Higher Maths Past Papers

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1 Introduction

It is hoped that this document will make it easier to find past paper questions, both for teachers and students. I will do my best to keep this document up to date and include new past paper questions as they become available. If you spot any mistakes, or want to suggest any improvements, send me an email at john.macbeath@wickhigh.org.uk. I am more than happy to send you the Tex file used to produce the document so that you can modify it as you wish.

Sincere thanks are given to Mr. C. Davie at Glenrothes High School for his extensive assistance in producing this document.

2 How to Use

The table on the next page contains links to questions sorted by topic and year. Clicking on a link will take you to the page of that question. The marking instructions follow directly after each question or section of questions where more than one question is on a page. To return to the table click on **Back to Table** at the top or bottom of any page. Trying to navigate the document without doing this is tedious.

Remember - Paper 1 questions are non-calculator, while Paper 2 questions are calculator questions. Note also that Specimen Papers (SP) are put together from previous past paper questions and so it is possible you may see the same question twice.

Before starting any past paper questions I recommend that you **print** a paper copy of the **Higher Maths Sheet** to avoid wasting time. A copy of the Formula Sheet is available on page 3.

Topic	18 P1	18 P2	17 P1	17 P2	16 P1	16 P2	15 P1	15 P2	SP P1	SP P2
Circles	4	5c, 12	2	3, 10	4, 8	4	11, 14	5	2, 7	12b
Differentiation & Optimisation		3,9	8, 15c	4, 7	2, 9	7	2, 7	8	11	6b, 6c, 9
Exp's & Logs	6,11	11	12	9	14	6	6		10	13
Functions & Graphs	2,11,15	6	1,6,15a		6,10,12		4,5,13	2	15	3, 7
Further Calculus	3,14		3,13		5	10,11b		7a		
Integration	10	1	10, 15b			3b, 9	12, 15	4	4, 13	2,5,10
Polynomials & Quadratics	7	4, 7a, 10	4	2	15	2, 3a	3, 8		6, 8	6a
Recurrence Relations		7b, 7c	9	8	3			3	9	
Straight Lines	1, 8	5a, 5b	7, 11	1	1	1	9	1	1, 3	1
Trig. Formulae & Equations	13			6,11	13	8b, 11a	10	7b	12, 14	8, 11
Vectors	5, 9 ,12	2	5	5	7,11	5	1	6	5	4, 12a
Wave Function		8	14			8a		9		

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product:

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(\mathrm{A} \pm \mathrm{B}) = \sin\mathrm{A}\cos\mathrm{B} \pm \cos\mathrm{A}\sin\mathrm{B}$$

$$\cos(\mathrm{A} \pm \mathrm{B}) = \cos\mathrm{A}\cos\mathrm{B} \mp \sin\mathrm{A}\sin\mathrm{B}$$

$$\sin 2\mathrm{A} = 2\sin\mathrm{A}\cos\mathrm{A}$$

$$\begin{aligned} \cos 2\mathrm{A} &= \cos^2\mathrm{A} - \sin^2\mathrm{A} \\ &= 2\cos^2\mathrm{A} - 1 \\ &= 1 - 2\sin^2\mathrm{A} \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

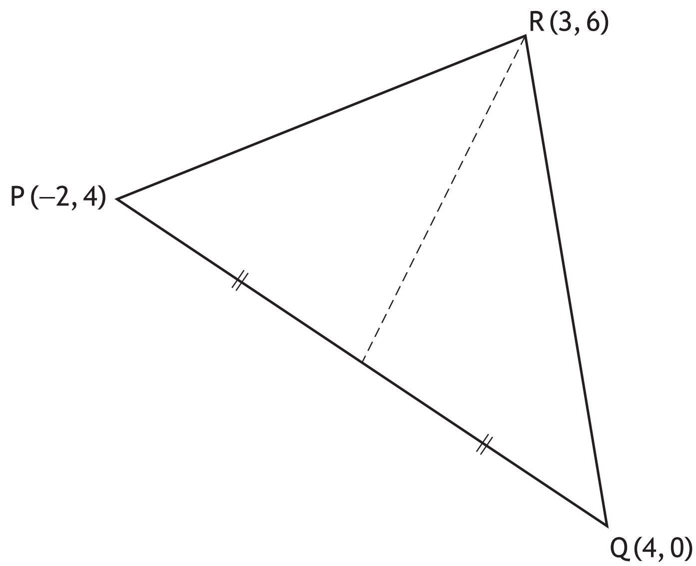
Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Attempt ALL questions **MARKS**

Total marks — 60

1. PQR is a triangle with vertices P $(-2, 4)$, Q $(4, 0)$ and R $(3, 6)$.



Find the equation of the median through R. **3**

2. A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by

$$g(x) = \frac{1}{5}x - 4.$$

Find the inverse function, $g^{-1}(x)$. **3**

3. Given $h(x) = 3\cos 2x$, find the value of $h'\left(\frac{\pi}{6}\right)$. **3**

[Turn over

Detailed marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	•[1] find mid-point of PQ •[2] find gradient of median •[3] determine equation of median	•[1] $(1,2)$ •[2] 2 •[3] $y=2x$	**3**

Notes:

1. •[2] is only available to candidates who use a midpoint to find a gradient.
2. •[3] is only available as a consequence of using the mid-point and the point R, or any other point which lies on the median, eg $(2,4)$.
3. At •[3] accept any arrangement of a candidate's equation where constant terms have been simplified.
4. •[3] is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate A - Perpendicular Bisector of PQ $M_{PQ}(1,2)$ •[1] ✓ $m_{PQ}=-\frac{2}{3}\Rightarrow m_{\perp}=\frac{3}{2}$ •[2] ✗ $2y=3x+1$ •[3] $\boxed{\checkmark 2}$ For other perpendicular bisectors award 0/3	**Candidate B - Altitude through R** $m_{PQ}=-\frac{2}{3}$ •[1] ^ $m_{\perp}=\frac{3}{2}$ •[2] ✗ $2y=3x+3$ •[3] $\boxed{\checkmark 2}$
Candidate C - Median through P $M_{QR}(3\cdot5,3)$ •[1] ✗ $m_{PM}=-\frac{2}{11}$ •[2] $\boxed{\checkmark 1}$ $11y+2x=40$ •[3] $\boxed{\checkmark 2}$	**Candidate D - Median through Q** $M_{PR}(0\cdot5,5)$ •[1] ✗ $m_{QM}=-\frac{10}{7}$ •[2] $\boxed{\checkmark 1}$ $7y+10x=40$ •[3] $\boxed{\checkmark 2}$

Question	Generic scheme	Illustrative scheme	Max mark
2.	**Method 1**	**Method 1**	3
	$\bullet^1$ equate composite function to x	$\bullet^1$ $g\left(g^{-1}(x)\right)=x$	
	$\bullet^2$ write $g(g^{-1}(x))$ in terms of $g^{-1}(x)$	$\bullet^2$ $\frac{1}{5}g^{-1}(x)-4=x$	
	$\bullet^3$ state inverse function	$\bullet^3$ $g^{-1}(x)=5(x+4)$	
	Method 2	**Method 2**	
	$\bullet^1$ write as $y=\frac{1}{5}x-4$ and start to rearrange	$\bullet^1$ $y+4=\frac{1}{5}x$	
	$\bullet^2$ express x in terms of y	$\bullet^2$ eg $x=5(y+4)$ or $x=\frac{(y+4)}{\frac{1}{5}}$	
	$\bullet^3$ state inverse function	$\bullet^3$ $g^{-1}(x)=5(x+4)$	
	Method 3	**Method 3**	
	$\bullet^1$ interchange variables	$\bullet^1$ $x=\frac{1}{5}y-4$	
	$\bullet^2$ express y in terms of x	$\bullet^2$ eg $y=5(x+4)$ or $y=\frac{(x+4)}{\frac{1}{5}}$	
	$\bullet^3$ state inverse function	$\bullet^3$ $g^{-1}(x)=5(x+4)$	

Notes:

1. $y=5(x+4)$ does not gain $\bullet^3$.
2. At $\bullet^3$ stage, accept g^{-1} written in terms of any dummy variable eg $g^{-1}(y)=5(y+4)$.
3. $g^{-1}(x)=5(x+4)$ with no working gains 3/3.

Commonly Observed Responses:

Candidate A

$x \rightarrow \frac{1}{5}x \rightarrow \frac{1}{5}x-4=g(x)$

$\div 5 \rightarrow -4$

$\therefore\ +4 \rightarrow \times 5$ — $\bullet^1$ ✓ awarded for knowing to perform inverse operations in reverse order

$5(x+4)$ — $\bullet^2$ ✓

$g^{-1}(x)=5(x+4)$ — $\bullet^3$ ✓

Candidate B - BEWARE	Candidate C
$g'(x)=\ldots$ $\quad\bullet^3$ ✘	$g^{-1}(x)=5x+4$ $\quad$ with no working Award 0/3

Question	Generic scheme	Illustrative scheme	Max mark
3.	•¹ start to differentiate •² complete differentiation •³ evaluate derivative	•¹ $-3\sin 2x\ldots$ stated or implied by •² •² $\ldots\times 2$ •³ $-3\sqrt{3}$	**3**

Notes:

1. Ignore the appearance of $+c$ at any stage.
2. •³ is available for evaluating an attempt at finding the derivative at $\frac{\pi}{6}$.
3. For $h'\left(\frac{\pi}{6}\right)=3\cos\left(2\times\frac{\pi}{6}\right)=\frac{3}{2}$ award 0/3.

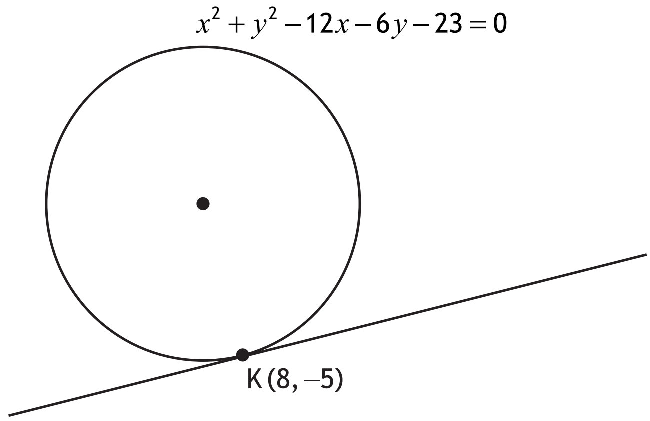
Commonly Observed Responses:

Candidate A		**Candidate B**		**Candidate C**	
$-3\sin 2x\ldots$	•¹ ✓	$3\sin 2x\ldots$	•¹ ✗	$3\sin 2x\ldots$	•¹ ✗
$\ldots\times\frac{1}{2}$	•² ✗	$\ldots\times 2$	•² ✓	$\ldots\times\frac{1}{2}$	•² ✗
$-\frac{3\sqrt{3}}{4}$	•³ ✓1	$3\sqrt{3}$	•³ ✓1	$\frac{3\sqrt{3}}{4}$	•³ ✓1

Candidate D		**Candidate E**		**Candidate F**	
$\pm 6\cos 2x$	•¹ ✗	$\pm 3\cos 2x\ldots$	•¹ ✗	$6\sin 2x$	•¹ ✗
	•² ✗	$\ldots\times 2$	•² ✓1		•² ✓
± 3	•³ ✓1	± 3	•³ ✓1	$3\sqrt{3}$	•³ ✓1

MARKS

4. The point K (8, –5) lies on the circle with equation $x^2 + y^2 - 12x - 6y - 23 = 0$.



Find the equation of the tangent to the circle at K. **4**

5. A (–3, 4, –7), B (5, t, 5) and C (7, 9, 8) are collinear.

(a) State the ratio in which B divides AC. **1**

(b) State the value of t. **1**

6. Find the value of $\log_5 250 - \frac{1}{3}\log_5 8$. **3**

Question	Generic scheme	Illustrative scheme	Max mark
4.	$\bullet^1$ state centre of circle $\bullet^2$ find gradient of radius $\bullet^3$ state gradient of tangent $\bullet^4$ state equation of tangent	$\bullet^1$ $(6,3)$ $\bullet^2$ -4 $\bullet^3$ $\frac{1}{4}$ $\bullet^4$ $y=\frac{1}{4}x-7$	4

