main Grid

4) Centre $= (3,4) = (-g, -f)$ $\therefore$ $g = -3$

$f = -4$

$2g = -K$

$-6 = -K$

$\underline{\underline{K = 6}}$

$r = \sqrt{g^2 + f^2 - c} = \sqrt{-3^2 + -4^2 + 12}$

$= \sqrt{37}$

$= \underline{\underline{6.1}}$

Main Grid

5. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. **4**

F

Solution

Main Grid

5) $\int 4x - 6x^2 \quad = \quad 2x^2 - 2x^3 \quad + c$

at $(-1, 9)$: $\quad 9 = 2\times -1^2 - 2\times -1^3 \quad + c$

$$9 = 2 + 2 + c$$

$$\underline{\underline{c = 5}}$$

$$\underline{\underline{y = 2x^2 - 2x^3 + 5}}$$

Main Grid

6. P is the point (–1, 2, –1) and Q is (3, 2, –4).

(*a*) Write down $\overrightarrow{PQ}$ in component form. **1**

(*b*) Calculate the length of $\overrightarrow{PQ}$. **1**

(*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$. **1**

F

Solution **Main Grid**

3a) $\overrightarrow{PQ} = q - p = \begin{pmatrix}3\\2\\-4\end{pmatrix} - \begin{pmatrix}-1\\2\\-1\end{pmatrix} = \begin{pmatrix}4\\0\\-3\end{pmatrix}$

b) $|\overrightarrow{PQ}| = \sqrt{4^2+0^2+-3^2} = \sqrt{25} = \underline{\underline{5}}$

c) $\frac{1}{5}\begin{pmatrix}4\\0\\-3\end{pmatrix} = \underline{\underline{\begin{pmatrix}\frac{4}{5}\\0\\\frac{-3}{5}\end{pmatrix}}}$

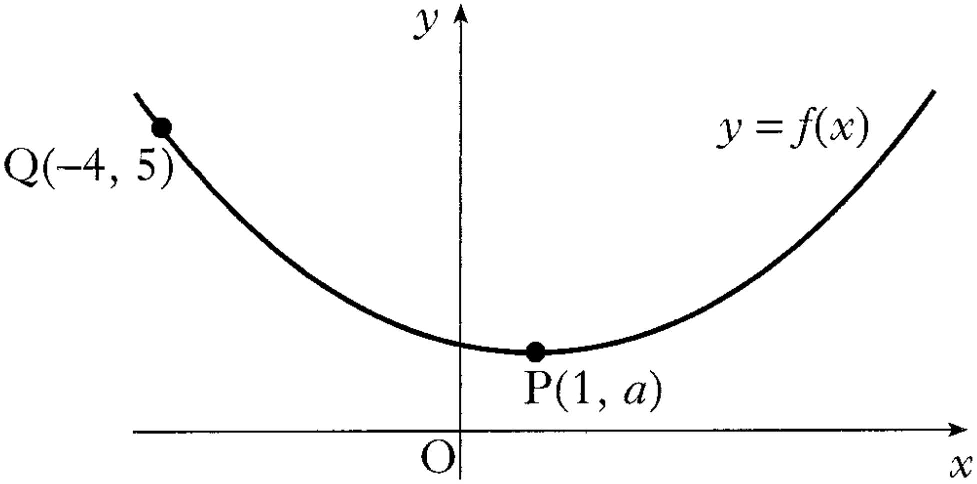
Main Grid

7. The diagram shows the graph of a function $y = f(x)$.

Copy the diagram and on it sketch the graphs of:

(*a*) $y = f(x - 4)$; **2**

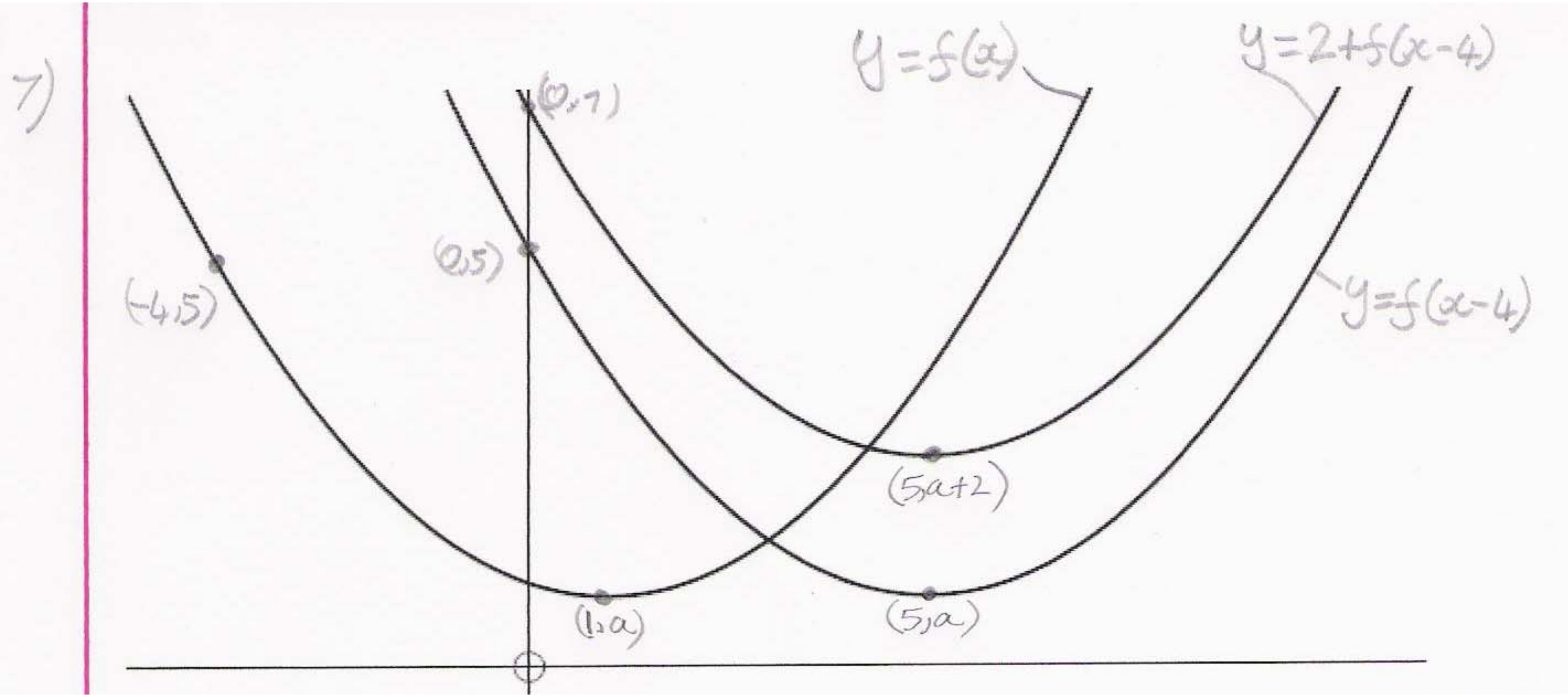
(*b*) $y = 2 + f(x - 4)$. **2**



F

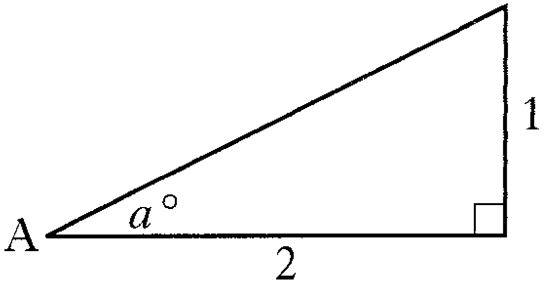
Solution

Main Grid



Main Grid

8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of a° at A.



(*a*) Find the exact values of:

(i) $\sin a^\circ$;

(ii) $\sin 2a^\circ$. **4**

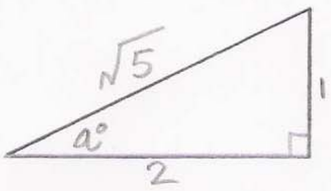
(*b*) By expressing $\sin 3a^\circ$ as $\sin(2a + a)^\circ$, find the exact value of $\sin 3a^\circ$. **4**

F

Solution

Main Grid

8) a)



$\sin a° = \frac{1}{\sqrt{5}}$ $\cos a° = \frac{2}{\sqrt{5}}$

i) $\sin a° = \frac{1}{\sqrt{5}}$

ii) $\sin 2a° = 2\sin a° \cos a°$

$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \underline{\underline{\frac{4}{5}}}$

b) $\sin(2a + a) = \sin 2a \cos a + \cos 2a \sin a$

$\cos 2a = \cos^2 a - \sin^2 a = \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2$

$= \frac{4}{5} - \frac{1}{5} = \underline{\underline{\frac{3}{5}}}$

$\sin(2a + a) = \frac{4}{5} \times \frac{2}{\sqrt{5}} + \frac{3}{5} \times \frac{1}{\sqrt{5}}$

$= \frac{8}{5\sqrt{5}} + \frac{3}{5\sqrt{5}} = \underline{\underline{\frac{11}{5\sqrt{5}}}}$

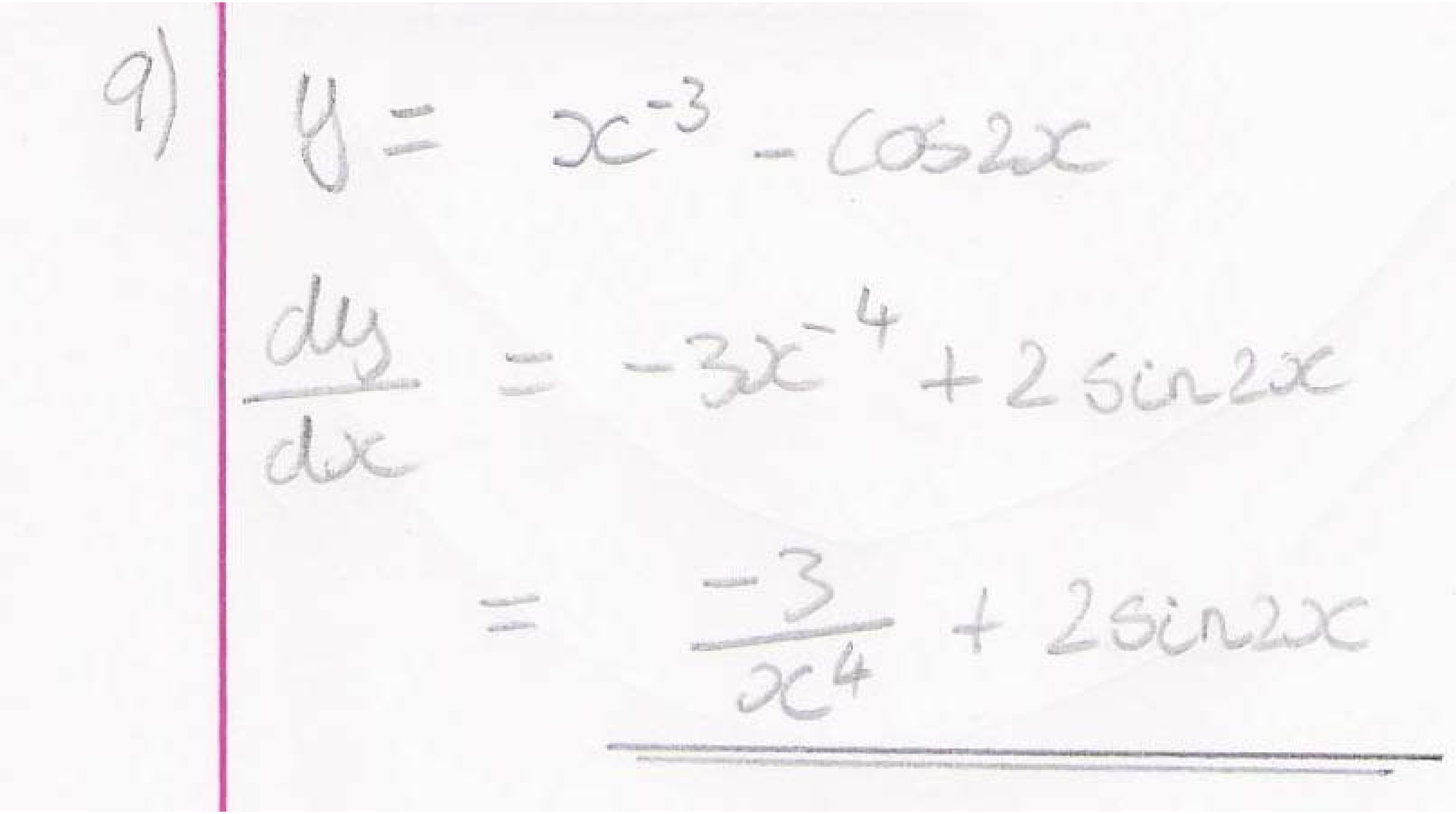
Main Grid

9. If $y = \dfrac{1}{x^3} - \cos 2x, \ \ x \neq 0$, find $\dfrac{dy}{dx}$. **4**

F

Solution

Main Grid



9) $y = x^{-3} - \cos 2x$

$$\frac{dy}{dx} = -3x^{-4} + 2\sin 2x$$

$$= \frac{-3}{x^4} + 2\sin 2x$$

Main Grid

10. A curve has equation $y = 7\sin x - 24\cos x$.

(*a*) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$. **4**

(*b*) Hence find, in the interval $0 \leq x \leq \pi$, the x-coordinate of the point on the curve where the gradient is 1. **3**

F

Solution

Main Grid

10)a) $7\sin x - 24\cos x = K\sin(x-a)$

$$= K(\sin x\cos a - \cos x\sin a)$$

$$= K\sin x\cos a - K\cos x\sin a$$

equate coefficients: $K\cos a = 7$, $K\sin a = 24$

square & add: $K^2\cos^2 a + K^2\sin^2 a = 7^2 + 24^2$

$$K^2(\cos^2 a + \sin^2 a) = 625$$

$$K^2 = 625$$

$$\underline{\underline{K = 25}}$$

More Solution

Main Grid

$$\frac{\cancel{K}\sin a}{\cancel{K}\cos a} = \tan a = \frac{24}{7}$$

sin is +ve, cos is +ve ∴ 1st quadrant.

$$a = \tan^{-1}\left(\frac{24}{7}\right) = \underline{\underline{1.29}} \text{ RADS.}$$

$$\therefore\ 7\sin x - 24\cos x = \underline{\underline{25\sin(x - 1.29)}}$$

More Solution

Main Grid

10) b)

$$y = 25\sin(x - 1.29)$$

$$\frac{dy}{dx} = 25\cos(x - 1.29) \times 1 \quad = 1$$

$$= 25\cos(x - 1.29) = 1$$

$$\cos(x - 1.29) = \frac{1}{25}$$

$$(x - 1.29) = \cos^{-1}\left(\frac{1}{25}\right)$$

$$x - 1.29 = 1.53$$

$$x = 1.53 + 1.29$$

$$\underline{x = 2.82 \text{ RADS}}$$

Main Grid

11. It is claimed that a wheel is made from wood which is over 1000 years old.

To test this claim, carbon dating is used.