Notes:

1. Accept $-\frac{8}{2}$ for $\bullet^2$.
2. The perpendicular gradient must be simplified at the $\bullet^3$ or $\bullet^4$ stage for $\bullet^3$ to be available.
3. $\bullet^4$ is only available as a consequence of trying to find and use a perpendicular gradient.
4. At $\bullet^4$ accept $y-\frac{1}{4}x+7=0$, $4y=x-28$, $x-4y-28=0$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

<table>
<tr><th>Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>5. (a)</td><td>•1 state ratio explicitly</td><td>•1 $4:1$</td><td>1</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">1. The only acceptable variations for •1 must be related explicitly to AB and BC.
For $\frac{\text{BC}}{\text{AB}}=\frac{1}{4}$, $\frac{\text{AB}}{\text{BC}}=\frac{4}{1}$ or $\text{BC}:\text{AB}=1:4$ award 1/1.
2. For $\text{BC}=\frac{1}{4}\text{AB}$ award 0/1.</td></tr>
<tr><td colspan="4">Commonly Observed Responses:</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td>(b)</td><td>•2 state value of t</td><td>•2 8</td><td>1</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">3. The answer to part (b) must be consistent with a ratio stated in part (a) unless a valid strategy which does not require the use of their ratio from part (a) is used.</td></tr>
<tr><td colspan="4">Commonly Observed Responses:</td></tr>
<tr><td colspan="2">Candidate A
$1:4$ •1 ✗
$t=8$ •2 ✗</td><td colspan="2">Candidate B
$1:4$ •1 ✗
$t=5$ •2 $\boxed{✓1}$</td></tr>
</table>

Question	Generic scheme	Illustrative scheme	Max mark
6.	•¹ apply $m\log_5 x = \log_5 x^m$ •² apply $\log_5 x - \log_5 y = \log_5 \frac{x}{y}$ •³ evaluate log	•¹ $\log_5 8^{\frac{1}{3}}$ •² $\log_5 \left(\frac{250}{8^{\frac{1}{3}}}\right)$ •³ 3	3

Notes:

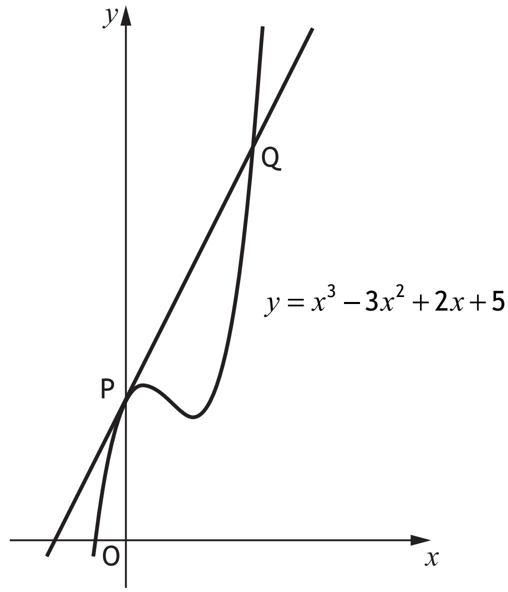
1. Each line of working must be equivalent to the line above within a valid strategy, however see Candidate B.
2. Do not penalise the omission of the base of the logarithm at •¹ or •².
3. For '3' with no working award 0/3.

Commonly Observed Responses:

Candidate A		Candidate B	
$\log_5 250 - \log_5 \frac{8}{3}$	•¹ ✗	$\frac{1}{3}\log_5(250 \div 8)$	
$\log_5 \frac{250}{\frac{8}{3}}$	•² ✓ 1	$\frac{1}{3}\log_5 \frac{125}{4}$	
$\log_5 \frac{375}{4}$	•³ ✓ 2	$\log_5 \left(\frac{125}{4}\right)^{\frac{1}{3}}$	Award 1/3 ✓ 1 ✗ ^
			•¹ is awarded for the final two lines of working

MARKS

7. The curve with equation $y = x^3 - 3x^2 + 2x + 5$ is shown on the diagram.



(a) Write down the coordinates of P, the point where the curve crosses the y-axis . **1**

(b) Determine the equation of the tangent to the curve at P. **3**

(c) Find the coordinates of Q, the point where this tangent meets the curve again. **4**

8. A line has equation $y - \sqrt{3}\,x + 5 = 0$.

Determine the angle this line makes with the positive direction of the x-axis. **2**

[Turn over

Question		Generic scheme	Illustrative scheme	Max mark
7.	(a)	•[1] state coordinates of P	•[1] $(0,5)$	**1**

Notes:

1. Accept '$x=0,\ y=5$'.
2. '$y=5$' alone or '5' does not gain •[1].

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	•[2] differentiate •[3] calculate gradient •[4] state equation of tangent	•[2] $3x^2-6x+2$ •[3] 2 •[4] $y=2x+5$	**3**

Notes:

3. At •[4] accept $y-2x=5$, $2x-y+5=0$, $y-5=2x$ or any other rearrangement of the equation where the constant terms have been simplified.
4. •[4] is only available if an attempt has been made to find the gradient from differentiation.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
7.	(c)	•5 set $y_{\text{line}} = y_{\text{curve}}$ and arrange in standard form	•5 $x^3 - 3x^2 = 0$	4
		•6 factorise	•6 $x^2(x-3)$	
		•7 state x-coordinate of Q	•7 3	
		•8 calculate y-coordinate of Q	•8 11	

Notes:

5. •5 is only available if '$=0$' appears at either •5 or •6 stage.
6. •7 and •8 are only available as a consequence of solving a cubic equation and a linear equation simultaneously.
7. For an answer of $(3,11)$ with no working award 0/4.
8. For an answer of $(3,11)$ verified in both equations award 3/4.
9. For an answer of $(3,11)$ verified in both equations along with a statement such as 'same point on both line and curve so Q is $(3,11)$' award 4/4.
10. For candidates who work with a derivative, no further marks are available.
11. $x = 3$ must be supported by valid working for •7 and •8 to be awarded.

Commonly Observed Responses:

Candidate A

$x^3 - 3x^2 = 0$	•5 ✓
$x - 3 = 0$	•6 ✓
$x = 3$	•7 ✓
$y = 11$	•8 ✓

Dividing by x^2 is valid since $x \neq 0$ at •6

Question	Generic scheme	Illustrative scheme	Max mark
8.	•¹ determine the gradient of the line	•¹ $m=\sqrt{3}$ or $\tan\theta=\sqrt{3}$	2
	•² determine the angle	•² $60°$ or $\frac{\pi}{3}$	

Notes:

1. Do not penalise the omission of units at •².
2. For $60°$ or $\frac{\pi}{3}$ without working award 2/2.

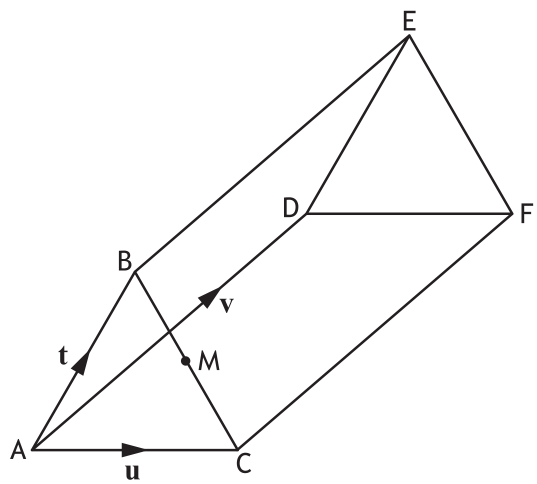
Commonly Observed Responses:

Candidate A		Candidate B	
$y=\sqrt{3}x+5$	Ignore incorrect processing of the constant term	$m=\sqrt{3}$	•¹ ✓
$m=\sqrt{3}$	•¹ ✓	$\theta=\tan\sqrt{3}$	•² ✗
$60°$	•² ✓	$\theta=60°$	
		Stating $\tan$ rather than $\tan^{-1}$ See general marking principle (g)	

MARKS

9. The diagram shows a triangular prism ABC,DEF.

$\overrightarrow{AB} = \mathbf{t}$, $\overrightarrow{AC} = \mathbf{u}$ and $\overrightarrow{AD} = \mathbf{v}$.



(a) Express $\overrightarrow{BC}$ in terms of $\mathbf{u}$ and $\mathbf{t}$. **1**

M is the midpoint of BC.

(b) Express $\overrightarrow{MD}$ in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$. **2**

10. Given that

- $\frac{dy}{dx} = 6x^2 - 3x + 4$, and
- $y = 14$ when $x = 2$,

express y in terms of x. **4**

Question	Generic scheme	Illustrative scheme	Max mark
9. (a)	$\bullet^1$ identify pathway	$\bullet^1$ $-\mathbf{t}+\mathbf{u}$	**1**
Notes:			
Commonly Observed Responses:			
(b)	$\bullet^2$ state an appropriate pathway $\bullet^3$ express pathway in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$	$\bullet^2$ eg $\frac{1}{2}\overrightarrow{BC}+\overrightarrow{CA}+\overrightarrow{AD}$ stated or implied by $\bullet^3$ $\bullet^3$ $-\frac{1}{2}\mathbf{t}-\frac{1}{2}\mathbf{u}+\mathbf{v}$	**2**
Notes:			
1. There is no need to simplify the expression at $\bullet^3$. Eg $\frac{1}{2}(-\mathbf{t}+\mathbf{u})-\mathbf{u}+\mathbf{v}$. 2. $\bullet^3$ is only available for using a valid pathway. 3. The expression at $\bullet^3$ must be consistent with the candidate's expression at $\bullet^1$. 4. If the pathway in $\bullet^1$ is given in terms of a single vector $\mathbf{t}$, $\mathbf{u}$ or $\mathbf{v}$, then $\bullet^3$ is not available.			
Commonly Observed Responses:			
Candidate A $\overrightarrow{MD}=-\frac{1}{2}\mathbf{t}+\mathbf{v}-\mathbf{u}$ $\bullet^2$ ^ $\bullet^3$ ✗			

Question	Generic scheme	Illustrative scheme	Max mark
10.	•1 know to and integrate one term	•1 eg $2x^3 \ldots$	**4**
	•2 complete integration	•2 eg $\ldots -\frac{3}{2}x^2+4x+c$	
	•3 substitute for x and y	•3 $14=2(2)^3-\frac{3}{2}(2)^2+4(2)+c$	
	•4 state equation	•4 $y=2x^3-\frac{3}{2}x^2+4x-4$ stated explicitly	

Notes:

1. For candidates who make no attempt to integrate to find y in terms of x award 0/4.
2. For candidates who omit $+c$, only •1 is available.
3. Candidates must attempt to integrate both terms containing x for •3 and •4 to be available. See Candidate B.
4. For candidates who differentiate any term, •2 •3 and •4 are not available.
5. •4 is not available for ‘ $f(x)=\ldots$ ’.
6. Candidates must simplify coefficients in **<u>their</u>** final line of working for the last mark available in that line of working to be awarded.