The formula $A(t) = A_0 e^{-0 \cdot 000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years.

For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree.

Is the claim true? **5**

F

Solution

Main Grid

11)

$$\frac{A(t)}{A_0} = 0.88$$

$$\therefore e^{-0.000124t} = 0.88$$

$$-0.000124t = \ln 0.88$$

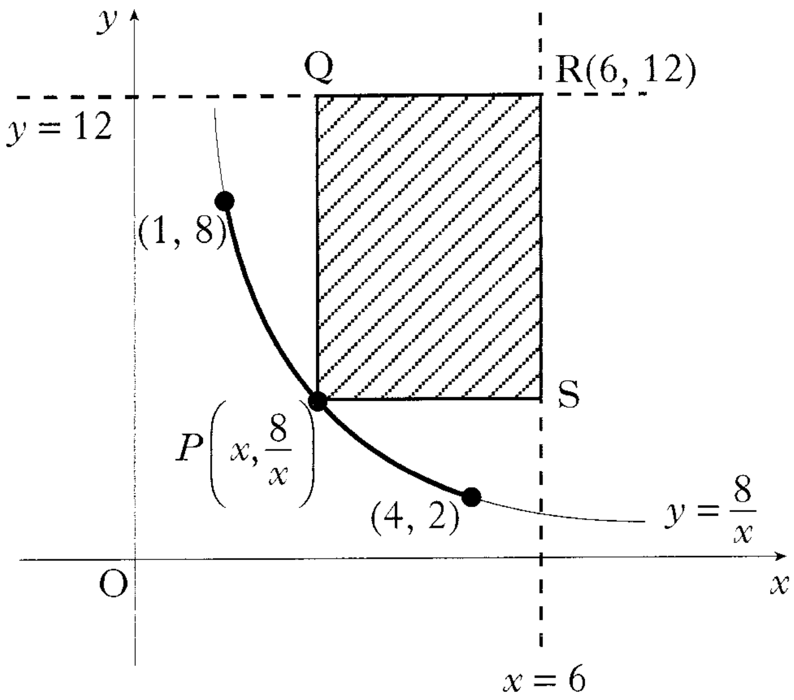
$$t = \frac{\ln 0.88}{-0.000124} = 1031 \text{ years}$$

$\therefore$ $1031 > 1000$ so claim is true.

Main Grid

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by

$$A = 80 - 12x - \frac{48}{x}.$$ **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

Solution

Main Grid

F

12) a) i) $PS = 6 - x$ $\qquad RS = 12 - \frac{8}{x}$

ii) $A = lb = (6-x)\left(12 - \frac{8}{x}\right)$

$= 72 - \frac{48}{x} - 12x + 8$

$= \underline{\underline{80 - 12x - \frac{48}{x}}}$

More Solution

Main Grid

b) Check stationary points and end poin

between $x = 1$, and $x = 4$.

St. Pt. at $\frac{dy}{dx} = 0$

$$\therefore -12 + \frac{48}{x^2} = 0 \quad (\times x^2)$$

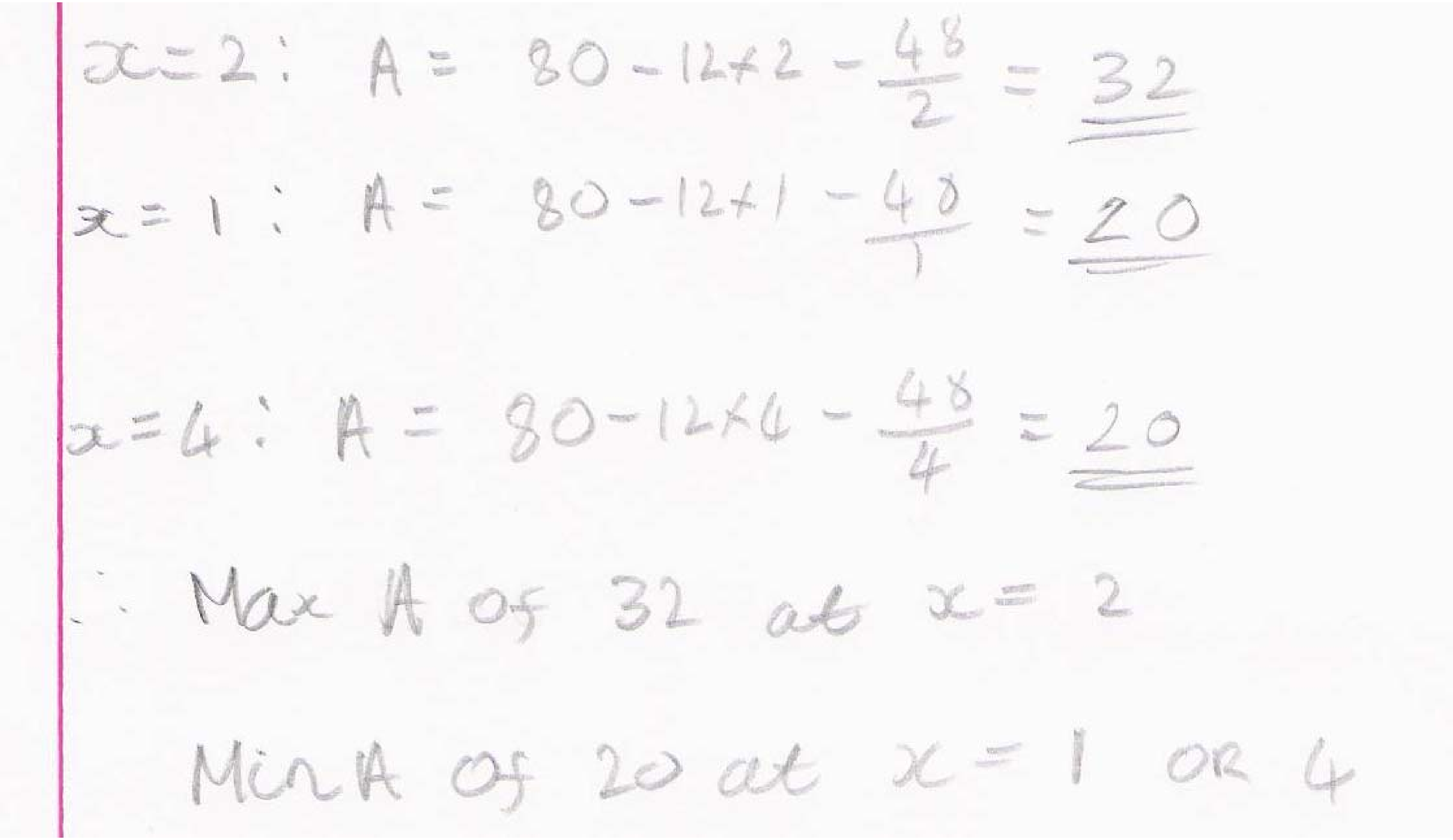
$$-12x^2 + 48 = 0$$

$$x^2 = \frac{-48}{-12} = 4$$

$$x = 2 \text{ or } \cancel{-2}$$ - OUTSIDE RANGE

More Solution

Main Grid



$x=2$: $A = 80-12\times 2-\frac{48}{2} = \underline{\underline{32}}$

$x=1$: $A = 80-12\times 1-\frac{48}{1} = \underline{\underline{20}}$

$x=4$: $A = 80-12\times 4-\frac{48}{4} = \underline{\underline{20}}$

∴ Max A of 32 at $x = 2$

Min A of 20 at $x = 1$ OR 4

Main Grid

1. Find the equation of the line through the point (–1, 4) which is parallel to the line with equation $3x - y + 2 = 0$. **3**

F

Solution

Main Grid

1) $y = 3x + 2 \quad \therefore M = 3 \quad , (a, b) = (-1, 4)$

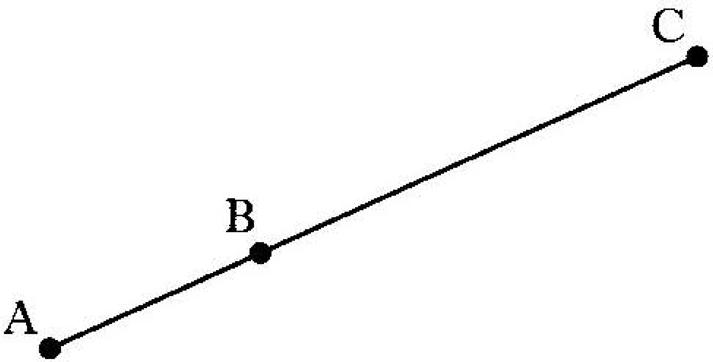
$$y - 4 = 3(x + 1)$$
$$y - 4 = 3x + 3$$
$$\underline{\underline{y = 3x + 7}}$$

Main Grid

2. Relative to a suitable coordinate system A and B are the points (–2, 1, –1) and (1, 3, 2) respectively.

A, B and C are collinear points and C is positioned such that BC = 2AB.

Find the coordinates of C.



4

F

Solution

Main Grid

2)

$$\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix}1\\3\\2\end{pmatrix} - \begin{pmatrix}-2\\1\\-1\end{pmatrix} = \begin{pmatrix}3\\2\\3\end{pmatrix}$$

$$2\overrightarrow{AB} = 2\begin{pmatrix}3\\2\\3\end{pmatrix} = \begin{pmatrix}6\\4\\6\end{pmatrix}$$

$$\begin{pmatrix}1\\3\\2\end{pmatrix} + \begin{pmatrix}6\\4\\6\end{pmatrix} = \begin{pmatrix}7\\7\\8\end{pmatrix} \qquad \therefore \ \underline{\underline{C(7,7,8)}}$$

Main Grid

3. Functions f and g, defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.

Find:

(*a*) $g(f(x))$; **2**

(*b*) $g(g(x))$. **2**

F

Solution

Main Grid

3) a) $g(f(x)) = 1 - 2(x^2+1) = \underline{\underline{-2x^2 - 1}}$

b) $g(g(x)) = 1 - 2(1-2x) = 1 - 2 + 4x$

$= \underline{\underline{4x - 1}}$

Main Grid

4. Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots. **4**

F

Solution

Main Grid

4) For no real roots $b^2-4ac<0$

$a=k \qquad b=-1 \qquad c=-1$

$$-1^2-(4\times k\times -1)<0$$

$$1+4k<0$$

$$4k<-1$$

$$\underline{\underline{k<-1/4}}$$

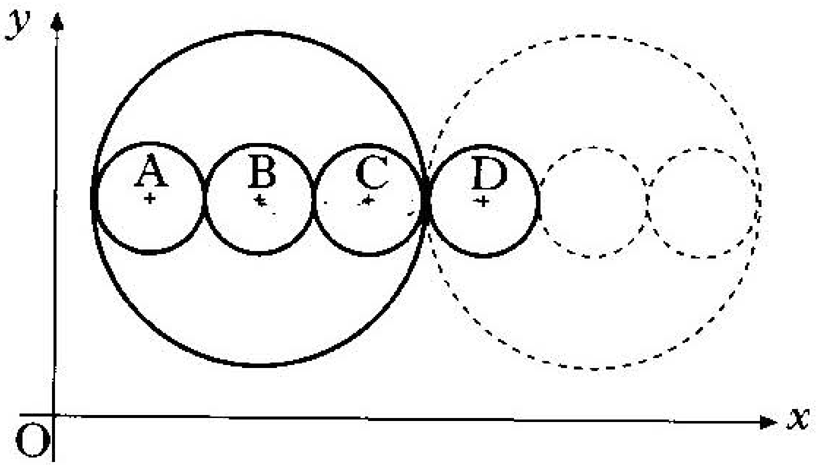
Main Grid

5. The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.



5

F

Solution

Main Grid

5)

$2g = -14 \qquad 2f = -16$

$\underline{\underline{g = -7}} \qquad \underline{\underline{f = -8}} \qquad \text{Centre} = (7, 8)$

$r = \sqrt{g^2 + f^2 - c} = \sqrt{-7^2 + -8^2 - 77} = \sqrt{36} = \underline{\underline{6}}$

$\therefore$ radius of small circles $= \underline{\underline{2}}$

$D_x = 7 + 2 + 2 + 2 + 2 = 15$

$D_y = 8 \qquad D(15, 8)$

$$\underline{\underline{(x - 15)^2 + (y - 8)^2 = 4}}$$

Main Grid

6. Solve the equation $\sin 2x° = 6\cos x°$ for $0 \le x \le 360$. **4**

6)

$$\sin 2x - 6\cos x = 0$$

$$2\sin x\cos x - 6\cos x = 0$$

$$2\cos x(\sin x - 3) = 0$$

$$\cos x = 0$$

$$\sin x = 3$$ NO SOLUTIONS

$$x^\circ = 90^\circ,\ 270^\circ$$

Main Grid

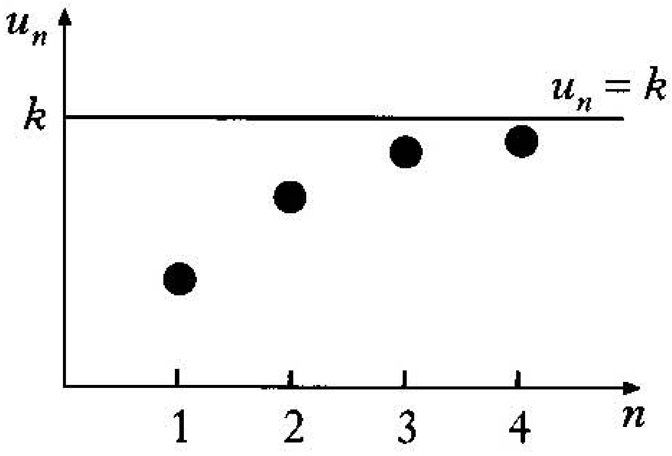
7. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16, \ u_0 = 0.$$

(*a*) Calculate the values of u_1, u_2 and u_3. **3**

Four terms of this sequence, u_1, u_2, u_3 and u_4 are plotted as shown in the graph.