Commonly Observed Responses:

Candidate A

$y=2x^3-\frac{3}{2}x^2+4x+c$	•1 ✓ •2 ✓
$y=2(2)^3-\frac{3}{2}(2)^2+4(2)+c$	
$c=-4$	•3 ✓ substitution for y implied by $c=-4$
	•4 ^

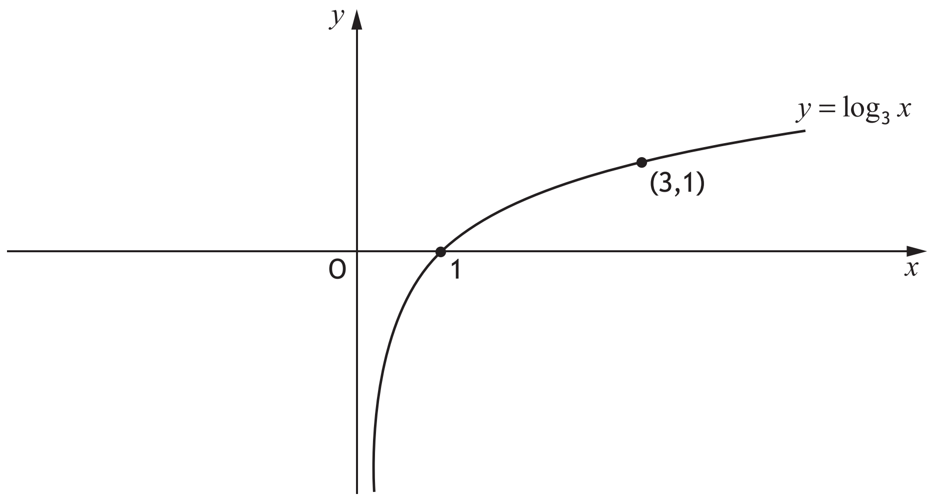
Candidate B - partial integration

$y=2x^3-\frac{3}{2}x^2+4+c$	•1 ✓ •2 ✗
$14=2(2)^3-\frac{3}{2}(2)^2+4+c$	•3 $\boxed{✓1}$
$c=0$	
$y=2x^3-\frac{3}{2}x^2+4$	•4 $\boxed{✓1}$

Back to Table

MARKS

11. The diagram shows the curve with equation $y = \log_3 x$.



(a) On the diagram in your answer booklet, sketch the curve with equation $y = 1 - \log_3 x$. **2**

(b) Determine the exact value of the x-coordinate of the point of intersection of the two curves. **3**

12. Vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}$.

(a) Express $2\mathbf{a} + \mathbf{b}$ in component form. **1**

(b) Hence find the values of p for which $|2\mathbf{a} + \mathbf{b}| = 7$. **3**

[Turn over for next question

Back to Table

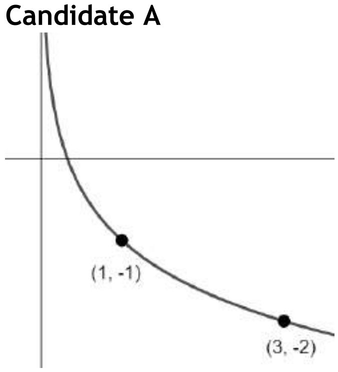
Question	Generic scheme	Illustrative scheme	Max mark
11. (a)	•[1] curve reflected in x-axis and translated 1 unit vertically •[2] accurate sketch	•[1] a generally decreasing curve above the x-axis for $1<x<3$ •[2] asymptote at $x=0$ **and** passing through $(3,0)$ **and** continuing to decrease for $x \geq 3$	**2**

Notes:

1. For any attempt which involves a horizontal translation or reflection in the y-axis award 0/2.
2. For a single transformation award 0/2.
3. For any attempt involving a reflection in the line $y=x$ award 0/2

Commonly Observed Responses:

Candidate A



Award 1/2

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•[3] set '$y=y$' •[4] start to solve •[5] state x coordinate	•[3] $\log_3 x = 1 - \log_3 x$ •[4] $\log_3 x = \frac{1}{2}$ or $\log_3 x^2 = 1$ •[5] $\sqrt{3}$ or $3^{\frac{1}{2}}$	**3**

Notes:

4. •[3] may be implied by $\log_3 x = \frac{1}{2}$ from symmetry of the curves.
5. Do not penalise the omission of the base of the logarithm at •[3] or •[4].
6. For a solution which equates a to $\log_3 a$, the final mark is not available.
7. If a candidate considers and then does not discard $-\sqrt{3}$ in their final answer, •[5] is not available.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
12. (a)	$\bullet^1$ find components	$\bullet^1$ $\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	1

Notes:

1. Accept $6\mathbf{i}-3\mathbf{j}+(4+p)\mathbf{k}$ for $\bullet^1$.
2. Do not accept $\begin{pmatrix} 6\mathbf{i} \\ -3\mathbf{j} \\ (4+p)\mathbf{k} \end{pmatrix}$ or $6\mathbf{i}-3\mathbf{j}+4\mathbf{k}+p\mathbf{k}$ for $\bullet^1$. However $\bullet^2$, $\bullet^3$ and $\bullet^4$ are still available.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(b)	$\bullet^2$ find an expression for magnitude	$\bullet^2$ $\sqrt{6^2+(-3)^2+(4+p)^2}$	3
	$\bullet^3$ start to solve	$\bullet^3$ $45+(4+p)^2=49 \Rightarrow (4+p)^2=4$ or $p^2+8p+12=0$	
	$\bullet^4$ find values of p	$\bullet^4$ $p=-2,\ p=-6$	

Notes:

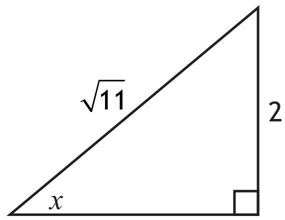
3. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. Eg $\sqrt{6^2+-3^2+(4+p)^2}$ or $\sqrt{6^2-3^2+(4+p)^2}$ leading to $\sqrt{45+(4+p)^2}$, $\bullet^2$ is awarded.
4. $\bullet^4$ is only available for two distinct values of p.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	$\bullet^1$ ✓	$\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	$\bullet^1$ ✓
$\sqrt{6^2-3^2+(4+p)^2}$	$\bullet^2$ ×	$\sqrt{6^2+(-3)^2+p^2}$	$\bullet^2$ ×
$27+(4+p)^2=49$		$45+p^2=49$	$\bullet^3$ $\boxed{✓\ 2}$
$(4+p)^2=22$	$\bullet^3$ $\boxed{✓\ 1}$	$p=\pm 2$	$\bullet^4$ $\boxed{✓\ 1}$
$p=-4\pm\sqrt{22}$	$\bullet^4$ $\boxed{✓\ 1}$		

MARKS

13. The right-angled triangle in the diagram is such that $\sin x = \dfrac{2}{\sqrt{11}}$ and $0 < x < \dfrac{\pi}{4}$.



(a) Find the exact value of:

(i) $\sin 2x$ — **3**

(ii) $\cos 2x$. — **1**

(b) By expressing $\sin 3x$ as $\sin(2x + x)$, find the exact value of $\sin 3x$. — **3**

14. Evaluate $\displaystyle\int_{-4}^{9} \frac{1}{\sqrt[3]{(2x+9)^2}}\,dx$. — **5**

15. A cubic function, f, is defined on the set of real numbers.

- $(x+4)$ is a factor of $f(x)$
- $x = 2$ is a repeated root of $f(x)$
- $f'(-2) = 0$
- $f'(x) > 0$ where the graph with equation $y = f(x)$ crosses the y-axis

Sketch a possible graph of $y = f(x)$ on the diagram in your answer booklet. — **4**

[END OF QUESTION PAPER]

Question	Generic scheme	Illustrative scheme	Max mark
13. (a) (i)	•¹ find the value of $\cos x$ •² substitute into the formula for $\sin 2x$ •³ simplify	•¹ $\frac{\sqrt{7}}{\sqrt{11}}$ stated or implied by •² •² $2\times\frac{2}{\sqrt{11}}\times\frac{\sqrt{7}}{\sqrt{11}}$ •³ $\frac{4\sqrt{7}}{11}$	**3**
(ii)	•⁴ evaluate $\cos 2x$	•⁴ $\frac{3}{11}$	**1**

Notes:

1. Where a candidate substitutes an incorrect value for $\cos x$ at •², •² may be awarded if the candidate has previously stated this incorrect value or it can be implied by a diagram.
2. •³ is only available as a consequence of substituting into a valid formula at •².
3. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•⁵ expand using the addition formula •⁶ substitute in values •⁷ simplify	•⁵ $\sin 2x\cos x+\cos 2x\sin x$ stated or implied by •⁶ •⁶ $\frac{4\sqrt{7}}{11}\times\frac{\sqrt{7}}{\sqrt{11}}+\frac{3}{11}\times\frac{2}{\sqrt{11}}$ •⁷ $\frac{34}{11\sqrt{11}}$	**3**

Notes:

4. For any attempt to use $\sin(2x+x)=\sin 2x+\sin x$, •⁵ •⁶ and •⁷ are not available

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
14.	$\bullet^1$ write in integrable form $\bullet^2$ start to integrate $\bullet^3$ complete integration $\bullet^4$ process limits $\bullet^5$ evaluate integral	$\bullet^1$ $(2x+9)^{-\frac{2}{3}}$ $\bullet^2$ $\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}\ldots$ $\bullet^3$ $\ldots\times\dfrac{1}{2}$ $\bullet^4$ $\frac{3}{2}(2(9)+9)^{\frac{1}{3}}-\frac{3}{2}(2(-4)+9)^{\frac{1}{3}}$ $\bullet^5$ 3	5

Notes:

1. For candidates who differentiate throughout, only $\bullet^1$ is available.
2. For candidates who 'integrate the denominator' without attempting to write in integrable form award 0/5.
3. $\bullet^2$ may be awarded for the appearance of $\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}$ in the line of working where the candidate first attempts to integrate. See Candidate F.
4. If candidates start to integrate individual terms within the bracket or attempt to expand a bracket or use another invalid approach no further marks are available.
5. For $\bullet^2$ to be awarded the integrand must contain a non-integer power.
6. Do not penalise the inclusion of '$+c$'.
7. $\bullet^4$ and $\bullet^5$ are not available to candidates who substitute into the original function.
8. The integral obtained must contain a non-integer power for $\bullet^5$ to be available.
9. $\bullet^5$ is only available to candidates who deal with the coefficient of x at the $\bullet^3$ stage. See Candidate A.

Commonly Observed Responses:

Candidate A		Candidate B	
$(2x+9)^{-\frac{2}{3}}$	$\bullet^1$ ✓	$(2x+9)^{\frac{2}{3}}$	$\bullet^1$ ✗
$\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}$	$\bullet^2$ ✓ $\bullet^3$ ^	$\dfrac{(2x+9)^{\frac{5}{3}}}{\frac{5}{3}}\times\dfrac{1}{2}$	$\bullet^2$ $\boxed{✓1}$ $\bullet^3$ ✓
$3(2(9)+9)^{\frac{1}{3}}-3(2(-4)+9)^{\frac{1}{3}}$	$\bullet^4$ $\boxed{✓1}$	$\frac{3}{10}(2(9)+9)^{\frac{5}{3}}-\frac{3}{10}(2(-4)+9)^{\frac{5}{3}}$	$\bullet^4$ $\boxed{✓1}$
6	$\bullet^5$ $\boxed{✓2}$ see note 9	$\dfrac{363}{5}$	$\bullet^5$ $\boxed{✓1}$

Back to Table

Question	Generic scheme	Illustrative scheme	Max mark
15.	•¹ root at $x = -4$ identifiable from graph	•¹	**4**
	•² stationary point touching x-axis when $x = 2$ identifiable from graph	•²	
	•³ stationary point when $x = -2$ identifiable from graph	•³	
	•⁴ identify orientation of the cubic curve and $f'(0) > 0$ identifiable from graph	•⁴ 	

Notes:

1. For a diagram which does not show a cubic curve award 0/4.
2. For candidates who identify the roots of the cubic at '$x = -4, -2$ and 2' or at '$x = -2, 2$ and 4' •⁴ is unavailable.

Commonly Observed Responses:

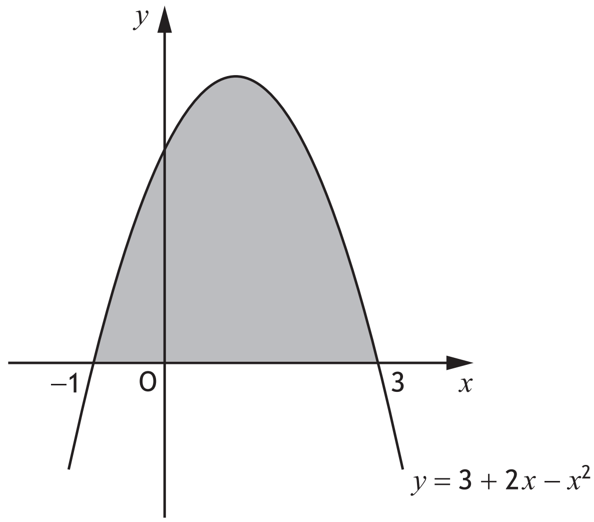
[END OF MARKING INSTRUCTIONS]

MARKS

Attempt ALL questions

Total marks — 70

1. The diagram shows the curve with equation $y = 3 + 2x - x^2$.



Calculate the shaded area. **4**

2. Vectors $\mathbf{u}$ and $\mathbf{v}$ are defined by $\mathbf{u} = \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -7 \\ 8 \\ 5 \end{pmatrix}$.

(a) Find $\mathbf{u.v}$. **1**

(b) Calculate the acute angle between $\mathbf{u}$ and $\mathbf{v}$. **4**

3. A function, f, is defined on the set of real numbers by $f(x) = x^3 - 7x - 6$.

Determine whether f is increasing or decreasing when $x = 2$. **3**

4. Express $-3x^2 - 6x + 7$ in the form $a(x+b)^2 + c$. **3**

[Turn over

Detailed marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	$\bullet^1$ state an integral to represent the shaded area	$\bullet^1$ $\int_{-1}^{3}\left(3+2x-x^2\right)dx$	4
	$\bullet^2$ integrate	$\bullet^2$ $3x+\frac{2x^2}{2}-\frac{x^3}{3}$	
	$\bullet^3$ substitute limits	$\bullet^3$ $\left(3\times 3+\frac{2\times 3^2}{2}-\frac{3^3}{3}\right)-\left(3\times(-1)+\frac{2\times(-1)^2}{2}-\frac{(-1)^3}{3}\right)$	
	$\bullet^4$ evaluate integral	$\bullet^4$ $\frac{32}{3}$ (units²)	

Notes:

1. $\bullet^1$ is not available to candidates who omit 'dx'.
2. Limits must appear at the $\bullet^1$ stage for $\bullet^1$ to be awarded.
3. Where a candidate differentiates one or more terms at $\bullet^2$, then $\bullet^3$ and $\bullet^4$ are unavailable.
4. Candidates who substitute limits without integrating, do not gain $\bullet^3$ or $\bullet^4$.
5. Do not penalise the inclusion of '$+c$'.
6. Do not penalise the continued appearance of the integral sign after $\bullet^1$.
7. If $\bullet^4$ is only given as a decimal then it must be given correct to 1 decimal place.

Commonly Observed Responses:

Candidate A		Candidate B	
$\int_{-1}^{3} 3+2x-x^2$	$\bullet^1$ ✘	$\int\left(3+2x-x^2\right)dx$	$\bullet^1$ ✘
$=3x+\frac{2x^2}{2}-\frac{x^3}{3}$	$\bullet^2$ ✓	$=3x+\frac{2x^2}{2}-\frac{x^3}{3}$	$\bullet^2$ ✓
	$\bullet^3$ ^	$=9-\left(-\frac{5}{3}\right)$	$\bullet^3$ ✓
$=\frac{32}{3}$	$\bullet^4$ $\boxed{✓\ 1}$	$=\frac{32}{3}$	$\bullet^4$ ✓

Question	Generic scheme	Illustrative scheme	Max mark
2. (a)	$\bullet^1$ find $\mathbf{u.v}$	$\bullet^1$ 24	1

Notes:

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(b)	$\bullet^2$ find $\lvert\mathbf{u}\rvert$	$\bullet^2$ $\sqrt{26}$	4
	$\bullet^3$ find $\lvert\mathbf{v}\rvert$	$\bullet^3$ $\sqrt{138}$	
	$\bullet^4$ apply scalar product	$\bullet^4$ $\cos\theta° = \dfrac{24}{\sqrt{26}\sqrt{138}}$	
	$\bullet^5$ calculate angle	$\bullet^5$ $66\cdot38...°$ or $1\cdot16...$radians	

Notes:

1. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. Eg $\sqrt{-1^2+4^2-3^2}=\sqrt{26}$ or $\sqrt{-1^2+4^2+-3^2}=\sqrt{26}$, $\bullet^2$ is awarded.
2. $\bullet^4$ is not available to candidates who simply state the formula $\cos\theta° = \dfrac{\mathbf{u.v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}$.
3. Accept answers which round to $66°$ or $1\cdot2$ radians (or 73·8 gradians).
4. Do not penalise the omission or incorrect use of units.
5. $\bullet^5$ is only available for a single angle.
6. For a correct answer with no working award 0/4.

Commonly Observed Responses:

Candidate A

$\lvert\mathbf{u}\rvert=\sqrt{26}$	$\bullet^2$ ✓	
$\lvert\mathbf{v}\rvert=\sqrt{138}$	$\bullet^3$ ✓	
$\dfrac{24}{\sqrt{26}\sqrt{138}}$	$\bullet^4$ ^	
$\theta = 66\cdot38...°$	$\bullet^5$ $\boxed{✓ 1}$	

Question	Generic scheme	Illustrative scheme	Max mark
3.	$\bullet^1$ differentiate	$\bullet^1$ $3x^2-7$	**3**
	$\bullet^2$ evaluate derivative at $x=2$	$\bullet^2$ 5	
	$\bullet^3$ interpret result	$\bullet^3$ $(f \text{ is})$ increasing	

Notes:

1. $\bullet^2$ and $\bullet^3$ are only available as a consequence of working with a derivative.
2. Accept $f'(2)>0$ for $\bullet^2$.
3. $f'(x)>0$ with no evidence of evaluating the derivative at $x=2$ does not gain $\bullet^2$ or $\bullet^3$. See candidate B.
4. Do not penalise candidates who use y in place of $f(x)$.

Commonly Observed Responses:

Candidate A		Candidate B	
$3x^2-7$	$\bullet^1$ ✓	$3x^2-7$	$\bullet^1$ ✓
$\begin{array}{c\|c} x & 2 \\ \hline f'(x) & + \end{array}$	$\bullet^2$ ✓	$f'(x)>0$	$\bullet^2$ ^
increasing	$\bullet^3$ ✓	f is increasing	$\bullet^3$ ^

Question	Generic scheme	Illustrative scheme	Max mark
4.	**Method 1**	**Method 1**	3
	•1 identify common factor	•1 $-3\left(x^2+2x\ldots\right.$ stated or implied by •2	
	•2 complete the square	•2 $-3(x+1)^2\ldots$	
	•3 process for c	•3 $-3(x+1)^2+10$	
	Method 2	**Method 2**	
	•1 expand completed square form	•1 $ax^2+2abx+ab^2+c$	
	•2 equate coefficients	•2 $a=-3$, $2ab=-6$ $ab^2+c=7$	
	•3 process for b and c and write in required form	•3 $-3(x+1)^2+10$	

Notes:

1. $-3(x+1)^2+10$ with no working gains •1 and •2 only; however, see Candidate E.
2. •3 is only available for a calculation involving both multiplication and addition of integers.

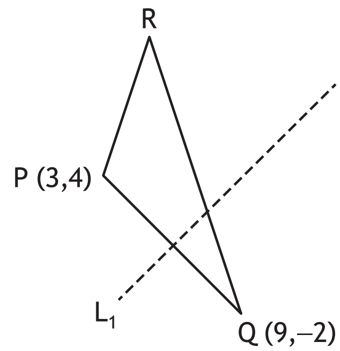
Commonly Observed Responses:

Candidate A		**Candidate B**	
$-3(x^2+2)+7$	exception in General marking principle (h)	$-3\left(\left(x^2-6x\right)+7\right)$	•1 ✗
$-3\left((x+1)^2-1\right)+7$	•1 ✓•2 ✓	$-3\left((x-3)^2-9\right)+7$	•2 $\boxed{✓ 1}$
$-3(x+1)^2+10$	•3 ✓	$-3(x-3)^2+34$	•3 $\boxed{✓ 1}$

Candidate C		**Candidate D**	
$a(x+b)^2+c=ax^2+2abx+ab^2+c$	•1 ✓	$ax^2+2abx+ab^2+c$	•1 ✓
$a=-3,\ \ 2ab=-6,\ \ ab^2+c=7$	•2 ✓	$a=-3,\ \ 2ab=-6,\ \ ab^2+c=7$	•2 ✓
$b=1,\ c=10$	•3 ✓	$b=1,\ c=10$	•3 ✗
•3 is awarded as all working relates to completed square form		•3 is lost as no reference is made to completed square form	

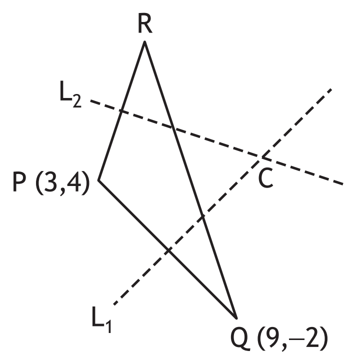
MARKS

5. PQR is a triangle with P(3,4) and Q(9,–2).



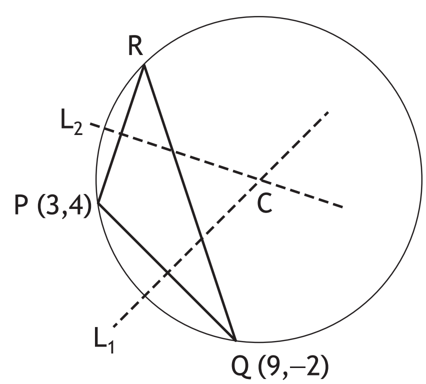
(a) Find the equation of L_1, the perpendicular bisector of PQ. **3**

The equation of L_2, the perpendicular bisector of PR is $3y + x = 25$.



(b) Calculate the coordinates of C, the point of intersection of L_1 and L_2. **2**

C is the centre of the circle which passes through the vertices of triangle PQR.



(c) Determine the equation of this circle. **2**

Question		Generic scheme	Illustrative scheme	Max mark
5.	(a)	•¹ find the midpoint of PQ •² calculate m_{PQ} and state perp. gradient •³ find equation of L_1 in a simplified form	•¹ $(6,1)$ •² $-1 \Rightarrow m_{perp} = 1$ •³ $y = x - 5$	3

Notes:

1. •³ is only available as a consequence of using a perpendicular gradient and a midpoint.
2. The gradient of the perpendicular bisector must appear in simplified form at •² or •³ stage for •³ to be awarded.
3. At •³, accept $x - y - 5 = 0$, $y - x = -5$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	•⁴ determine y coordinate •⁵ state x coordinate	•⁴ 5 •⁵ 10	2

Notes:

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(c)	•⁶ calculate radius of the circle •⁷ state equation of the circle	•⁶ $\sqrt{50}$ stated or implied by •⁷ •⁷ $(x-10)^2 + (y-5)^2 = 50$	2

Notes:

4. Where candidates have calculated the coordinates of C incorrectly, •⁶ and •⁷ are available for using either PC or QC for the radius.
5. Where incorrect coordinates for C appear without working, only •⁷ is available.
6. Do not accept $\left(\sqrt{50}\right)^2$ for •⁷.

Commonly Observed Responses:

MARKS

6. Functions, f and g, are given by $f(x) = 3 + \cos x$ and $g(x) = 2x$, $x \in \mathbb{R}$.

(a) Find expressions for

(i) $f(g(x))$ and **2**

(ii) $g(f(x))$. **1**

(b) Determine the value(s) of x for which $f(g(x)) = g(f(x))$ where $0 \leq x < 2\pi$. **6**

7. (a) (i) Show that $(x-2)$ is a factor of $2x^3 - 3x^2 - 3x + 2$. **2**

(ii) Hence, factorise $2x^3 - 3x^2 - 3x + 2$ fully. **2**

The fifth term, u_5, of a sequence is $u_5 = 2a - 3$.

The terms of the sequence satisfy the recurrence relation $u_{n+1} = au_n - 1$.

(b) Show that $u_7 = 2a^3 - 3a^2 - a - 1$. **1**

For this sequence, it is known that

- $u_7 = u_5$
- a limit exists.