As $n \to \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence.



(*b*) (i) Give a reason why this sequence has a limit.

(ii) Find the exact value of k. **3**

Solution

Main Grid

7) $U_1 = 0 + 16 = 16$

$U_2 = \frac{1}{4} \times 16 + 16 = 20$

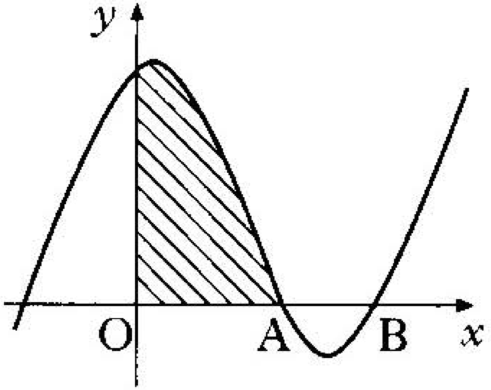
$U_3 = \frac{1}{4} \times 20 + 16 = 21$

b) i) limit exists because $-1 < \frac{1}{4} < 1$.

ii) $L = \dfrac{b}{1-a} = \dfrac{16}{1-\frac{1}{4}} = \dfrac{16}{\frac{3}{4}} = \underline{21\frac{1}{3}}$

Main Grid

8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.



(*a*) Show that the graph cuts the x-axis at (3, 0). **1**

(*b*) Hence or otherwise find the coordinates of A. **3**

(*c*) Find the shaded area. **5**

F

Solution

Main Grid

8) a)

$$\begin{array}{c|cccc} 3 & 1 & -4 & 1 & 6 \\ & & 3 & -3 & -6 \\ \hline & 1 & -1 & -2 & \underline{\underline{0}} \end{array}$$

Remainder $= 0$

so 3 must be a root.

b) $x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2 \qquad x = -1$

$\therefore \underline{\underline{A(2, 0)}}$

More Solution

Main Grid

c) $\int_0^2 x^3 - 4x^2 + x + 6 \, dx$

$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x\right]_0^2$$

$$= \left(\frac{2^4}{4} - \frac{4\times 2^3}{3} + \frac{2^2}{2} + 6\times 2\right) - 0$$

$$= 4 - \frac{32}{3} + 2 + 12 = \underline{\underline{7\tfrac{1}{3}\ \text{units}^2}}$$

Main Grid

9. A function f is defined by the formula $f(x) = 3x - x^3$.

(*a*) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. **2**

(*b*) Find the coordinates of the stationary points of the function and determine their nature. **7**

(*c*) Sketch the graph of $y = f(x)$. **1**

F

Solution

Main Grid

9) a) $x : y = 0,$ $3x - x^3 = 0$

$$x(3 - x^2) = 0$$

$$\underline{\underline{x = 0}} \qquad 3 - x^2 = 0$$

$$\underline{\underline{x = \pm\sqrt{3}}}$$

More Solution

Main Grid

b) Stationary Points at $f'(x) = 0$

$$f'(x) = 3 - 3x^2 = 0$$

$$x^2 = 1$$

$$\underline{\underline{x = 1 \text{ and } -1}}$$

$$f(1) = 3\times1 - 1^3 = 2 \qquad \underline{\underline{(1, 2)}}$$

$$f(-1) = 3\times-1 - -1^3 = -2 \qquad \underline{\underline{(-1, -2)}}$$

More Solution

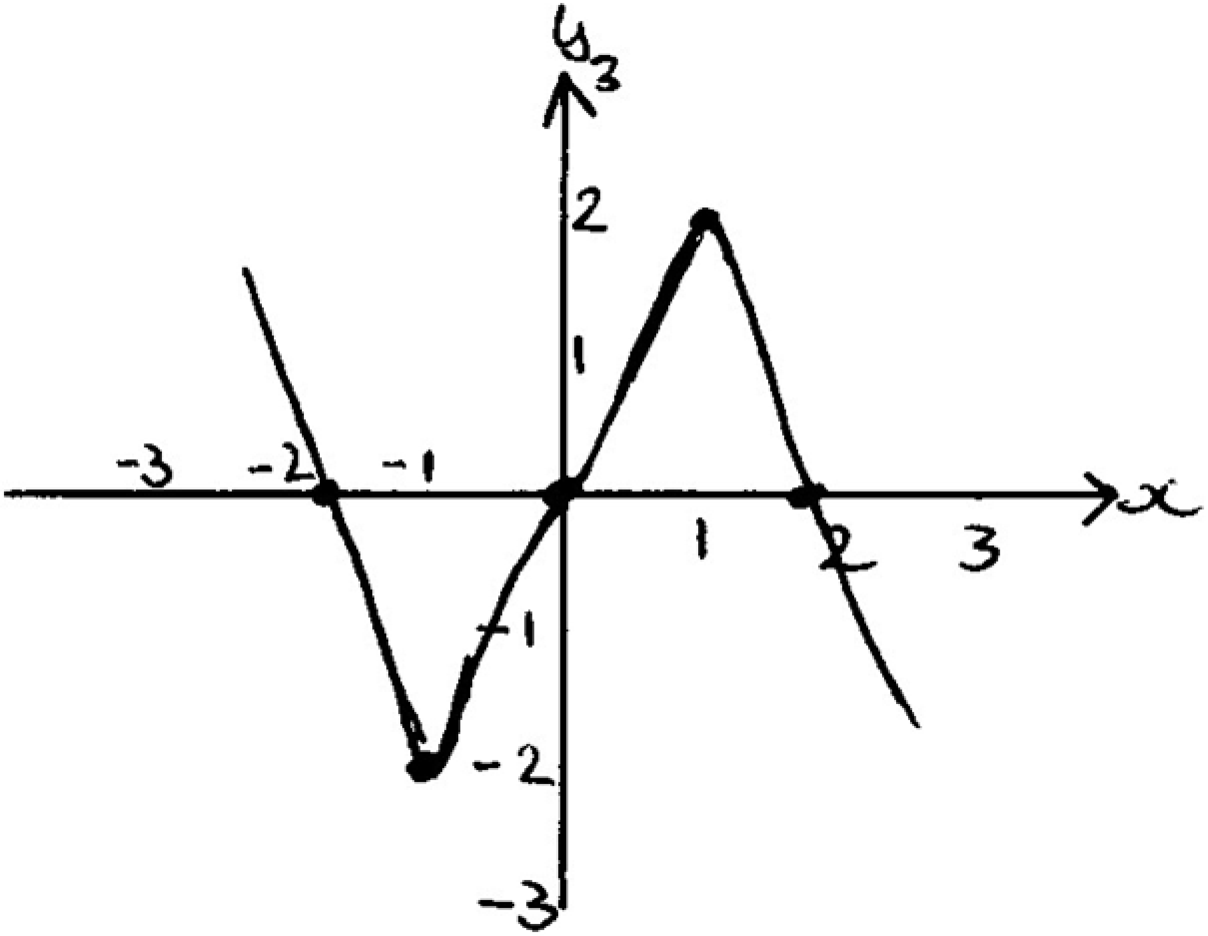
Main Grid

x	-2	-1	0	0	1	2
$f'(x)$	$-$	0	$+$	$+$	0	$-$
SHAPE	╲	—	╱	╱	—	╲
	MIN AT $(-1,-2)$			MAX AT $(1,2)$		

More Solution

Main Grid

9) c)



Main Grid

10. Given that $y = \sqrt{3x^2 + 2}$, find $\dfrac{dy}{dx}$. **3**

F

Solution

Main Grid

10)

$$y = (3x^2+2)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(3x^2+2)^{-1/2} \times 6x$$

$$= 3x(3x^2+2)^{-1/2} = \underline{\underline{\frac{3x}{\sqrt{3x^2+2}}}}$$

Main Grid

11. (*a*) Express $f(x) = \sqrt{3}\cos x + \sin x$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$. **4**

F

Solution

Main Grid

11) a) $\sqrt{3}\cos x + \sin x = K\cos x\cos a + K\sin x\sin a$

$K\cos a = \sqrt{3} \quad K\sin a = 1$

squaring and adding:

$$K^2\cos^2 a + K^2\sin^2 a = \sqrt{3}^2 + 1^2$$

$$K^2(\cos^2 a + \sin^2 a) = 3 + 1 \quad (\cos^2 a + \sin^2 a = 1)$$

$$K^2 = 4$$

$$\underline{K = 2}$$

Now: $\dfrac{K\sin a}{K\cos a} = \dfrac{1}{\sqrt{3}} = \tan a$

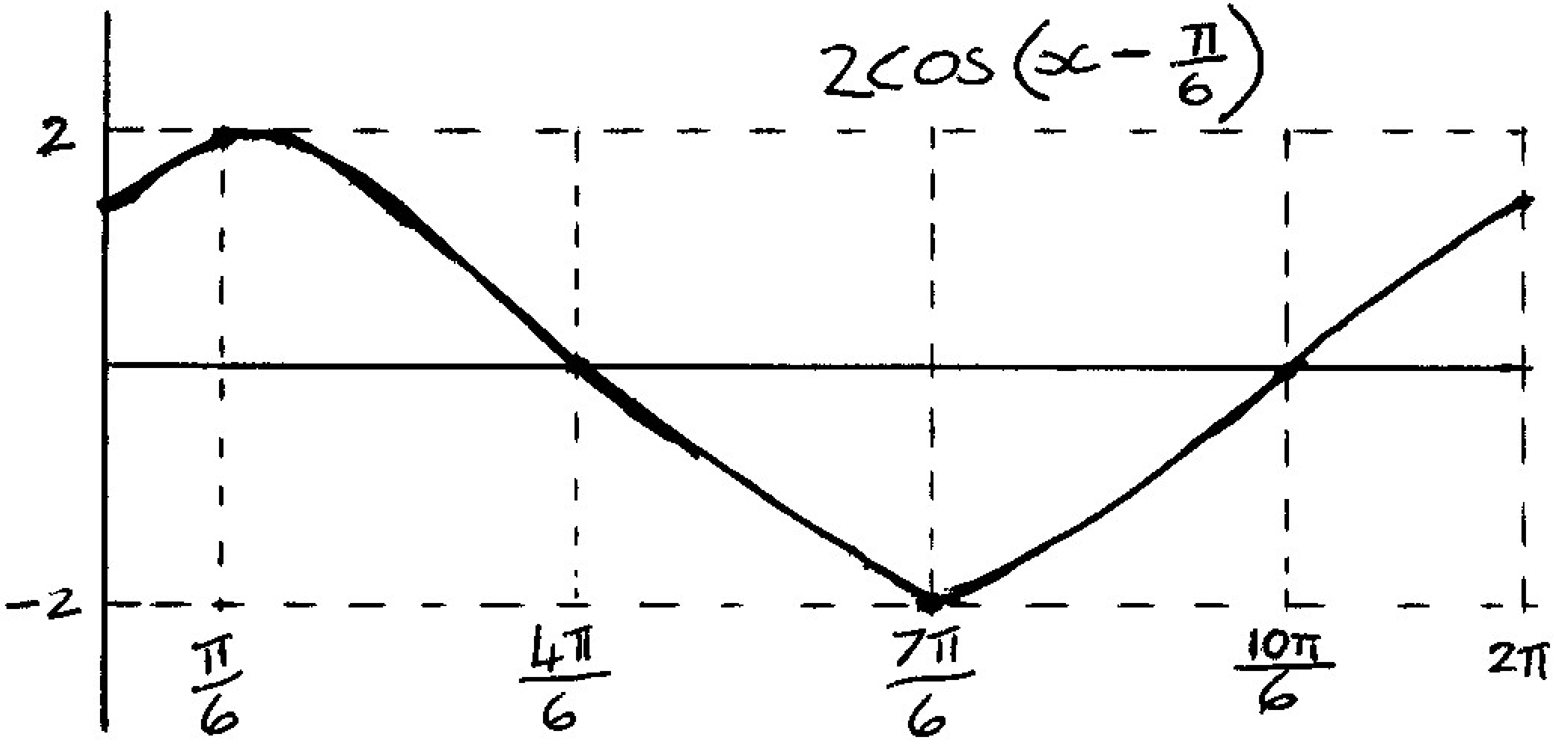
sin is +ve, cos is +ve a is in 1st quadrant

$$a = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \underline{\underline{\frac{\pi}{6}}} \quad \therefore \underline{\underline{2\cos\left(x - \frac{\pi}{6}\right)}}$$

More Solution

Main Grid

11) b)

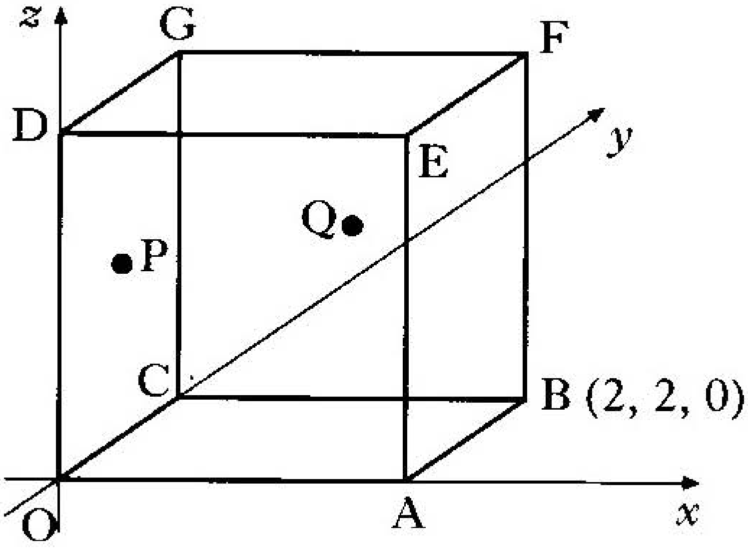


Main Grid

1. OABCDEFG is a cube with side 2 units, as shown in the diagram.

B has coordinates (2, 2, 0).