(c) (i) Determine the value of a. **3**

(ii) Calculate the limit. **1**

[Turn over

Question	Generic scheme	Illustrative scheme	Max mark
6. (a) (i)	•¹ start composite process •² substitute into expression	•¹ $f(2x)$ •² $3+\cos 2x$	**2**
(ii)	•³ state second composite	•³ $2(3+\cos x)$	**1**

Notes:

1. For $3+\cos 2x$ without working, award both •¹ and •².
2. Candidates who interpret the composite function as either $g(x)\times f(x)$ or $g(x)+f(x)$ do not gain any marks.

Commonly Observed Responses:

Candidate A - interpret $f(g(x))$ as $g(f(x))$		Candidate B - interpret $f(g(x))$ as $g(f(x))$	
(i) $2(3+\cos x)$	•¹ ✗ •² $\boxed{\checkmark\ 1}$	(i) $f(2x)=2(3+\cos x)$	•¹ ✓ •² ✗
(ii) $3+\cos 2x$	•³ $\boxed{\checkmark\ 1}$	(ii) $3+\cos(2x)$	•³ $\boxed{\checkmark\ 1}$

Question	Generic scheme	Illustrative scheme	Max mark
6. (b)	•[4] equate expressions from (a)	•[4] $3+\cos 2x = 2(3+\cos x)$	**6**
	•[5] substitute for $\cos 2x$ in equation	•[5] $3+2\cos^2 x - 1 = 2(3+\cos x)$	
	•[6] arrange in standard quadratic form	•[6] $2\cos^2 x - 2\cos x - 4 = 0$	
	•[7] factorise	•[7] $2(\cos x - 2)(\cos x + 1)$	
	•[8] solve for $\cos x$	•[8] $\cos x = 2$ •[8] $\cos x = -1$	
	•[9] solve for x	•[9] $\cancel{\cos x = 2}$ or eg 'no solution' •[9] $x = \pi$	

Notes:

3. Do not penalise absence of common factor at •[7].
4. •[5] cannot be awarded until the equation reduces to a quadratic in $\cos x$.
5. Substituting $2\cos^2 \mathrm{A} - 1$ or $2\cos^2 \alpha - 1$ at •[5] stage should be treated as bad form provided the equation is written in terms of x at •[6] stage. Otherwise, •[5] is not available.
6. '$=0$' must appear by •[7] stage for •[6] to be awarded. However, for candidates using the quadratic formula to solve the equation, '$=0$' must appear at •[6] stage for •[6] to be awarded.
7. For candidate who do not arrange in standard quadratic form, eg $-2\cos x + 2\cos^2 x - 4 = 0$ •[6] is only available if •[7] has been awarded.
8. •[7] •[8] and •[9] are only available as a consequence of solving a quadratic with distinct real roots.
9. •[7] •[8] and •[9] are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$.
10. •[9] is not available to candidates who work in degrees and do not convert their solution(s) into radian measure.
11. Answers written as decimals should be rounded to no fewer than 2 significant figures.
12. •[9] is not available for any solution containing angles outwith the interval $0 \le x < 2\pi$.

Question	Generic scheme	Illustrative scheme	Max mark
7. (a) (i)	$\bullet^1$ use ‘2’ in synthetic division or in evaluation of cubic $\bullet^2$ complete division/evaluation and interpret result	$\bullet^1$ $\begin{array}{r\|rrrr} 2 & 2 & -3 & -3 & 2 \\ \hline & 2 & & & \end{array}$ or $2\times(2)^3-3(2)^2-3\times(2)+2$ $\bullet^2$ $\begin{array}{r\|rrrr} 2 & 2 & -3 & -3 & 2 \\ & & 4 & 2 & -2 \\ \hline & 2 & 1 & -1 & \boxed{0} \end{array}$ Remainder $=0 \therefore (x-2)$ is a factor or $f(2)=0 \therefore (x-2)$ is a factor	**2**
(ii)	$\bullet^3$ state quadratic factor $\bullet^4$ complete factorisation	$\bullet^3$ $2x^2+x-1$ $\bullet^4$ $(x-2)(2x-1)(x+1)$ stated explicitly	**2**

Notes:

1. Communication at $\bullet^2$ must be consistent with working at that stage i.e. a candidate’s working must arrive legitimately at 0 before $\bullet^2$ can be awarded.
2. Accept any of the following for $\bullet^2$:
   - ‘$f(2)=0$ so $(x-2)$ is a factor’
   - ‘since remainder = 0, it is a factor’
   - the 0 from any method linked to the word ‘factor’ by e.g. ‘so’, ‘hence’, ‘$\therefore$’, ‘$\rightarrow$’, ‘$\Rightarrow$’
3. Do not accept any of the following for $\bullet^2$:
   - double underlining the zero or boxing the zero without comment
   - ‘$x=-2$ is a factor’, ‘$(x+2)$ is a factor’, ‘$(x+2)$ is a root’, ‘$x=2$ is a root’, ‘$(x-2)$ is a root’, ‘$x=-2$ is a root’
   - the word ‘factor’ only, with no link.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
7. (b)	$\bullet^5$ demonstrate result	$\bullet^5$ $u_6=a(2a-3)-1=2a^2-3a-1$ leading to $u_7=a(2a^2-3a-1)-1$ $=2a^3-3a^2-a-1$	**1**

Notes:

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
7. (c) (i)	•6 equate u_5 and u_7 **and** arrange in standard form •7 solve cubic •8 discard invalid solutions for a	•6 $2a^3-3a^2-3a+2=0$ •7 $a=2,\ a=\frac{1}{2},\ a=-1$ •8 $a=\frac{1}{2}$	**3**
(ii)	•9 calculate limit	•9 -2	**1**

Notes:

4. Where •6 has been awarded, •7 is available for solutions in terms of x appearing in a(ii). However, see Candidates B and C. BEWARE: Candidates who make a second attempt at factorising the cubic obtained in c(i) and do so incorrectly cannot be awarded mark 7 for solutions appearing in a(ii).
5. •8 is only available as a result of a valid strategy at •7.
6. $x=\frac{1}{2}$ does not gain •8.
7. For candidates who do not state the cubic equation at •6, and adopt a guess and check approach, using solutions for x found in a(ii), may gain 3/3. See Candidate D.

Commonly Observed Responses:

Candidate A

$2a^3-3a^2-3a+2=0$ •6 ✓

$x=2,\ x=\frac{1}{2},\ x=-1$ in a(ii) •7 ✓ •8 ^

Candidate B - missing '$=0$' from equation

$2a^3-3a^2-3a+2$ •6 ^

$\cancel{x=2},\ x=\frac{1}{2},\ \cancel{x=-1}$ in a(ii) •7 $\boxed{✓ 1}$

$a=\frac{1}{2}$ •8 $\boxed{✓ 1}$

Candidate C - missing '$=0$' from equation

$2a^3-3a^2-3a+2$ •6 ^

$\cancel{x=2},\ x=\frac{1}{2},\ \cancel{x=-1}$ in a(ii) •7 ^

$\frac{1}{2}$ •8 ^

No clear link between a and x.

Candidate D - $x=-1$, $x=\frac{1}{2}$ and $x=2$ identified in a(ii)

$u_5=2\left(\frac{1}{2}\right)-3=-2$ •6 ✓

$u_7=2\left(\frac{1}{2}\right)^3-3\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)-1=-2$ •7 ✓

$a=\frac{1}{2}$ because $-1<a<1$ •8 ✓

MARKS

8. (a) Express $2\cos x° - \sin x°$ in the form $k\cos(x-a)°$, $k>0$, $0<a<360$. **4**

(b) Hence, or otherwise, find

(i) the minimum value of $6\cos x° - 3\sin x°$ and **1**

(ii) the value of x for which it occurs where $0 \le x < 360$. **2**

9. A sector with a particular fixed area has radius x cm.

The perimeter, P cm, of the sector is given by

$$P = 2x + \frac{128}{x}.$$

Find the minimum value of P. **6**

10. The equation $x^2 + (m-3)x + m = 0$ has two real and distinct roots.

Determine the range of values for m. **4**

11. A supermarket has been investigating how long customers have to wait at the checkout.

During any half hour period, the percentage, P%, of customers who wait for less than t minutes, can be modelled by

$P = 100(1 - e^{kt})$, where k is a constant.

(a) If 50% of customers wait for less than 3 minutes, determine the value of k. **4**

(b) Calculate the percentage of customers who wait for 5 minutes or longer. **2**

Question	Generic scheme	Illustrative scheme	Max mark
8. (a)	$\bullet^1$ use compound angle formula	$\bullet^1$ $k\cos x^\circ\cos a^\circ + k\sin x^\circ\sin a^\circ$ stated explicitly	4
	$\bullet^2$ compare coefficients	$\bullet^2$ $k\cos a^\circ = 2$ and $k\sin a^\circ = -1$ stated explicitly	
	$\bullet^3$ process for k	$\bullet^3$ $k=\sqrt{5}$	
	$\bullet^4$ process for a and express in required form	$\bullet^4$ $\sqrt{5}\cos\left(x-333\cdot4\ldots\right)^\circ$	

Notes:

1. Accept $k\left(\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ\right)$ for $\bullet^1$. Treat $k\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
2. Do not penalise the omission of degree signs.
3. $\sqrt{5}\cos x^\circ\cos a^\circ + \sqrt{5}\sin x^\circ\sin a^\circ$ or $\sqrt{5}\left(\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ\right)$ is acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $k\cos x^\circ = 2$, $k\sin x^\circ = -1$, however $\bullet^4$ may still be gained.
5. $\bullet^3$ is only available for a single value of k, $k>0$.
6. $\bullet^4$ is not available for a value of a given in radians.
7. Accept any value of a which rounds to 333°
8. Candidates may use any form of the wave function for $\bullet^1$, $\bullet^2$ and $\bullet^3$, however, $\bullet^4$ is only available if the wave is interpreted in the form $k\cos(x-a)^\circ$.
9. Evidence for $\bullet^4$ may not appear until part (b).

Commonly Observed Responses:

Responses with missing information in working:

Candidate A	Candidate B	Candidate C
$\bullet^1$ ^	$k\cos x^\circ\cos a^\circ + k\sin x^\circ\sin a^\circ$ $\bullet^1$ ✓	$\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ$ $\bullet^1$ ✗
$\sqrt{5}\cos a^\circ = 2$	$\cos a^\circ = 2$	$\cos a^\circ = 2$
$\sqrt{5}\sin a^\circ = -1$ $\bullet^2$ ✓ $\bullet^3$ ✓	$\sin a^\circ = -1$ $\bullet^2$ ✗	$\sin a^\circ = -1$ $\bullet^2$ ✗
		$k=\sqrt{5}$ $\bullet^3$ ✓
$\tan a^\circ = -\frac{1}{2}$	$\tan a^\circ = -\frac{1}{2}$	$\tan a^\circ = -\frac{1}{2}$
$a = 333\cdot4$	$a = 333\cdot4$ Not consistent with equations at $\bullet^2$.	$a = 333\cdot4$
$\sqrt{5}\cos\left(x-333\cdot4\right)^\circ$ $\bullet^4$ ✓	$\sqrt{5}\cos\left(x-333\cdot4\right)^\circ$ $\bullet^3$ ✓ $\bullet^4$ ✗	$\sqrt{5}\cos\left(x-333\cdot4\right)^\circ$ $\bullet^4$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
8.	(b)	(i)	•[5] state minimum value	•[5] $-3\sqrt{5}$ or $-\sqrt{45}$	1
		(ii)	**Method 1** •[6] start to solve •[7] state value of x **Method 2** •[6] start to solve •[7] state value of x	**Method 1** •[6] $x-333\cdot4=180$ leading to $x=513\cdot4$ •[7] $x=153\cdot4\ldots$ **Method 2** •[6] $x-333\cdot4=-180$ •[7] $x=153\cdot4\ldots$	2

Notes:

10. •[7] is only available for a single value of x.
11. •[7] is only available in cases where $a<-180$ or $a>180$. See Candidate J

Commonly Observed Responses:

Candidate J - from $\sqrt{5}\cos(x-26\cdot6)^\circ$	Candidate K - from 'minimum' of eg $-\sqrt{5}$
$x-26\cdot6=180$ $x=206\cdot6$ •[6] ✓ 1 •[7] ✓ 2 Similarly for $\sqrt{5}\cos(x-116\cdot6)^\circ$	$3\sqrt{5}\cos(x-333\cdot4)^\circ=-\sqrt{5}$ $\cos(x-333\cdot4)^\circ=-\frac{1}{3}$ $x-333\cdot4=109\cdot5,\ 250\cdot5$ $x=442\cdot9,\ 583\cdot9$ •[6] ✓ 1 $x=82\cdot9,\ 223\cdot9$ •[7] ✗

Question	Generic scheme	Illustrative scheme	Max mark
9.	•[1] express P in differentiable form	•[1] $2x+128x^{-1}$	**6**
	•[2] differentiate	•[2] $2-\frac{128}{x^2}$	
	•[3] equate expression for derivative to 0	•[3] $2-\frac{128}{x^2}=0$	
	•[4] process for x	•[4] 8	
	•[5] verify nature	•[5] table of signs for a derivative (see next page) $\therefore$ minimum or $P''(8)=\frac{1}{2}>0 \quad \therefore$ minimum	
	•[6] evaluate P	•[6] $P=32$ or min value $=32$	

Notes:

1. For a numerical approach award 0/6.
2. For candidates who integrate any term at the •[2] stage, only •[3] is available on follow through for setting their ‘derivative’ to 0.
3. •[4], •[5] and •[6] are only available for working with a derivative which contains an index ≤ -2.
4. At •[2] accept $2-128x^{-2}$.
5. Ignore the appearance of -8 at •[4].
6. $\sqrt{\frac{128}{2}}$ must be simplified at •[4] or •[5] for •[4] to be awarded.
7. •[5] is not available to candidates who consider a value of $x \leq 0$ in the neighbourhood of 8.
8. •[6] is still available in cases where a candidate’s table of signs does not lead legitimately to a minimum at •[5].
9. •[5] and •[6] are not available to candidates who state that the minimum exists at a negative value of x.

Commonly Observed Responses:

Candidate A - differentiating over more than one line		Candidate B - differentiating over more than one line	
	•[1] ^	$P(x)=2x+128x^{-1}$	•[1] ✓
$P'(x)=2+128x^{-1}$		$P'(x)=2+128x^{-1}$	
$P'(x)=2-128x^{-2}$	•[2] ✗	$P'(x)=2-128x^{-2}$	•[2] ✗
$2-128x^{-2}=0$	•[3] $\boxed{✓ 1}$	$2-128x^{-2}=0$	•[3] $\boxed{✓ 1}$

Question	Generic scheme	Illustrative scheme	Max mark
10.	•¹ use the discriminant •² identify roots of quadratic expression •³ apply condition •⁴ state range with justification	•¹ $(m-3)^2-4\times1\times m$ •² 1, 9 •³ $(m-3)^2-4\times1\times m>0$ •⁴ $m<1,\ m>9$ with eg sketch or table of signs	4

Notes:

1. If candidates have the condition '$\text{discriminant}<0$', '$\text{discriminant}\le0$' or '$\text{discriminant}\ge0$', then •³ is lost but •⁴ is available.
2. Ignore the appearance of $b^2-4ac=0$ where the correct condition has subsequently been applied.
3. For candidates who have identified expressions for a, b, and c and then state $b^2-4ac>0$ award •³. See Candidate A.
4. For the appearance of x in any expression for •¹, award 0/4.

Commonly Observed Responses:

Candidate A

$(m-3)^2-4\times1\times m$	•¹ ✓
$m^2-10m+9=0$	
$m=1,\ m=9$	•² ✓
$b^2-4ac>0$	•³ ✓
$m<1,\ m>9$	•⁴ ^

Expressions for a, b, and c implied at •¹

Back to Table

Question	Generic scheme	Illustrative scheme	Max mark
11. (a)	•¹ substitute for P and t	•¹ $50=100\left(1-e^{3k}\right)$	4
	•² arrange equation in the form $A=e^{kt}$	•² $0\cdot5=e^{3k}$ or $-0\cdot5=-e^{3k}$	
	•³ simplify	•³ $\ln 0\cdot5=3k$	
	•⁴ solve for k	•⁴ $k=-0\cdot231$	

Notes:

1. •² may be assumed by •³.
2. Any base may be used at •³ stage. See Candidate D.
3. Accept any answer which rounds to $-0\cdot2$.
4. •³ must be consistent with the equation of the form $A=e^{kt}$ at its first appearance.
5. For candidates whose working would (or should) arrive at $\log(\text{negative})$ •⁴ is not available.
6. Where candidates use a ‘rule’ masquerading as a law of logarithms, •³ and •⁴ is not available.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$50=100\left(1-e^{3k}\right)$	•¹ ✓	$0\cdot5=100\left(1-e^{3k}\right)$	•¹ ✗
$0\cdot5=-e^{3k}$	•² ✗	$0\cdot995=e^{3k}$	•² $\boxed{\checkmark\ 1}$
$\ln(0\cdot5)=3k$	•³ ✗	$\ln(0\cdot995)=3k$	•³ $\boxed{\checkmark\ 1}$
$k=-0\cdot231$	•⁴ ✗	$k=-0\cdot0017\ldots$	•⁴ $\boxed{\checkmark\ 1}$
68·5	•⁵ $\boxed{\checkmark\ 1}$	$P=0\cdot8319\ldots$	•⁵ $\boxed{\checkmark\ 1}$
31·5% still queueing	•⁶ $\boxed{\checkmark\ 1}$	99·2% still queuing	•⁶ $\boxed{\checkmark\ 1}$
Candidate C		**Candidate D**	
$50=100\left(1-e^{3k}\right)$	•¹ ✓	$50=100\left(1-e^{3k}\right)$	•¹ ✓
$-0\cdot5=-e^{3k}$	•² ✓	$0\cdot5=e^{3k}$	•² ✓
$\ln(-0\cdot5)=\ln\left(-e^{3k}\right)$	•³ ✗	$\log_{10}(0\cdot5)=3k\log_{10}e$	•³ ✓
$k=-0\cdot231$	•⁴ ✗	$k=-0\cdot231$	•⁴ ✓
68·5	•⁵ $\boxed{\checkmark\ 1}$		
31·5% still queueing	•⁶ $\boxed{\checkmark\ 1}$		

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•⁵ evaluate P for $t=5$	•⁵ 68·5	2
	•⁶ interpret result	•⁶ 31·5% still queueing	

Notes:

7. •⁵ and •⁶ are not available where $k\geq0$.
8. •⁶ is only available where the value of P in •⁵ was obtained by substituting into an exponential expression.

Commonly Observed Responses:

Candidate D - $k=-0\cdot2$	
63·2	•⁵ ✓
36·8% still queueing	•⁶ ✓

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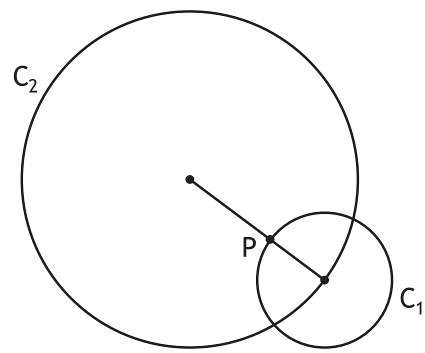
MARKS

12. Circle C_1 has equation $(x-13)^2+(y+4)^2=100$.

Circle C_2 has equation $x^2+y^2+14x-22y+c=0$.



(a) (i) Write down the coordinates of the centre of C_1. **1**

(ii) The centre of C_1 lies on the circumference of C_2.

Show that $c=-455$. **1**

The line joining the centres of the circles intersects C_1 at P.

(b) (i) Determine the ratio in which P divides the line joining the centres of the circles. **2**

(ii) Hence, or otherwise, determine the coordinates of P. **2**

P is the centre of a third circle, C_3.

C_2 touches C_3 internally.

(c) Determine the equation of C_3. **1**

[END OF QUESTION PAPER]

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Question	Generic scheme	Illustrative scheme	Max mark
12. (a) (i)	•[1] write down coordinates of centre	•[1] $(13,-4)$	**1**
(ii)	•[2] substitute coordinates and process for c	•[2] $13^2+(-4)^2+14\times13-22\times(-4)\ldots$ leading to $c=-455$	**1**
Notes:			
1. Accept $x=13,\ y=-4$ for •[1]. 2. Do not accept $g=13,\ f=-4$ or $13,\ -4$ for •[1]. 3. For those who substitute into $r=\sqrt{g^2+f^2-c}$, working to find r must be shown for •[2] to be awarded.			
Commonly Observed Responses:			
(b) (i)	•[3] calculate two key distances •[4] state ratio	•[3] two from $r_2=25$, $r_1=10$ and $r_2-r_1=15$ •[4] $3:2$ or $2:3$	**2**
(ii)	•[5] identify centre of C_2 •[6] state coordinates of P	•[5] $(-7,11)$ or $\begin{pmatrix}-7\\11\end{pmatrix}$ •[6] $(5,2)$	**2**
Notes:			
4. The ratio must be consistent with the working for r_2-r_1 5. Evidence for •[3] may appear on a sketch. 6. For $3:2$ or $2:3$ with no working, award 0/2. 7. At •[6], the ratio used to identify the coordinates of P must be consistent with the sizes of the circles in the original diagram for •[6] to be available.			
Commonly Observed Responses:			
(c)	•[7] state equation	•[7] $(x-5)^2+(y-2)^2=1600$ or $x^2+y^2-10x-4y-1571=0$	**1**
Notes:			
Commonly Observed Responses:			

[END OF MARKING INSTRUCTIONS]

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ATTEMPT ALL QUESTIONS | MARKS

Total marks — 60

1. Functions f and g are defined on suitable domains by $f(x) = 5x$ and $g(x) = 2\cos x$.

   (a) Evaluate $f(g(0))$. **1**

   (b) Find an expression for $g(f(x))$. **2**

2. The point P $(-2, 1)$ lies on the circle $x^2 + y^2 - 8x - 6y - 15 = 0$.

   Find the equation of the tangent to the circle at P. **4**

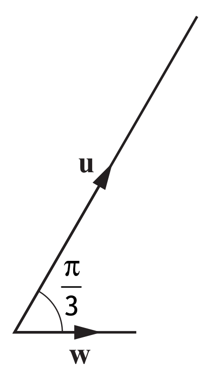
3. Given $y = (4x-1)^{12}$, find $\frac{dy}{dx}$. **2**

4. Find the value of k for which the equation $x^2 + 4x + (k-5) = 0$ has equal roots. **3**

5. Vectors $\mathbf{u}$ and $\mathbf{v}$ are $\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$ respectively.

   (a) Evaluate $\mathbf{u}.\mathbf{v}$. **1**

   (b)



   Vector $\mathbf{w}$ makes an angle of $\frac{\pi}{3}$ with $\mathbf{u}$ and $|\mathbf{w}| = \sqrt{3}$.

   Calculate $\mathbf{u}.\mathbf{w}$. **3**

[Turn over

Specific marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] evaluate expression	•[1] 10	1

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
1.	(b)		•[2] interpret notation •[3] state expression for $g\left(f\left(x\right)\right)$	•[2] $g(5x)$ •[3] $2\cos 5x$	2

Notes:

1. For $2\cos 5x$ without working, award both •[2] and •[3].
2. Candidates who interpret the composite function as either $g\left(x\right)\times f\left(x\right)$ or $g\left(x\right)+f\left(x\right)$ do not gain any marks.
3. $g\left(f\left(x\right)\right)=10\cos x$ award •[2]. However, $10\cos x$ with no working does not gain any marks.
4. $g\left(f\left(x\right)\right)$ leading to $2\cos\left(5x\right)$ followed by incorrect 'simplification' of the function award •[2] and •[3].