P is the centre of face OCGD and Q is the centre of face CBFG.



(*a*) Write down the coordinates of G. 1

(*b*) Find ***p*** and ***q***, the position vectors of points P and Q. 2

(*c*) Find the size of angle POQ. 5

F

Solution

Main Grid

1) a) $G\ (0, 2, 2)$

b) $P\ (0, 1, 1)$ $\quad Q\ (1, 2, 1)$

$$p = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad q = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

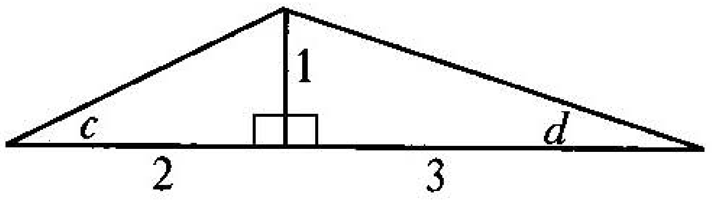
c) $$\cos\theta = \frac{p.q}{|p||q|}$$

$$= \frac{0 + 2 + 1}{\sqrt{2}\ \sqrt{6}} = \frac{3}{\sqrt{12}} = \frac{3}{2\sqrt{3}} = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \underline{\underline{30^\circ = POQ}}$$

Main Grid

2. The diagram shows two right-angled triangles with angles c and d marked as shown.



(*a*) Find the exact value of $\sin(c + d)$. **4**

(*b*) (i) Find the exact value of $\sin 2c$.

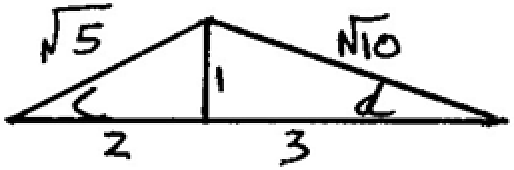
(ii) Show that $\cos 2d$ has the same exact value. **4**

F

Solution

Main Grid

2) a)



$\sin c = \frac{1}{\sqrt{5}}$ $\qquad$ $\cos c = \frac{2}{\sqrt{5}}$

$\sin d = \frac{1}{\sqrt{10}}$ $\qquad$ $\cos d = \frac{3}{\sqrt{10}}$

$$\begin{aligned}
\sin(c+d) &= \sin c \cos d + \cos c \sin d \\
&= \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}} \\
&= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}} = \frac{5}{\sqrt{50}} = \frac{5}{5\sqrt{2}} = \underline{\underline{\frac{1}{\sqrt{2}}}}
\end{aligned}$$

More Solution **Main Grid**

b) i) $\sin 2C = 2\sin C \cos x$

$$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \underline{\underline{\frac{4}{5}}}$$

ii) $\cos 2d = 2\cos^2 d - 1$

$$= 2\left(\frac{3}{\sqrt{10}}\right)^2 - 1$$

$$= 2\left(\frac{9}{10}\right) - \frac{10}{10} = \frac{18}{10} - \frac{10}{10} = \frac{8}{10} = \underline{\underline{\frac{4}{5}}}$$

Main Grid

3. Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. **6**

F

Solution

Main Grid

3) $x^2 + (6-2x)^2 + 6x - 4(6-2x) - 7 = 0$

$x^2 + 36 - 24x + 4x^2 + 6x - 24 + 8x - 7 = 0$

$5x^2 - 10x + 5 = 0$ (÷5)

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$\underline{\underline{x = 1}}$ One root, one point of contact therefore tangent.

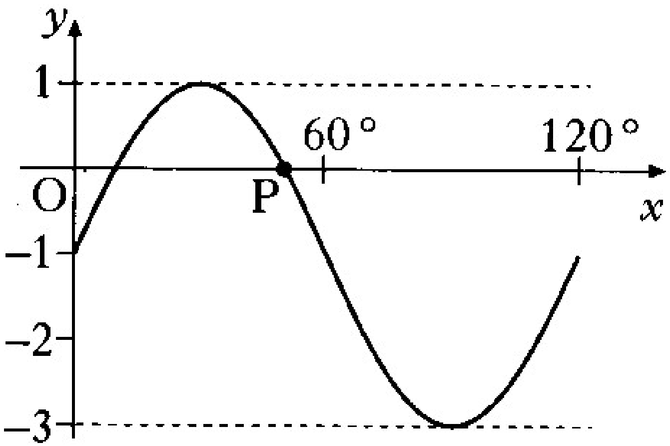
at $x=1$: $y = 6 - 2\times 1 = 4$ $\therefore \underline{\underline{(1, 4)}}$

Main Grid

4. The diagram shows part of the graph of a function whose equation is of the form $y = a\sin(bx°) + c$.

(*a*) Write down the values of a, b and c. **3**

(*b*) Determine the exact value of the x-coordinate of P, the point where the graph intersects the x-axis as shown in the diagram. **3**



F

Solution

Main Grid

4) a) $a = 2$, $b = 3$, $c = -1$

b) $2 \sin 3x - 1 = 0$

$\sin 3x = 1/2$

$3x = 30^\circ \qquad 150^\circ$

$x = 10^\circ \qquad 50^\circ$

$\therefore \underline{\underline{P_x = 50^\circ}}$

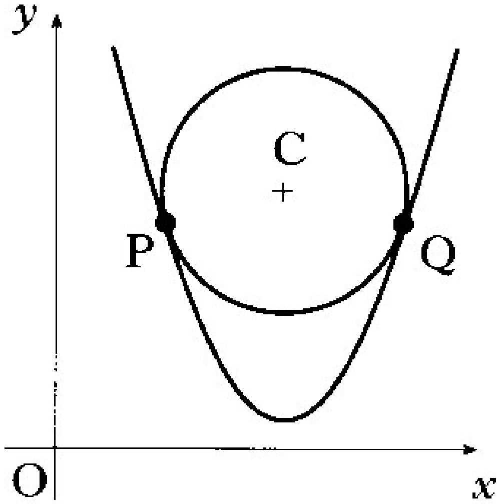
Main Grid

5. A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.

(*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. 5

(*b*) Find the coordinates of P. 2

(*c*) Find the coordinates of C, the centre of the circle. 2



F

Solution

Main Grid

5) a) $M = \frac{dy}{dx} = x - 8 = 4$

$$\underline{\underline{x = 12}}$$

at $x = 12$: $y = \frac{1}{2} \times 12^2 - 8 \times 12 + 34 = 10$

$$\therefore \underline{\underline{Q\,(12, 10)}}$$

b) $M_P = -4$

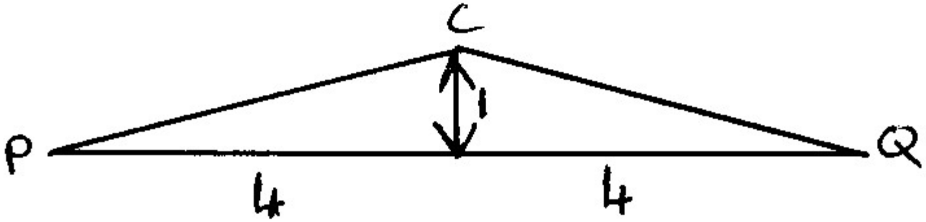
$x - 8 = -4$

$$\underline{\underline{x = 4}}$$

More Solution

Main Grid

2) At P $M_t = -4$ so $M_r = \frac{1}{4}$ because $M_t M_r = -1$



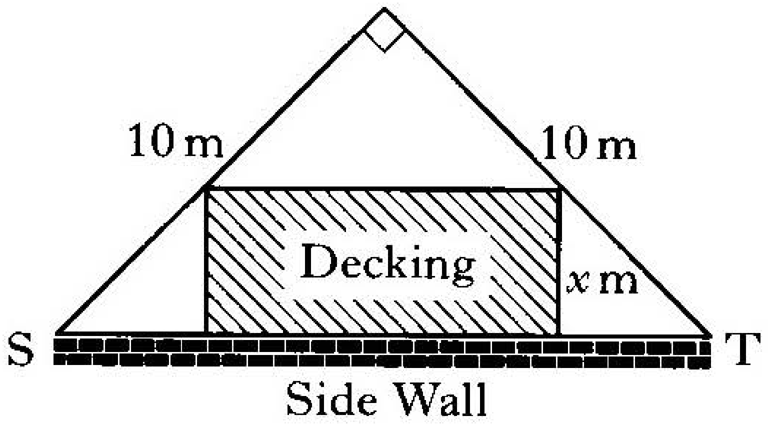
∴ $\underline{\underline{C(8,11)}}$

If $M = 1/4$ then 4 along for 1 up.

Main Grid

6. A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



(*a*) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = \left(10\sqrt{2}\right)x - 2x^2.$$

3

(*b*) Find the dimensions of the decking which maximises its area. **5**

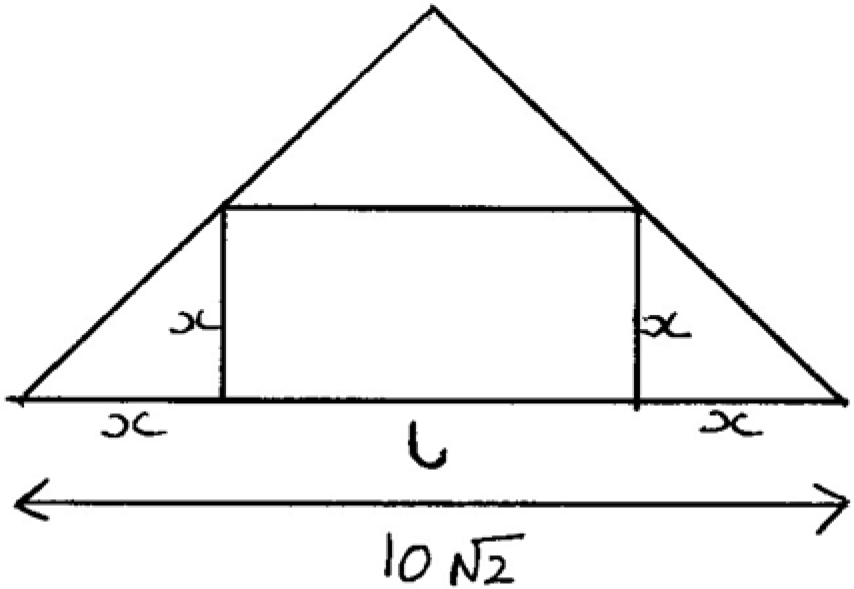
F

Solution

Main Grid

6) a) $\sqrt{10^2+10^2} = \sqrt{200} = \underline{\underline{10\sqrt{2}}}$

ii)



$l = 10\sqrt{2} - 2x$

$A = lb = x(10\sqrt{2} - 2x)$

$= \underline{\underline{10\sqrt{2}\,x - 2x^2}}$

More Solution

Main Grid

b) $$\frac{dA}{dx} = 10\sqrt{2} - 4x = 0$$

$$x = \frac{10\sqrt{2}}{4} = \underline{\underline{\frac{5\sqrt{2}}{2}}}$$

x	3	$\frac{5\sqrt{2}}{2}$	4
dA/dx	+	0	−
SHAPE	/	‾	\

$\therefore$ Max at $\underline{\underline{x = \frac{5\sqrt{2}}{2}}}$

$$l = 10\sqrt{2} - 2\left(\frac{5\sqrt{2}}{2}\right) = \underline{\underline{5\sqrt{2}}}$$

Dimensions are $\underline{\underline{5\sqrt{2} \text{ by } \frac{5\sqrt{2}}{2}}}$

7. Find the value of $\int_0^2 \sin(4x+1)\,dx$. **4**

F

Solution

Main Grid

7) $$\int_0^2 \sin(4x+1)\,dx = \left[-\frac{1}{4}\cos(4x+1)\right]_0^2$$

* CALCULATOR IN RADIANS *

$$= -\frac{1}{4}\cos(9) + \frac{1}{4}\cos(1)$$

$$= \frac{0.911}{4} + \frac{0.540}{4}$$

$$= 0.22775 + 0.135$$

$$= \underline{\underline{0.36275}}$$

Main Grid

8. The curve with equation $y = \log_3(x - 1) - 2{\cdot}2$, where $x > 1$, cuts the x-axis at the point $(a, 0)$.

Find the value of a. **4**

F

Solution

Main Grid

8) at $(a,0)$: $\log_3(a-1) - 2.2 = 0$

$$\log_3(a-1) - \log_3 3^{2.2} = \log_3 1$$

$$\log_3 \frac{a-1}{3^{2.2}} = \log_3 1$$

$$\frac{a-1}{3^{2.2}} = 1$$

$$a-1 = 3^{2.2}$$

$$a = 3^{2.2}+1$$

$$\underline{\underline{a = 12.21}}$$

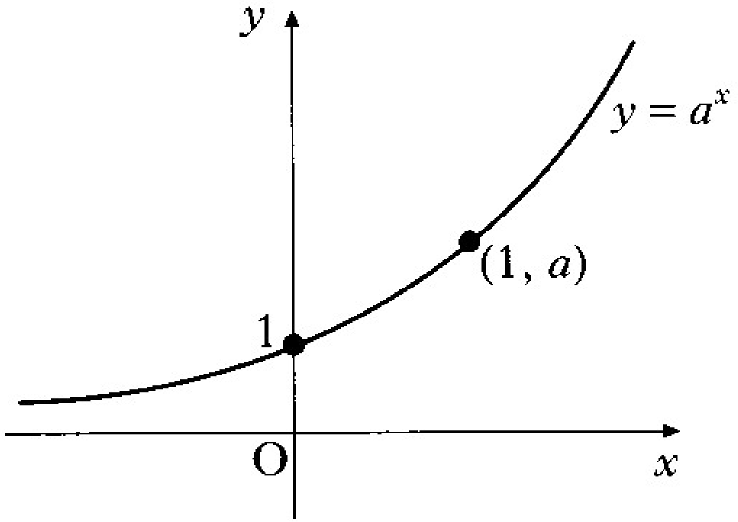
Main Grid

9. The diagram shows the graph of $y = a^x$, $a > 1$.

On separate diagrams, sketch the graphs of:

(*a*) $y = a^{-x}$; **2**

(*b*) $y = a^{1-x}$. **2**



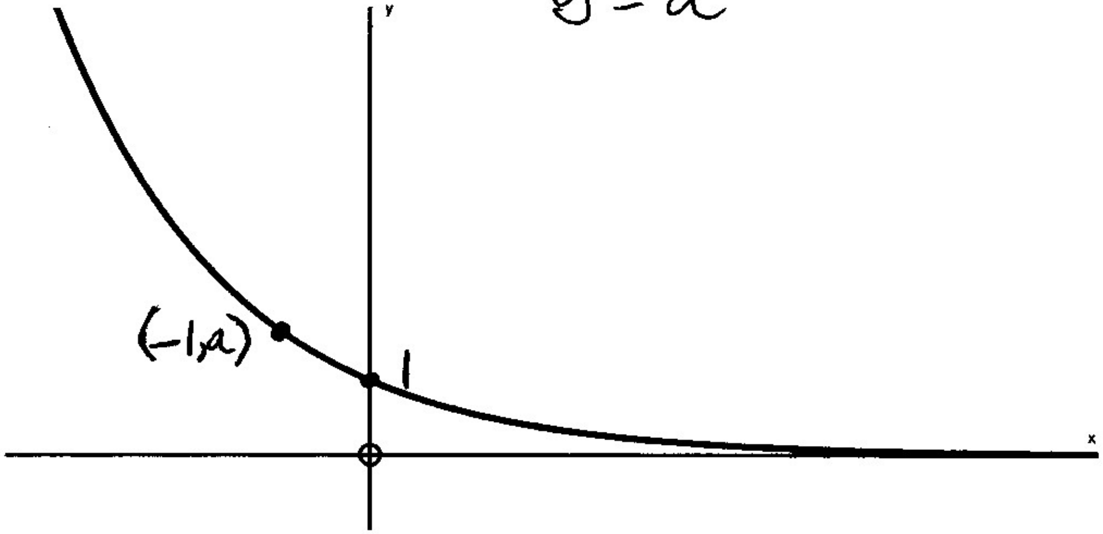
F

Solution

Main Grid

9) a)

$y = a^{-x}$



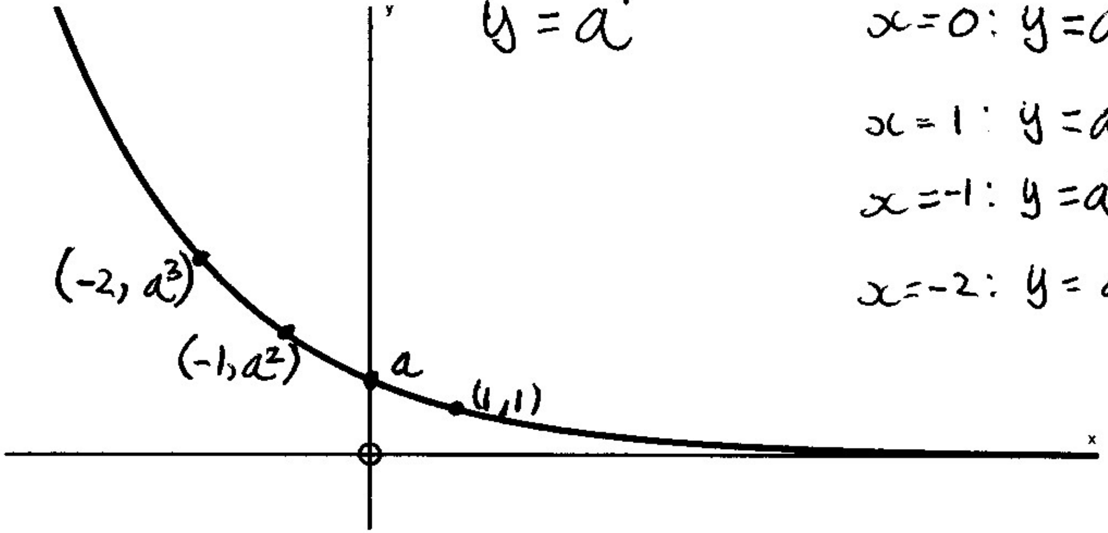
$y = a^{1-x}$

$x = 0: y = a^1 = a$

$x = 1: y = a^0 = 1$

$x = -1: y = a^2$

$x = -2: y = a^3$

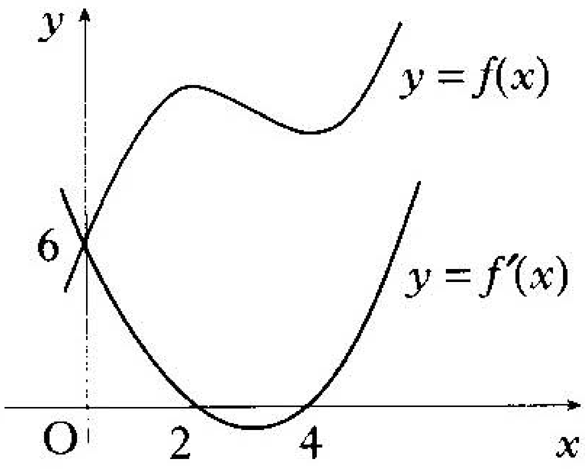


Main Grid

10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point $(0, 6)$.

The graph of $y = f'(x)$ also passes through the points $(2, 0)$ and $(4, 0)$.



(*a*) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:

(i) write down the values of a and b;

(ii) find the value of k. **3**

(*b*) Find the equation of the graph of the cubic function $y = f(x)$. **4**

F

Solution

Main Grid

10) a) i) $a = 2$ $\qquad b = 4$

ii) $f'(x) = k(x-2)(x-4)$

at $(0,6)$ $6 = k \times -2 \times -4$

$6 = 8k$

$\underline{\underline{k = 3/4}}$

More Solution

Main Grid

$$b) \quad f'(x) = \frac{3}{4}(x-2)(x-4)$$

$$= \frac{3}{4}(x^2 - 6x + 8)$$

$$= \frac{3}{4}x^2 - \frac{9}{2}x + 6$$

$$f(x) = \int \frac{3}{4}x^2 - \frac{9}{2}x + 6 \; dx$$

$$= \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + c$$

at $(0,6)$ $\quad \underline{\underline{c = 6}}$

$$\therefore \quad \underline{\underline{f(x) = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + 6}}$$

Main Grid

11. Two variables x and y satisfy the equation $y = 3 \times 4^x$.

(*a*) Find the value of a if $(a, 6)$ lies on the graph with equation $y = 3 \times 4^x$. **1**

(*b*) If $(-\frac{1}{2}, b)$ also lies on the graph, find b. **1**

(*c*) A graph is drawn of $\log_{10}y$ against x. Show that its equation will be of the form $\log_{10}y = Px + Q$ and state the gradient of this line. **4**

F

Solution

Main Grid

Main Grid

More Solution

a) $y = 3 \times 4^{x}$

$6 = 3 \times 4^{a}$

$2 = 4^{a}$

$4^{1/2} = 4^{a}$ $\quad \therefore \quad \underline{\underline{a = 1/2}}$

b) $b = 3 \times 4^{-1/2}$

$b = 3 \times \frac{1}{\sqrt{4}} = \underline{\underline{\frac{3}{2}}}$

c)

$$y = 3 \times 4^x$$

$$\log_{10} y = \log_{10}(3 \times 4^x)$$

$$\log_{10} y = \log_{10} 4^x + \log_{10} 3$$

$$\log_{10} y = x \log_{10} 4 + \log_{10} 3$$

$$\log_{10} y = 0.602x + 0.477$$

$$\therefore \quad \underline{\underline{M = 0.602}}$$

Main Grid

1. A sequence is defined by the recurrence relation

$$u_{n+1} = 0{\cdot}3u_n + 6 \text{ with } u_{10} = 10.$$

What is the value of u_{12}?

A 6·6

B 7·8

C 8·7

D 9·6

F

Solution

Main Grid

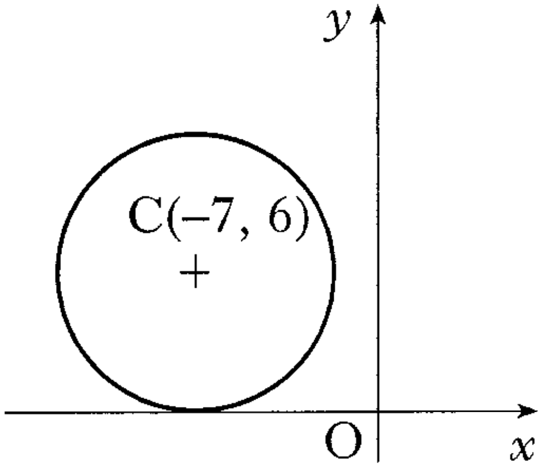
Q1 **C**

$$U_{11} = (0.3 \times 10) + 6 = 9$$

$$U_{12} = (0.3 \times 9) + 6 = 8.7$$

Main Grid

2. The x-axis is a tangent to a circle with centre (–7, 6) as shown in the diagram.



What is the equation of the circle?

A $(x + 7)^2 + (y - 6)^2 = 1$

B $(x + 7)^2 + (y - 6)^2 = 49$

C $(x - 7)^2 + (y + 6)^2 = 36$

D $(x + 7)^2 + (y - 6)^2 = 36$

F

Solution

Main Grid

Q2 **D**

$$(x - a)^2 + (y - b)^2 = r^2$$

Radius must be 6

$$(x + 7)^2 + (y - 6)^2 = 36$$

Main Grid

3. The vectors $\boldsymbol{u} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$ are perpendicular.

What is the value of k?

A 0

B 3

C 4

D 5

F

Solution

Main Grid

Q3 $\boxed{\textbf{C}}$

$u \bullet v = 0$ for perpendicular.

$(k \times 0) + (-1 \times 4) + (1 \times \mathrm{k}) = 0$

$-4 + k = 0$

$\underline{k = 4}$

Main Grid

4. A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n - 240$.

What is the limit of this sequence as $n \to \infty$?

A -800

B -400

C 200

D 400

F

Solution

Main Grid

Q4 $\boxed{\mathbf{B}}$

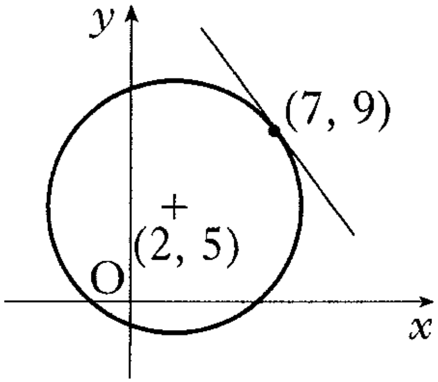
$$L=\frac{b}{1-a}$$

$$=\frac{-240}{1-0.4}=\frac{-240}{0.6}=\underline{\underline{-400}}$$

Main Grid

5. The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?



A $y - 9 = -\frac{5}{4}(x - 7)$

B $y + 9 = -\frac{4}{5}(x + 7)$

C $y - 7 = \frac{4}{5}(x - 9)$

D $y + 9 = \frac{5}{4}(x + 7)$

F

Solution

Main Grid

Q5 **A**

$$m_{radius} = \frac{9-5}{7-2} = \frac{4}{5}$$

$$m_{radius} \times m_{tangent} = -1 \qquad m_{tangent} = -\frac{5}{4}$$

$$y - b = m(x - a)$$

$$y - 9 = -\frac{5}{4}(x - 7)$$

Main Grid

6. What is the solution of the equation $2\sin x - \sqrt{3} = 0$ where $\frac{\pi}{2} \leq x \leq \pi$?

A $\frac{\pi}{6}$

B $\frac{2\pi}{3}$

C $\frac{3\pi}{4}$

D $\frac{5\pi}{6}$

F

Solution

Main Grid

Q6 $\boxed{\mathbf{B}}$

$$2\sin x - \sqrt{3} = 0$$

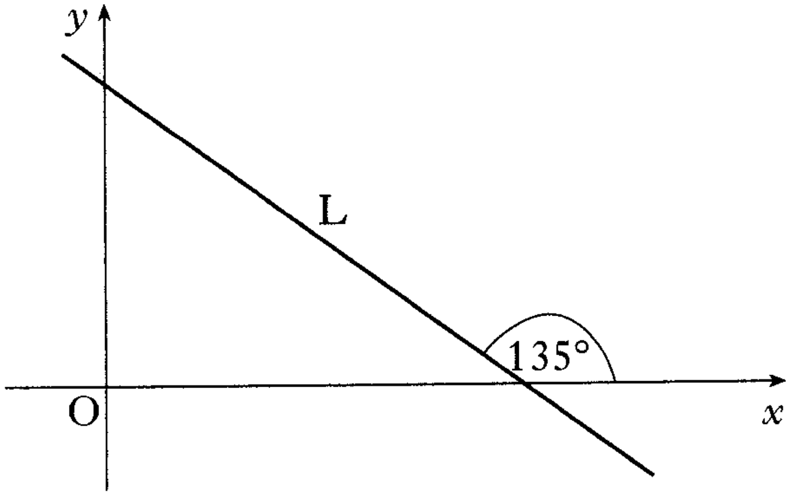
$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

2nd Quadrant $$x = \pi - \frac{\pi}{3} = \underline{\underline{\frac{2\pi}{3}}}$$

Main Grid

7. The diagram shows a line L; the angle between L and the positive direction of the x-axis is 135°, as shown.



What is the gradient of line L?

A $-\frac{1}{2}$

B $-\frac{\sqrt{3}}{2}$

C -1

D $\frac{1}{2}$

F

Solution

Main Grid

Q7

$$\boxed{\textbf{C}}$$

$$m = \tan\theta$$

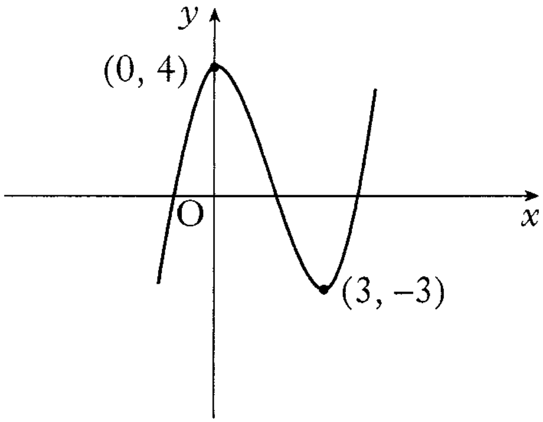
$$m = \tan 135^{o}$$

$$\tan 135^{o} = -\tan 45^{o} = \underline{\underline{-1}}$$

(Related angle of 135º is 45 º)

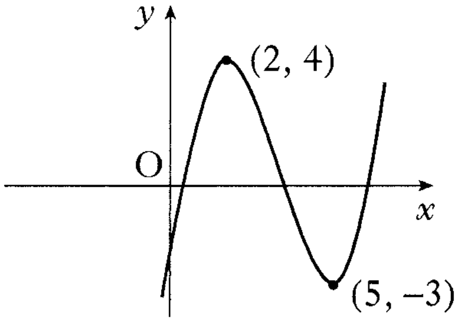
Main Grid

8. The diagram shows part of the graph of a function with equation $y = f(x)$.

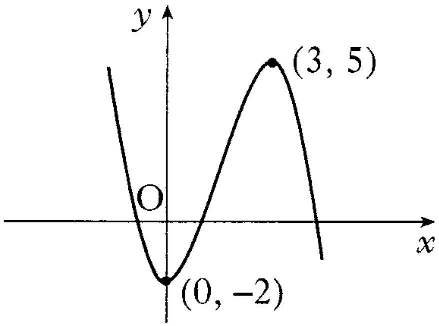


Which of the following diagrams shows the graph with equation $y = -f(x-2)$?

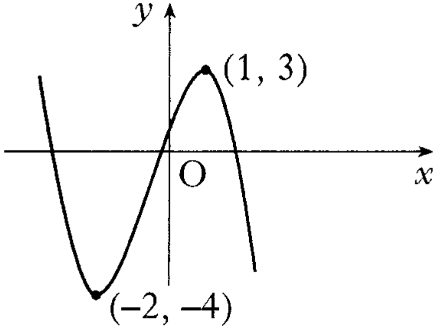
A



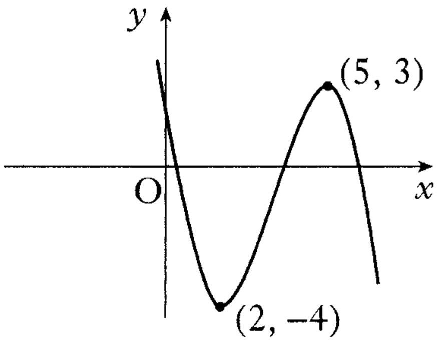
C



B



D



F

Solution

Main Grid

Q8 **D**

$-f$ means reflected in the x-axis.

$(x - 2)$ means moved 2 places to the left.

Main Grid

9. Given that $0 \leq a \leq \frac{\pi}{2}$ and $\sin a = \frac{3}{5}$, find an expression for $\sin(x + a)$.

A $\sin x + \frac{3}{5}$

B $\frac{4}{5}\sin x + \frac{3}{5}\cos x$

C $\frac{3}{5}\sin x - \frac{4}{5}\cos x$

D $\frac{2}{5}\sin x - \frac{3}{5}\cos x$

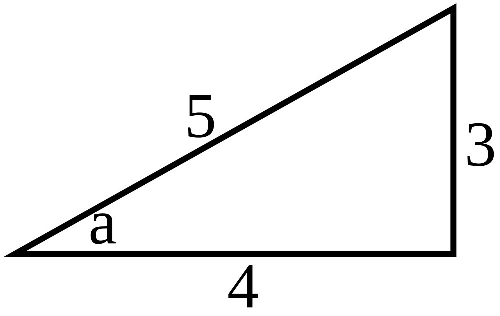
F

Solution

Main Grid

Q9

$$\boxed{\textbf{B}}$$



$$\sin a = \frac{3}{5} \qquad \cos a = \frac{4}{5}$$

$$\sin(x+a) = \sin x \cos a + \cos x \sin a$$

$$= \left(\sin x \times \frac{4}{5}\right) + \left(\cos x \times \frac{3}{5}\right)$$

$$= \underline{\underline{\frac{4}{5}\sin x + \frac{3}{5}\cos x}}$$

Main Grid

10. Here are two statements about the roots of the equation $x^2 + x + 1 = 0$:

(1) the roots are equal;

(2) the roots are real.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

F

Solution

Main Grid

Q10 **A**

$$a = 1 \quad b = 1 \quad c = 1$$

$$b^2 - 4ac = 1^2 - 4 \times 1 \times 1$$

$$\underline{\underline{= -3}}$$

$b^2 - 4ac \angle 0$ therefore no real roots.

Main Grid

11. E(–2, –1, 4), P(1, 5, 7) and F(7, 17, 13) are three collinear points.

P lies between E and F.

What is the ratio in which P divides EF?

A 1:1

B 1:2

C 1:4

D 1:6

F

Solution

Main Grid

Q11 **B**

–2 1 7

(3) (6)

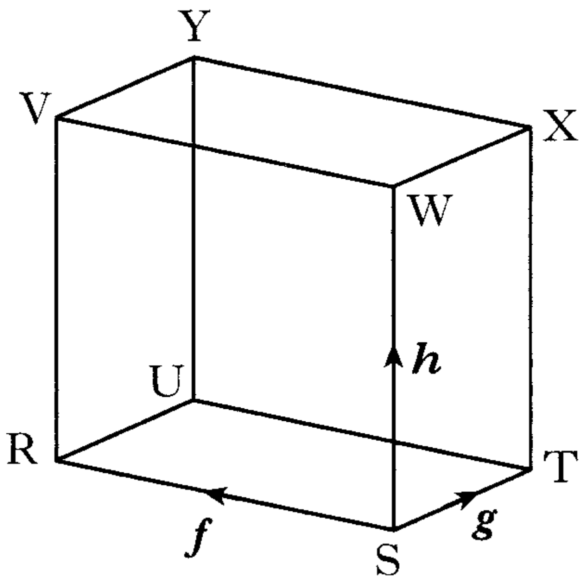
1:2

Main Grid

12. In the diagram RSTU, VWXY represents a cuboid.

$\overrightarrow{SR}$ represents vector $\boldsymbol{f}$, $\overrightarrow{ST}$ represents vector $\boldsymbol{g}$ and $\overrightarrow{SW}$ represents vector $\boldsymbol{h}$.

Express $\overrightarrow{VT}$ in terms of $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$.



A $\overrightarrow{VT} = \boldsymbol{f} + \boldsymbol{g} + \boldsymbol{h}$

B $\overrightarrow{VT} = \boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

C $\overrightarrow{VT} = -\boldsymbol{f} + \boldsymbol{g} - \boldsymbol{h}$

D $\overrightarrow{VT} = -\boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

F

Solution

Main Grid

Q12 $\boxed{\textbf{C}}$

$$\overrightarrow{VT} = \overrightarrow{VW} + \overrightarrow{WX} + \overrightarrow{XT}$$

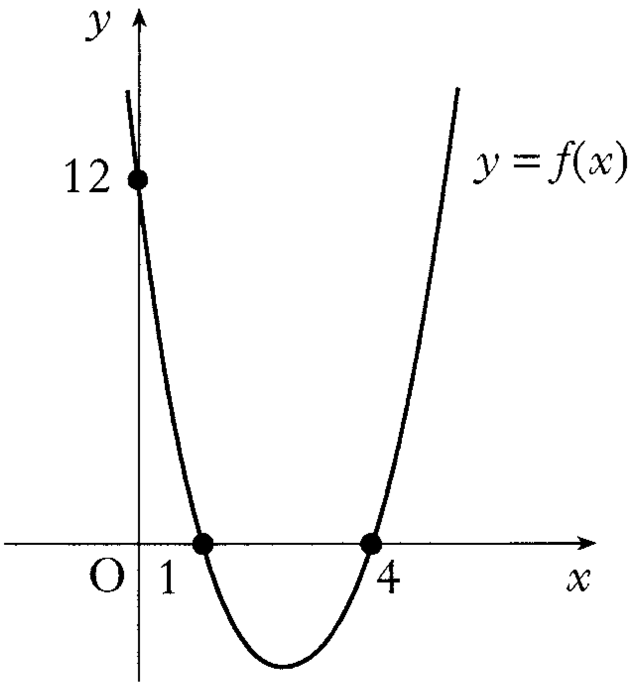
$$\overrightarrow{VT} = \underline{\underline{-f + g - h}}$$

Solution

Main Grid

13. The diagram shows part of the graph of a quadratic function $y = f(x)$.

The graph has an equation of the form $y = k(x - a)(x - b)$.



What is the equation of the graph?

A $y = 3(x - 1)(x - 4)$

B $y = 3(x + 1)(x + 4)$

C $y = 12(x - 1)(x - 4)$

D $y = 12(x + 1)(x + 4)$

F

Solution

Main Grid

Q13 **A**

$$y = k(x-a)(x-b)$$

$$y = k(x-1)(x-4)$$

$$\text{at } (0,12): 12 = k \times -1 \times -4$$

$$k = 3$$

$$\underline{\underline{y = 3(x-1)(x-4)}}$$

Main Grid

14. Find $\int 4\sin(2x+3)\,dx$.

A $-4\cos(2x+3)+c$

B $-2\cos(2x+3)+c$

C $4\cos(2x+3)+c$

D $8\cos(2x+3)+c$

F

Solution

Main Grid

Q14 $\boxed{\text{B}}$

$$\int 4\sin(2x+3)\,dx$$

$$=\frac{-4\cos(2x+3)}{2}+c$$

$$=\underline{\underline{-2\cos(2x+3)+c}}$$

Main Grid

15. What is the derivative of $(x^3 + 4)^2$?

A $(3x^2 + 4)^2$

B $\frac{1}{3}(x^3 + 4)^3$

C $6x^2(x^3 + 4)$

D $2(3x^2 + 4)^{-1}$

F

Solution

Main Grid

Q15 $\boxed{\textbf{C}}$

$$y = (x^3 + 4)^2$$

$$\frac{dy}{dx} = 2(x^3 + 4) \times 3x^2$$

$$\frac{dy}{dx} = \underline{\underline{6x^2(x^3 + 4)}}$$

Main Grid

16. $2x^2 + 4x + 7$ is expressed in the form $2(x + p)^2 + q$.

What is the value of q?

A 5

B 7

C 9

D 11

Q16 $\boxed{\textbf{A}}$

$$2x^2+4x+7=2(x+p)^2+2$$

$$2x^2+4x+7=2(x^2+2px+p^2)+q$$

$$2x^2+4x+7=2x^2+4px+2p^2+q$$

Compare coefficients

$$4p=4 \qquad\qquad 2p^2+q=7$$

$$p=1 \qquad\qquad 2+q=7$$

$$\underline{\underline{q=5}}$$

Main Grid

17. A function f is given by $f(x) = \sqrt{9 - x^2}$.

What is a suitable domain of f?

A $x \geq 3$

B $x \leq 3$

C $-3 \leq x \leq 3$

D $-9 \leq x \leq 9$

F

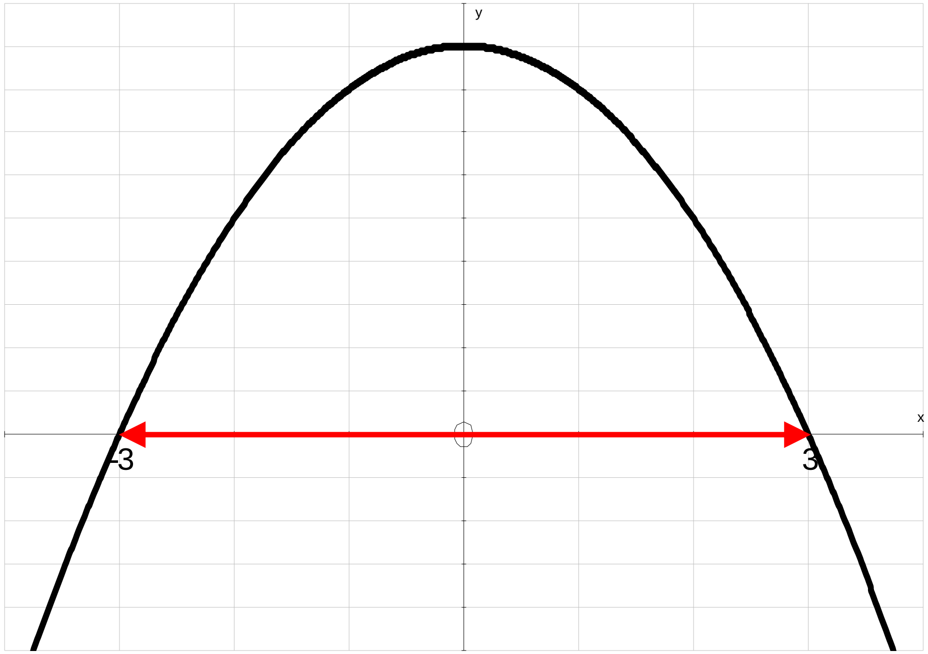
Solution

Main Grid

Q17 $\boxed{C}$

You can't square root a negative number. So:

$$9 - x^2 \geq 0$$



$$-3 \leq x \leq 3$$

Main Grid

18. Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $|\boldsymbol{p}| = 3$, $|\boldsymbol{q}| = 4$ and $\boldsymbol{p}.\boldsymbol{q} = 10$.