Commonly Observed Responses:

Candidate A

$$g\left(f\left(x\right)\right)=2\cos\left(5x\right) \quad \bullet^2 ✓ \quad \bullet^3 ✓$$
$$=10\cos\left(x\right)$$

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ state coordinates of centre	•¹ (4, 3)	
			•² find gradient of radius	•² $\frac{1}{3}$	
			•³ state perpendicular gradient	•³ -3	
			•⁴ determine equation of tangent	•⁴ $y=-3x-5$	4

Notes:

1. Accept $\frac{2}{6}$ for $\bullet^2$.
2. The perpendicular gradient must be simplified at $\bullet^3$ or $\bullet^4$ stage for $\bullet^3$ to be available.
3. $\bullet^4$ is only available as a consequence of trying to find and use a perpendicular gradient.
4. At $\bullet^4$, accept $y+3x+5=0$, $y+3x=-5$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.			$\bullet^1$ start to differentiate	$\bullet^1$ $12(4x-1)^{11}$.....	
			$\bullet^2$ complete differentiation	$\bullet^2$ ....$\times 4$	2

Notes:

1. $\bullet^2$ is awarded for correct application of the chain rule.

Commonly Observed Responses:

Candidate A	Candidate B
$\frac{dy}{dx}=12(4x-1)^{11}\times 4$ $\bullet^1$✓ $\bullet^2$✓ $\frac{dy}{dx}=36(4x-1)^{11}$ **Working subsequent to a correct answer: General Marking Principle (n)**	$\frac{dy}{dx}=36(4x-1)^{11}$ $\bullet^1$✗ $\bullet^2$✗ **Incorrect answer with no working**

Question			Generic scheme	Illustrative scheme	Max mark
4.			Method 1 •¹ use the discriminant •² apply condition and simplify •³ determine the value of k	Method 1 •¹ $4^2 - 4\times1\times(k-5)$ •² $36-4k=0$ or $36=4k$ •³ $k=9$	3
			Method 2 •¹ communicate and express in factorised form •² expand and compare •³ determine the value of k	Method 2 •¹ equal roots $\Rightarrow x^2+4x+(k-5)=(x+2)^2$ •² x^2+4x+4 leading to $k-5=4$ •³ $k=9$	

Notes:

1. At the •¹ stage, treat $4^2-4\times1\times k-5$ as bad form only if the candidate treats '$k-5$' as if it is bracketed in their next line of working. See Candidates A and B.
2. In Method 1 if candidates use any condition other than 'discriminant $=0$' then •² is lost and •³ is unavailable.

Commonly Observed Responses:

Candidate A		Candidate B		
$4^2-4\times1\times k-5$	•¹ ✓	$4^2-4\times1\underline{\times k-5}$	•¹ ✗	
$36-4k=0$	•² ✓	$11-4k=0$	•² $\boxed{\checkmark 1}$	
$k=9$	•³ ✓	$k=\frac{11}{4}$	•³ $\boxed{\checkmark 1}$	

Question	Generic scheme	Illustrative scheme	Max mark
5. (a)	$\bullet^1$ evaluate scalar product	$\bullet^1$ 1	**1**
Notes:			
Commonly Observed Responses:			

Question	Generic scheme	Illustrative scheme	Max mark
5. (b)	$\bullet^2$ calculate $\lvert \mathbf{u} \rvert$	$\bullet^2$ $\sqrt{27}$	
	$\bullet^3$ use scalar product	$\bullet^3$ $\sqrt{27} \times \sqrt{3} \times \cos\frac{\pi}{3}$	
	$\bullet^4$ evaluate $\mathbf{u.w}$	$\bullet^4$ $\frac{9}{2}$ or $4{\cdot}5$	**3**
Notes:			
1. Candidates who treat negative signs with a lack of rigour and arrive at $\sqrt{27}$ gain $\bullet^2$. 2. Surds must be fully simplified for $\bullet^4$ to be awarded.			
Commonly Observed Responses:			

MARKS

6. A function, h, is defined by $h(x) = x^3 + 7$, where $x \in \mathbb{R}$.

Determine an expression for $h^{-1}(x)$. **3**

7. A $(-3, 5)$, B $(7, 9)$ and C $(2, 11)$ are the vertices of a triangle.

Find the equation of the median through C. **3**

8. Calculate the rate of change of $d(t) = \frac{1}{2t}$, $t \neq 0$, when $t = 5$. **3**

9. A sequence is generated by the recurrence relation $u_{n+1} = m u_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. **2**

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(ii) Calculate this limit. **2**

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Question			Generic scheme	Illustrative scheme	Max mark
6.			**Method 1**	**Method 1**	
			•1 equate composite function to x	•1 $h\left(h^{-1}(x)\right)=x$	
			•2 write $h\left(h^{-1}(x)\right)$ in terms of $h^{-1}(x)$	•2 $\left(h^{-1}(x)\right)^3+7=x$	
			•3 state inverse function	•3 $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$	3
			Method 2	**Method 2**	
			•1 write as $y=x^3+7$ and start to rearrange	•1 $y-7=x^3$	
			•2 complete rearrangement	•2 $x=\sqrt[3]{y-7}$	
			•3 state inverse function	•3 $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$	3
			Method 3	**Method 3**	
			•1 interchange variables	•1 $x=y^3+7$	
			•2 complete rearrangement	•2 $y=\sqrt[3]{x-7}$	
			•3 state inverse function	•3 $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$	3

Notes:

1. $y=\sqrt[3]{x-7}$ $\left(\text{or } y=(x-7)^{\frac{1}{3}}\right)$ does not gain •3.

2. At •3 stage, accept h^{-1} expressed in terms of any dummy variable eg $h^{-1}(y)=\sqrt[3]{y-7}$.

3. $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$ with no working gains 3/3.

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Question			Generic scheme	Illustrative scheme	Max mark
7.			•¹ **find midpoint of AB** •² demonstrate the line is vertical •³ state equation	•¹ $(2,7)$ •² m_{median} undefined •³ $x=2$	 3

Notes:

1. $m_{median}=\frac{\pm 4}{0}$ alone is not sufficient to gain •². Candidates must use either 'vertical' or 'undefined'. However •³ is still available.
2. '$m_{median}=\frac{4}{0}$ ×' '$m_{median}=\frac{4}{0}$ impossible' '$m_{median}=\frac{4}{0}$ infinite' are **not** acceptable for •².
   However, if these are followed by either 'vertical' or 'undefined' then award •², and •³ is still available.
3. '$m_{median}=\frac{4}{0}=0$ undefined' '$m_{median}=\frac{-}{0}$ undefined' are **not** acceptable for •².
4. •³ is not available as a consequence of using a numeric gradient; however, see notes 5 and 6.
5. For candidates who find an incorrect midpoint (a,b), using the coordinates of A and B and find the 'median' through C without any further errors award 1/3. However, if $a=2$, then both •² and •³ are available.
6. For candidates who find $15y=2x+121$ (median through B) or $3y=2x+21$ (median through A) award 1/3.

Commonly Observed Responses:

Candidate A	Candidate B	Candidate C
$(2,7)$ •¹ ✓ $m=\frac{4}{0}$ $=0$ **undefined** •² ✗ $x=2$ •³ $\boxed{✓1}$	$(2,7)$ •¹ ✓ $m=\frac{4}{0}$ $=0$ •² ✗ $y=7$ •³ $\boxed{✓2}$	$(2,7)$ •¹ ✓ $m=\frac{4}{0}$ •² ^ $y-7=\frac{4}{0}(x-2)$ $0=4x-8$ $x=2$ •³ ✗
Candidate D $(2,7)$ •¹ ✓ Median passes through $(2,7)$ and $(2,11)$ •² ✗ $x=2$ •³ $\boxed{✓1}$	**Candidate E** $(2,7)$ •¹ ✓ Both coordinates have an x value $2\Rightarrow$ vertical line •² ✓ $x=2$ •³ ✓	

Question			Generic scheme	Illustrative scheme	Max mark
8.			$\bullet^1$ write in differentiable form	$\bullet^1$ $\frac{1}{2}t^{-1}$	
			$\bullet^2$ differentiate	$\bullet^2$ $-\frac{1}{2}t^{-2}$	
			$\bullet^3$ evaluate derivative	$\bullet^3$ $-\frac{1}{50}$	3

Notes:

1. Candidates who arrive at an expression containing more than one term at $\bullet^1$ award 0/3.
2. $\bullet^2$ is only available for differentiating a term containing a negative power of t.

Commonly Observed Responses:

Candidate A		Candidate B		Candidate C	
$2t^{-1}$	$\bullet^1$ ✗	$2t^{-1}$	$\bullet^1$ ✗	$-\frac{1}{2}t^{-2}$	$\bullet^1$ ✓ implied by $\bullet^2$✓
$-2t^{-2}$	$\bullet^2$ $\boxed{\checkmark 1}$	$-2t^{-2}$	$\bullet^2$ $\boxed{\checkmark 1}$		
$-\frac{2}{25}$	$\bullet^3$ $\boxed{\checkmark 1}$	$-\frac{1}{50}$	$\bullet^3$ ✗	$-\frac{1}{50}$	$\bullet^3$ ✓

Candidate D		Candidate E		Candidate F Bad form of chain rule		Candidate G	
$(2t)^{-1}$	$\bullet^1$ ✓	$(2t)^{-1}$	$\bullet^1$ ✓	$2t^{-1}$	$\bullet^1$ ✓	$2t^{-1}$	$\bullet^1$ ✗
$-(2t)^{-2}$	$\bullet^2$ ✗	$-(2t)^{-2}$	$\bullet^2$ ✗	$-2t^{-2}\times 2$	$\bullet^2$ ✓	$-2t^{-2}\times 2$	$\bullet^2$ ✗
$-\frac{1}{100}$	$\bullet^3$ $\boxed{\checkmark 1}$	$-\frac{2}{25}$	$\bullet^3$ ✗	$-\frac{1}{50}$	$\bullet^3$ ✓	$-\frac{4}{25}$	$\bullet^3$ $\boxed{\checkmark 1}$

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] interpret information •[2] state the value of m	•[1] $13 = 28m + 6$ **stated explicitly or in a rearranged form** •[2] $m = \frac{1}{4}$ or $m = 0\cdot 25$	**2**

Notes:

1. Stating '$m = \frac{1}{4}$' or simply writing '$\frac{1}{4}$' with no other working gains only •[2].

Commonly Observed Responses:

Candidate A		**Candidate B**	
$13 = 28u_n + 6$	•[1] ✗	$28 = 13m + 6$	•[1] ✗
$u_n = \frac{1}{4}$	•[2] ✓1	$m = \frac{22}{13}$	•[2] ✓1

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)	(i)	•3 communicate condition for limit to exist	•3 a limit exists as the recurrence relation is linear and $-1 < \frac{1}{4} < 1$	1

Notes:

2. For •3 accept:

   any of $-1 < \frac{1}{4} < 1$ or $\left|\frac{1}{4}\right| < 1$ or $0 < \frac{1}{4} < 1$ with no further comment;

   or statements such as:

   "$\frac{1}{4}$ lies between -1 and 1" or "$\frac{1}{4}$ is a proper fraction"

3. •3 is not available for:

   $-1 \leq \frac{1}{4} \leq 1$ or $\frac{1}{4} < 1$

   or statements such as:

   "It is between -1 and 1." or "$\frac{1}{4}$ is a fraction."

4. Candidates who state $-1 < m < 1$ can only gain •3 if it is **explicitly stated** that $m = \frac{1}{4}$ in part (a).
5. Do not accept '$-1 < a < 1$' for •3.