Find the value of $\boldsymbol{q}.(\boldsymbol{p} + \boldsymbol{q})$.

A 0

B 14

C 26

D 28

F

Solution

Main Grid

Q18 $\boxed{\boxed{\textbf{C}}}$

$$q.(p+q)$$

$$=q.p+q.q$$

$$=q.p+|q|^2$$

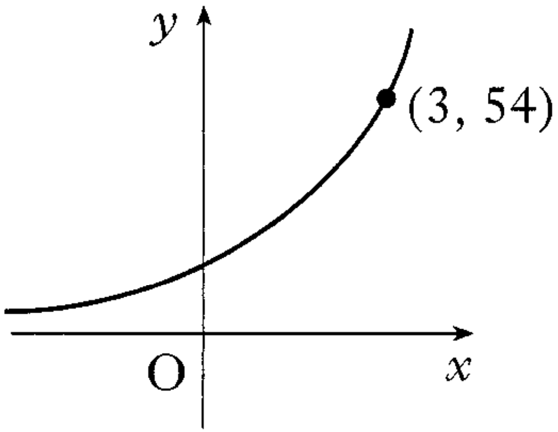
$$=10+16$$

$$=\underline{\underline{26}}$$

Main Grid

19. The diagram shows part of the graph whose equation is of the form $y = 2m^x$.

What is the value of m?



A 2

B 3

C 8

D 18

F

Solution

Main Grid

Q19 **B**

$$y = 2m^x$$

$$\text{at } (3,54): \quad 54 = 2m^3$$

$$m^3 = 27$$

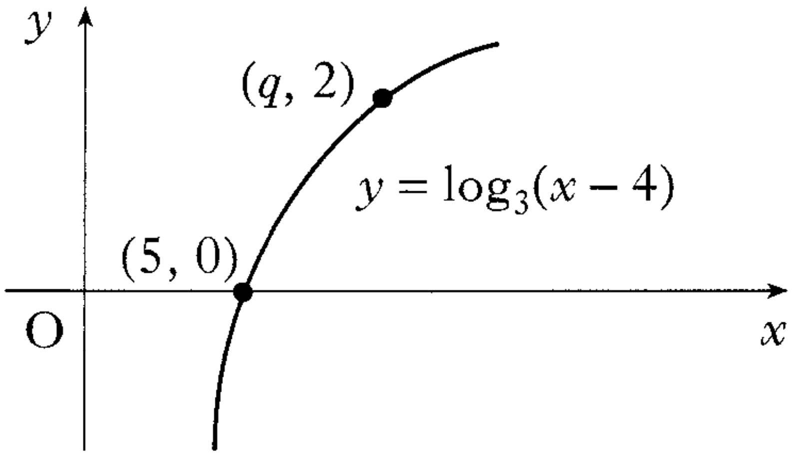
$$m = \sqrt[3]{27}$$

$$\underline{\underline{m = 3}}$$

Main Grid

20. The diagram shows part of the graph of $y = \log_3(x - 4)$.

The point $(q, 2)$ lies on the graph.



What is the value of q?

A 6

B 7

C 8

D 13

F

Solution

Main Grid

Q20 $\boxed{\textbf{D}}$

$$y = \log_3(x-4)$$

at $(q,2)$:

$$2 = \log_3(q-4)$$

$$\log_3 9 = \log_3(q-4)$$

$$9 = q-4$$

$$\underline{\underline{q = 13}}$$

Main Grid

21. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

(*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. **6**

(*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. **5**

(*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. **4**

F

Solution 21a

Solution 21b

Solution 21c

Main Grid

Q21 a

Stationary points at $f'(x) = 0$

$$f'(x) = 3x^2 - 3 = 0$$

$$x^2 = 1$$

$$\underline{\underline{x = \pm 1}}$$

$$f(1) = 1 - 3 + 3 = 0 \qquad \underline{\underline{(1,0)}}$$

$$f(-1) = -1 + 3 + 2 = 4 \qquad \underline{\underline{(-1,4)}}$$

x	-2	-1	0	0	1	2
dy/dx	+	0	–	–	0	+
Shape	╱	‾	╲	╲	_	╱
	Max at (-1,4)			Min at (1,0)		

Solution 21b

Solution 21c

Main Grid

Q21b

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \underline{0} \end{array}$$

Remainder = 0,
Therefore (x – 1)
is a factor.

$$(x-1)(x^2+x-2)$$

$$\underline{\underline{(x-1)(x-1)(x+2)}}$$

Solution 21c

Main Grid

Q21c

$$\text{Cuts } x \text{ at } y = 0 : x^3 - 3x + 2 = 0$$

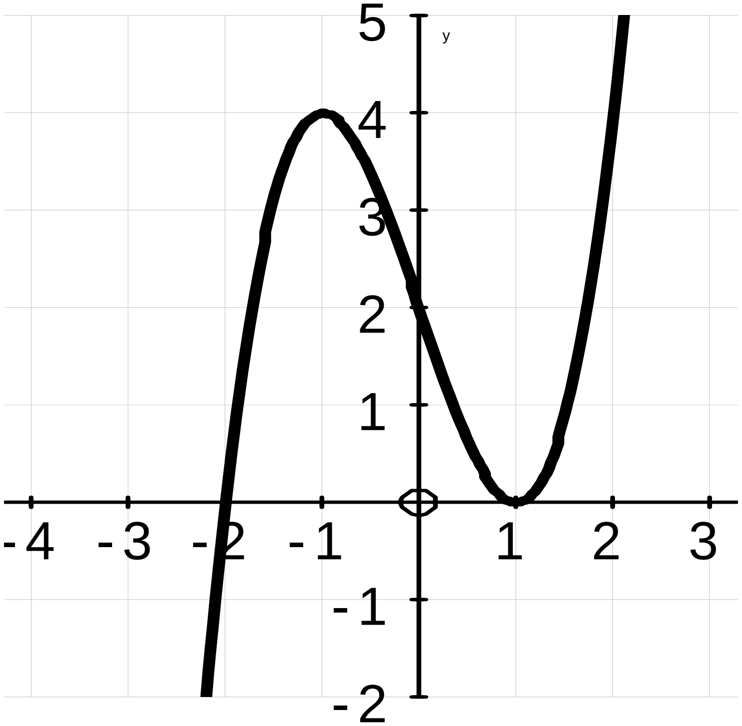
$$(x-1)(x-1)(x+2) = 0$$

$$x = 1 \qquad x = -2$$

$$\underline{\underline{(1,0)}} \qquad \underline{\underline{(-2,0)}}$$

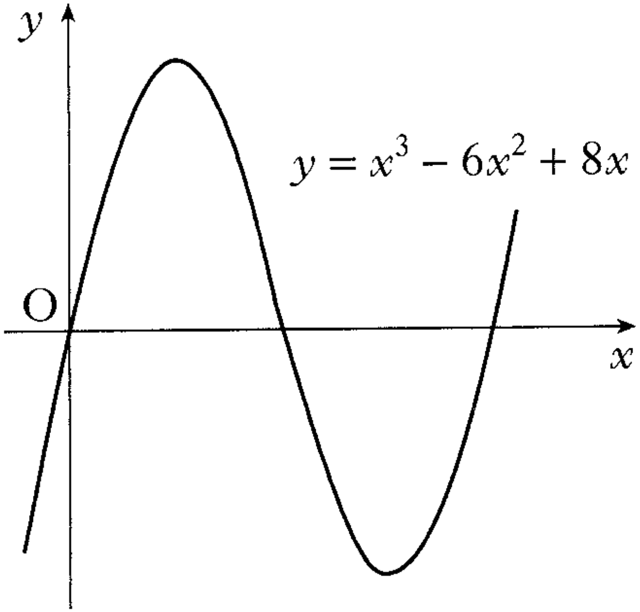
$$\text{Cuts } y \text{ at } x = 0 : 0^3 - 3 \times 0 + 2 = 2$$

$$\underline{\underline{(0,2)}}$$



Main Grid

22. The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.



(*a*) Find the coordinates of the points on the curve where the gradient of the tangent is -1. **5**

(*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. **2**

F

Solution

Main Grid

Q22a

$$\frac{dy}{dx} = -1: \quad \frac{dy}{dx} = 3x^2 - 12x + 8 = -1$$

$$3x^2 - 12x + 9 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$x = 1 \quad x = 3$$

$$\text{at } x = 1: y = 1^3 - (6\times1^2) + (8\times1) = 3 \qquad \underline{\underline{(1,3)}}$$

$$\text{at } x = 3: y = 3^3 - (6\times3^2) + (8\times3) = -3 \qquad \underline{\underline{(3,-3)}}$$

Q22b

$$\text{Try } (1,3): y = 4 - x$$

$$y = 4 - 1 = 3 \quad \text{Therefore A is (1,3).}$$

Main Grid

23. Functions f, g and h are defined on suitable domains by

$$f(x) = x^2 - x + 10,\ g(x) = 5 - x \text{ and } h(x) = \log_2 x.$$

(*a*) Find expressions for $h(f(x))$ and $h(g(x))$. **3**

(*b*) Hence solve $h(f(x)) - h(g(x)) = 3$. **5**

F

Solution

Main Grid

Q23a

$$h(f(x)) = \log_2(x^2 - x + 10)$$

$$h(g(x)) = \log_2(5 - x)$$

Q23b

$$h(f(x)) - h(g(x)) = 3$$

$$\log_2(x^2 - x + 10) - \log_2(5 - x) = 3$$

$$\log_2 \frac{x^2 - x + 10}{5 - x} = \log_2 8 \qquad \left[2^3 = 8\right]$$

$$\frac{x^2 - x + 10}{5 - x} = 8$$

$$x^2 - x + 10 = 8(5 - x)$$

$$x^2 - x + 10 = 40 - 8$$

$$x^2 + 7x - 30 = 0$$

$$(x - 3)(x - 10) = 0$$

$$\underline{\underline{x = 3}} \qquad \underline{\underline{x = -10}}$$

Main Grid

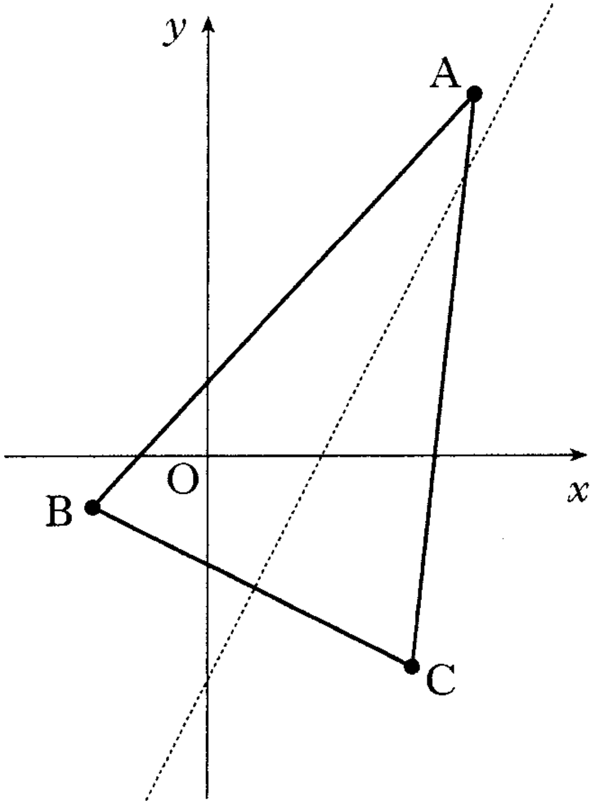
1. The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.

The broken line represents the perpendicular bisector of BC.

(*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. **4**

(*b*) Find the equation of the median from C. **3**

(*c*) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. **3**



F

Solution 1a

Solution 1b

Solution 1c

Main Grid

Q1a

$$\text{Mid-point of BC}:\left(\frac{5-3}{2},\frac{-5-1}{2}\right)=(1,-3)$$

$$m_{BC}=\frac{-5+1}{5+3}=\frac{-4}{8}=-\frac{1}{2}\qquad m_1m_2=-1\qquad \therefore \underline{m=2}$$

$$y-b=m(x-a)\qquad m=2\qquad (a,b)=(1,-3)$$

$$y+3=2(x-1)$$

$$y+3=2x-2$$

$$\underline{\underline{y=2x-5}}$$

Solution 1b

Solution 1c

Main Grid

Q1b

$$\text{Mid-point of AB}:\left(\frac{7-3}{2},\frac{9-1}{2}\right)=(2,4)$$

$$m_{median}=\frac{4+5}{2-5}=\frac{9}{-3}=-3$$

$$y-b=m(x-a) \qquad m=-3 \qquad (a,b)=(2,4)$$

$$y-4=-3(x-2)$$

$$y-4=-3x+6$$

$$\underline{\underline{y=-3x+10}}$$

Solution 1c

Main Grid

Q1c

$$y = 2x - 5 \ldots\ldots\ldots\ldots(1)$$

$$\underline{y = -3x + 10}\ldots\ldots\ldots(2)$$

$$(1)-(2) \qquad 0 = 5x - 15$$

$$\underline{x = 3}$$

$$y = 2 \times 3 - 5$$

$$\underline{y = 1}$$

$$\text{Point of intersection is } \underline{\underline{(3,1)}}$$

Main Grid

2. The diagram shows a cuboid OABC, DEFG.

F is the point (8, 4, 6).

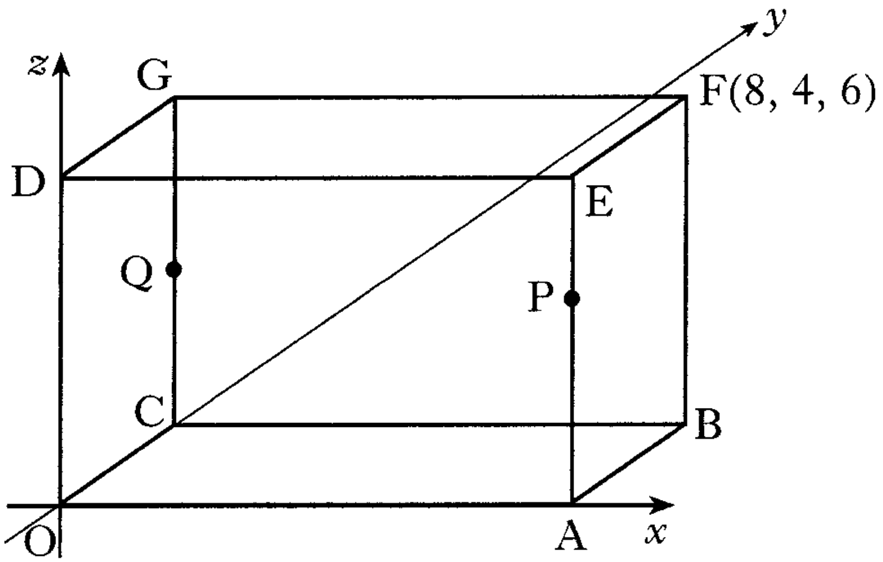
P divides AE in the ratio 2:1.

Q is the midpoint of CG.

(*a*) State the coordinates of P and Q. **2**

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$. **2**

(*c*) Find the size of angle QPA. **5**



F

Solution 2ab

Solution 2c

Main Grid

Q2a

P(8,0,4) Q(0,4,3)

Q2b

$$\overrightarrow{PQ} = q - p = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -8 \\ 4 \\ -1 \end{pmatrix}}}$$

A (8,0,0)

$$\overrightarrow{PA} = a - p = \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}}}$$

Solution 2c

Main Grid

Q2c

To find angle QPA we must use $\overrightarrow{\mathrm{PQ}}$ and $\overrightarrow{\mathrm{PA}}$

$$\cos A = \frac{\overrightarrow{PQ}.\overrightarrow{PA}}{\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PA}\right|}$$

$$\cos A = \frac{(-8\times 0)+(4\times 0)+(-1\times -4)}{\sqrt{64+16+1}\times\sqrt{0+0+16}}$$

$$\cos A = \frac{4}{9\times 4} = \frac{4}{36} = \frac{1}{9}$$

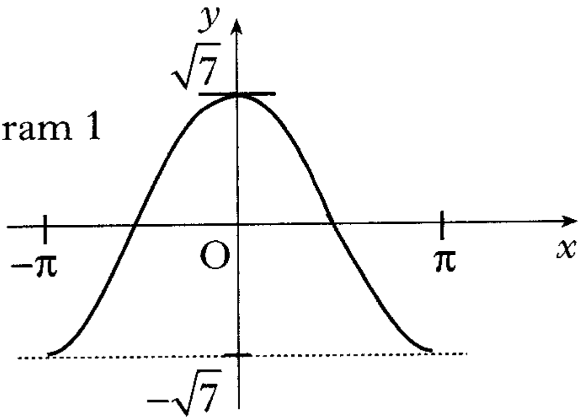
$$A = \cos^{-1}\left(\frac{1}{9}\right) = \underline{\underline{83.6^o}}$$

Main Grid

3. (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

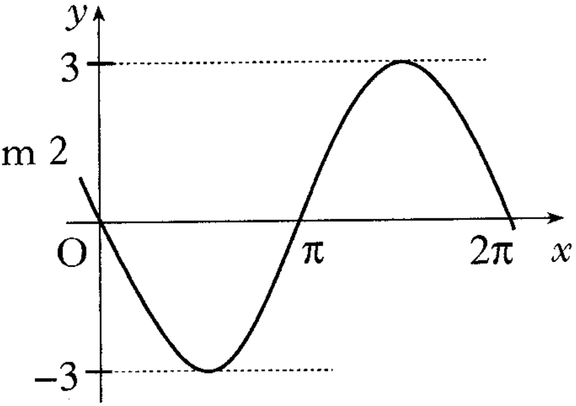
Diagram 1



(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q.

Diagram 2



2

(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. **2**

F

Solution 3ab **Solution 3c** **Main Grid**

Q3a i) $\sqrt{7}$ ii) $q=-3$

Q3b

$$\sqrt{7}\cos x-3\sin x=k\cos(x+a)$$

$$\sqrt{7}\cos x-3\sin x=k\cos x\cos a-k\sin x\sin a$$

comparing coefficients : $k\cos a=\sqrt{7}$ $\quad k\sin a=3$

squaring and adding :

$$k^2\cos^2 a+k^2\sin^2 a=\sqrt{7}^2+3^2$$

$$k^2(\cos^2 a+\sin^2 a)=7+9 \qquad *\cos^2 a+\sin^2 a=1*$$

$$k^2=16$$

$$\underline{\underline{k=4}}$$

$$\frac{k\sin a}{k\cos a}=\tan a=\frac{3}{\sqrt{7}}$$

sin is positive, cos is positive, 1st quadrant.

$$a=\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)=\underline{\underline{0.848\,rads}}$$

$$\underline{\underline{\sqrt{7}\cos x-3\sin x=4\cos(x+0.848)}}$$

Solution 3c

Main Grid

Q3c

$$\text{Differentiate } 4\cos(x + 0.848)$$

$$= -4\sin(x + 0.848) \times 1$$

$$= \underline{\underline{-4\sin(x + 0.848)}}$$

Main Grid

4. (*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. **2**

(*b*) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect. **4**

(*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.

Find the coordinates of the points of intersection of the two circles. **5**

F

Solution 4ab

Solution 4c

Main Grid

Q4a

$$2g = 8 \qquad 2f = 4$$

$$\underline{g = 4} \qquad \underline{f = 2}$$

$$C_1 = (-g, -f) = \underline{\underline{(-4, -2)}}$$

$$r_1 = \sqrt{g^2 + f^2 - c} = \sqrt{(-4)^2 + (-2)^2 + 38} = \sqrt{58} = \underline{\underline{7.6}}$$

Q4b

$$C_2 = (4,6)$$

$$dist = \sqrt{(4+4)^2 + (6+2)^2} = \sqrt{128} = \underline{11.3}$$

$$r_1 + r_2 = 7.6 + \sqrt{26} = \underline{12.7}$$

$$\underline{\underline{\text{Since } 11.3 \angle 12.7 \text{ the circles must intersect}}}$$

Solution 4c **Main Grid**

Q4c Substitute line into circle

$$x^2+(4-x)^2+8x+4(4-x)-38=0$$

$$x^2+16-8x+x^2+8x+16-4x-38=0$$

$$2x^2-4x-6=0$$

$$x^2-2x-3=0$$

$$(x-3)(x+1)=0$$

$$\underline{x=3}\quad \underline{x=-1}$$

Substitute x values into line equation

$$y=4-3=1 \qquad y=4+1=5$$

$$\underline{\underline{(3,1)}} \qquad \underline{\underline{(-1,5)}}$$

Main Grid

5. Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \leq x < 360$. **5**

F

Solution

Main Grid

Q5

$$\cos 2x^o + 2\sin x^o = \sin^2 x^o$$

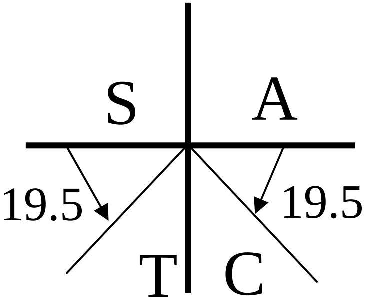
$$1 - 2\sin^2 x^o + 2\sin x^o = \sin^2 x^o$$

$$3\sin^2 x^o - 2\sin x^o - 1 = 0$$

$$(3\sin x^o + 1)(\sin x^o - 1) = 0$$

$$\sin x^o = -\frac{1}{3} \qquad\qquad \sin x^o = 1$$

$$RA = \sin^{-1}\left(\frac{1}{3}\right) = 19.5^o \qquad\qquad \underline{x^o = 90^o}$$



$$x^o = 180^o + 19.5^o = \underline{199.5^o}$$

$$x^o = 360^o - 19.5^o = \underline{340.5^o}$$

$$\underline{\underline{x^o = 90^o,\ 199.5^o,\ 340.5^o}}$$

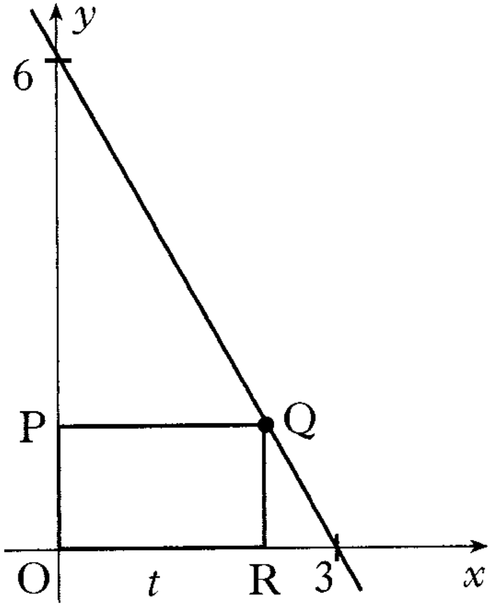
Main Grid

6. In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.

(*a*) Show that QR = $6 - 2t$. **3**

(*b*) Find the coordinates of Q for which the rectangle has a maximum area. **6**



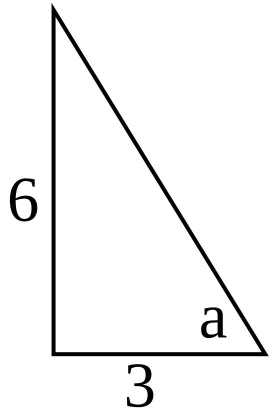
F

Solution 6a

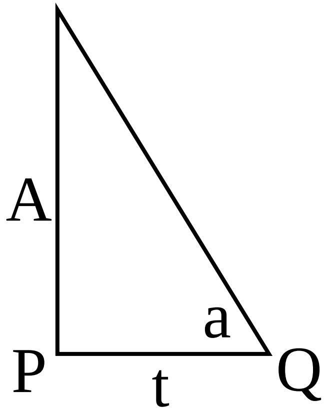
Solution 6b

Main Grid

Q6a



$$\tan a = \frac{6}{3} = 2$$



$$\tan a = \frac{A}{t}$$

$$A = t \tan a$$

$$A = 2t$$

$$\therefore \quad \underline{\underline{QR = 6 - 2t}}$$

Solution 6b

Main Grid

Q6b

$$A = t(6-2t) = \underline{6t-2t^2}$$

$$\text{Maximum at } \frac{dA}{dt} = 0$$

$$\frac{dA}{dt} = 6-4t = 0$$

$$t = \frac{6}{4} = \underline{\underline{\frac{3}{2}}}$$

t	1	3/2	2
dA/dt	+	0	–
Shape	╱	—	╲
	Max at t = 3/2		

$$y = -2x+6$$

$$\text{at } x = \frac{3}{2}: \quad y = -2\times\frac{3}{2}+6 = 3 \quad \therefore \text{ Max area at } Q = \underline{\underline{\left(\frac{3}{2}, 3\right)}}$$

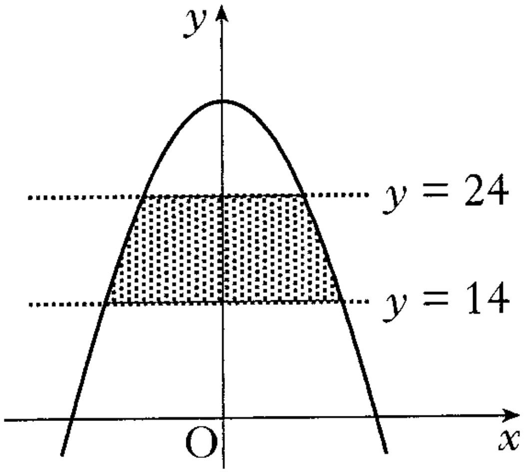
Main Grid

7. The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area.



8

F

Solution

Main Grid

Q7

Find the points of intersection.

$$32-2x^2=14 \qquad\qquad 32-2x^2=24$$

$$2x^2=18 \qquad\qquad 2x^2=8$$

$$x^2=9 \qquad\qquad x^2=4$$

$$\underline{\underline{x=\pm3}} \qquad\qquad \underline{\underline{x=\pm2}}$$

Because of symmetry integrate half the shaded area and then double your answer.

More Solution

Main Grid

Q7 contd

$$\int_0^3 32-2x^2-14dx=\int_0^3 18-2x^2 dx$$

$$=\left[18x-\frac{2x^3}{3}\right]_0^3=\left[(18\times 3)-\left(\frac{2\times 3^3}{3}\right)\right]-0=54-18=\underline{\underline{36}}$$

$$\int_0^2 32-2x^2-24dx=\int_0^2 8-2x^2 dx$$

$$=\left[8x-\frac{2x^3}{3}\right]_0^2=\left[(8\times 2)-\left(\frac{2\times 2^3}{3}\right)\right]-0=16-5.3=\underline{\underline{10.7}}$$

$$36-10.7=25.3$$

$$\text{Area}=2\times 25.3=\underline{\underline{50.6\text{ units}^2}}$$

Main Grid

Main Grid