Commonly Observed Responses:

Candidate C			Candidate D		
(a)	$m = \frac{1}{4}$	•1 ✓ •2 ✓	(a)	$\frac{1}{4}$	•1 ✓ •2 ✓
(b)	$-1 < m < 1$	•3 ✓	(b)	$-1 < m < 1$	•3 ✗

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)	(ii)	$\bullet^4$ know how to calculate limit $\bullet^5$ calculate limit	$\bullet^4$ $\dfrac{6}{1-\frac{1}{4}}$ or $L=\frac{1}{4}L+6$ $\bullet^5$ 8	2

Notes:

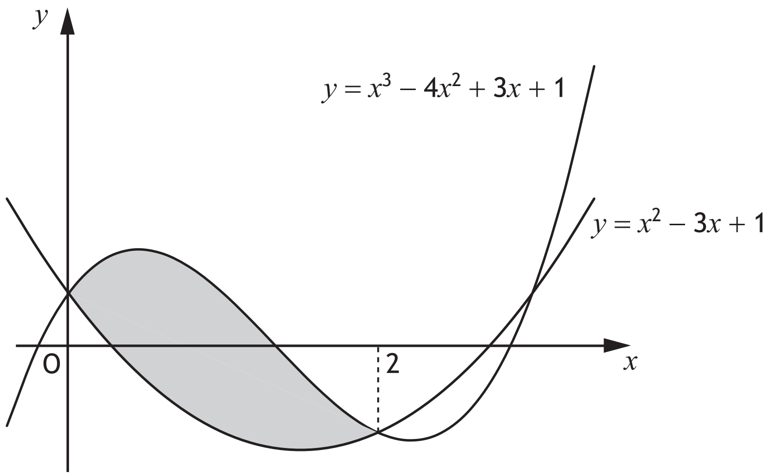
6. Do not accept $L=\dfrac{b}{1-a}$ with no further working for $\bullet^4$.
7. $\bullet^4$ and $\bullet^5$ are not available to candidates who conjecture that $L=8$ following the calculation of further terms in the sequence.
8. For $L=8$ with no working, award 0/2.
9. For candidates who use a value of m appearing ex nihilo or which is inconsistent with their answer in part (a) $\bullet^4$ and $\bullet^5$ are not available.

Commonly Observed Responses:

Candidate E - no valid limit **(a)** $m=4$ $\quad\bullet^1$ ✘ **(b)** $L=\dfrac{6}{1-4}$ $\quad\bullet^4$ $\boxed{\checkmark 1}$ $L=-2$ $\quad\bullet^5$ ✘	

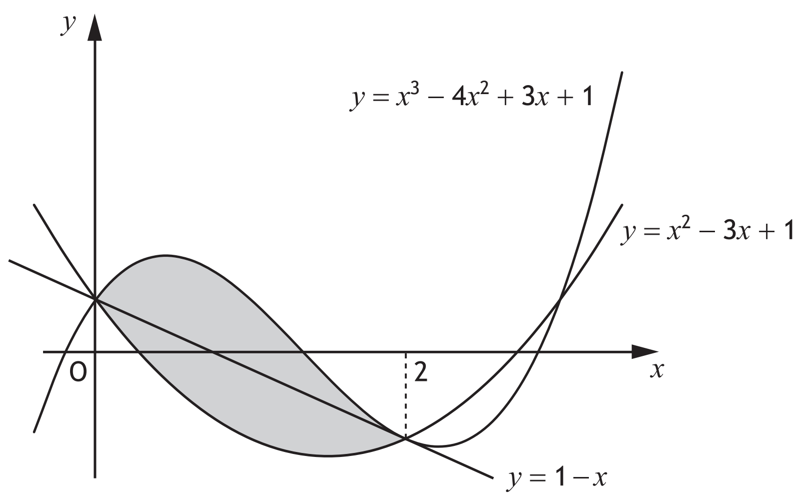
MARKS

10. Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.



(a) Calculate the shaded area. **5**

The line passing through the points of intersection of the curves has equation $y = 1 - x$.



(b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$. **4**

[Turn over

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)			**Method 1**	
			•¹ know to integrate between appropriate limits	•¹ $\int_0^2 \dots \, dx$	
			•² use "*upper - lower*"	•² $\int_0^2 \left(\left(x^3 - 4x^2 + 3x + 1\right) - \left(x^2 - 3x + 1\right)\right)$	
			•³ integrate	•³ $\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2$	
			•⁴ substitute limits	•⁴ $\left(\frac{2^4}{4} - \frac{5 \times 2^3}{3} + 3 \times 2^2\right) - (0)$	
			•⁵ evaluate area	•⁵ $\frac{8}{3}$	
				Method 2	
			•¹ know to integrate between appropriate limits for both integrals	•¹ $\int_0^2 \dots \, dx$ and $\int_0^2 \dots \, dx$	
			•² integrate both functions	•² $\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2} + x$ and $\frac{x^3}{3} - \frac{3x^2}{2} + x$	
			•³ substitute limits into both functions	•³ $\left(\frac{2^4}{4} - \frac{4\left(2^3\right)}{3} + \frac{3\left(2^2\right)}{2} + 2\right) - 0$ and $\left(\frac{2^3}{3} - \frac{3\left(2^2\right)}{2} + 2\right) - 0$	
			•⁴ evaluation of both functions	•⁴ $\frac{4}{3}$ and $\frac{-4}{3}$	
			•⁵ evidence of subtracting areas	•⁵ $\frac{4}{3} - \frac{-4}{3} = \frac{8}{3}$	**5**

Question			Generic scheme	Illustrative scheme	Max mark
10.	(b)			**Method 1**	
			$\bullet^6$ use "*line - quadratic*"	$\bullet^6$ $\int\left(\left(1-x\right)-\left(x^2-3x+1\right)\right)dx$	
			$\bullet^7$ integrate	$\bullet^7$ $-\frac{x^3}{3}+x^2$	
			$\bullet^8$ substitute limits and evaluate integral	$\bullet^8$ $\left(-\frac{2^3}{3}+2^2\right)-\left(0\right)=\frac{4}{3}$	
			$\bullet^9$ state fraction	$\bullet^9$ $\frac{1}{2}$	
				Method 2	
			$\bullet^6$ use "cubic - *line*"	$\bullet^6$ $\int\left(\left(x^3-4x^2+3x+1\right)-\left(1-x\right)\right)dx$	
			$\bullet^7$ integrate	$\bullet^7$ $\frac{x^4}{4}-\frac{4x^3}{3}+2x^2$	
			$\bullet^8$ substitute limits and evaluate integral	$\bullet^8$ $\left(\frac{2^4}{4}-4\times\frac{2^3}{3}+2\times2^2\right)-\left(0\right)=\frac{4}{3}$	
			$\bullet^9$ state fraction	$\bullet^9$ $\frac{1}{2}$	
				Method 3	
			$\bullet^6$ integrate line	$\bullet^6$ $\int\left(1-x\right)dx=\left[x-\frac{x^2}{2}\right]_0^2$	
			$\bullet^7$ substitute limits and evaluate integral	$\bullet^7$ $\left(2-\frac{2^2}{2}\right)-\left(0\right)=0$	
			$\bullet^8$ evidence of subtracting integrals	$\bullet^8$ $0-\left(-\frac{4}{3}\right)=\frac{4}{3}$ or $\frac{4}{3}-0$	
			$\bullet^9$ state fraction	$\bullet^9$ $\frac{1}{2}$	4

MARKS

11. A and B are the points $(-7, 2)$ and $(5, a)$.

AB is parallel to the line with equation $3y - 2x = 4$.

Determine the value of a. **3**

12. Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a. **3**

13. Find $\displaystyle\int \frac{1}{(5-4x)^{\frac{1}{2}}}\,dx,\ \ x < \frac{5}{4}$. **4**

14. (a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x-a)°$, where $k > 0$ and $0 < a < 360$. **4**

(b) Hence, or otherwise, sketch the graph with equation $y = \sqrt{3}\sin x° - \cos x°$, $0 \le x \le 360$. **3**

Use the diagram provided in the answer booklet.

Question			Generic scheme	Illustrative scheme	Max mark
11.				**Method 1**	
			•¹ determine the gradient of given line or of AB	•¹ $\frac{2}{3}$ or $\frac{a-2}{12}$	
			•² determine the other gradient	•² $\frac{a-2}{12}$ or $\frac{2}{3}$	
			•³ find a	•³ 10	
				Method 2	
			•¹ determine the gradient of given line	•¹ $\frac{2}{3}$ stated or implied by •²	
			•² equation of line and substitute	•² $y-2=\frac{2}{3}(x+7)$ $a-2=\frac{2}{3}(5+7)$	
			•³ solve for a	•³ 10	3

Notes:

Commonly Observed Responses:

Candidate A - using simultaneous equations	**Candidate B**	**Candidate C - Method 2**
$m_{\text{line}}=\frac{2}{3}$ •¹ ✓	$m_{AB}=\frac{a-2}{12}$ •¹ ✓	•¹ ✓
$\left.\begin{array}{l}3y=2x+20\\3y=2x-10+3a\end{array}\right\}$ •² ✓	$\frac{a-2}{12}=\underline{-2}$ •² ✗	$y-2=\frac{2}{3}(x+7)$
$0=0+30-3a$	$a=-22$ •³ $\boxed{✓1}$	$3y=2x+20$
$3a=30$		$3y=2\times5+20$ •² ✓
$a=10$ •³ ✓		$3y=30$
		$y=10$
		No mention of a •³ ^

Question			Generic scheme	Illustrative scheme	Max mark
12.			•1 use laws of logs	•1 $\log_a 9$	
			•2 write in exponential form	•2 $a^{\frac{1}{2}} = 9$	
			•3 solve for a	•3 81	3

Notes:

1. $\frac{36}{4}$ must be simplified at •1 or •2 stage for •1 to be awarded.
2. Accept $\log 9$ at •1.
3. •2 may be implied by •3.

Commonly Observed Responses:

Candidate A		Candidate B		Candidate C	
$\log_a 144$	•1 ✗	$\log_a 32$	•1 ✗	$\log_a 9$	•1 ✓
$a^{\frac{1}{2}} = 144$	•2 $\boxed{✓1}$	$a^{\frac{1}{2}} = 32$	•2 $\boxed{✓1}$	$a = 9^{\frac{1}{2}}$	•2 ✗
$a = 12$	•3 ✗		•3 ^	$a = 3$	•3 $\boxed{✓2}$

Candidate D

$2\log_a 36 - 2\log_a 4 = 1$

$\log_a 36^2 - \log_a 4^2 = 1$ •1 ✓

$\log_a \frac{36^2}{4^2} = 1$

$\log_a 81 = 1$ •2 ✓

$a = 81$ •3 ✓

Question			Generic scheme	Illustrative scheme	Max mark
13.			•[1] write in integrable form	•[1] $(5-4x)^{-\frac{1}{2}}$	
			•[2] start to integrate	•[2] $\dfrac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}}\ldots$	
			•[3] process coefficient of x	•[3] $\ldots\times\dfrac{1}{(-4)}$	
			•[4] complete integration and simplify	•[4] $-\dfrac{1}{2}(5-4x)^{\frac{1}{2}}+c$	4

Notes:

1. For candidates who differentiate throughout, only •[1] is available.
2. For candidates who 'integrate the denominator' without attempting to write in integrable form award 0/4.
3. If candidates start to integrate individual terms within the bracket or attempt to expand a bracket no further marks are available.
4. '$+c$' is required for •[4].

Commonly Observed Responses:

Candidate A		Candidate B	
$(5-4x)^{-\frac{1}{2}}$	•[1] ✓	$(5-4x)^{\frac{1}{2}}$	•[1] ✗
$\dfrac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}}$	•[2] ✓ •[3] ^	$\dfrac{(5-4x)^{\frac{3}{2}}}{\frac{3}{2}}\times\dfrac{1}{(-4)}$	•[2] $\boxed{✓1}$ •[3] ✓
$2(5-4x)^{\frac{1}{2}}+c$	•[4] $\boxed{✓2}$	$-\dfrac{(5-4x)^{\frac{3}{2}}}{6}+c$	•[4] $\boxed{✓1}$

Candidate C		Candidate D	
Differentiate in part:		Differentiate in part:	
$(5-4x)^{-\frac{1}{2}}$	•[1] ✓	$(5-4x)^{-\frac{1}{2}}$	•[1] ✓
$-\dfrac{1}{2}(5-4x)^{-\frac{3}{2}}\times\dfrac{1}{(-4)}$	•[2] ✗ •[3] ✓	$\dfrac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}}\times(-4)$	•[2] ✓ •[3] ✗
$\dfrac{1}{8}(5-4x)^{-\frac{3}{2}}+c$	•[4] $\boxed{✓1}$	$-8(5-4x)^{\frac{1}{2}}+c$	•[4] $\boxed{✓1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.	(a)		•¹ use compound angle formula	•¹ $k\sin x°\cos a° - k\cos x°\sin a°$ **stated explicitly**	
			•² compare coefficients	•² $k\cos a° = \sqrt{3},\ k\sin a° = 1$ **stated explicitly**	
			•³ process for k	•³ $k = 2$	
			•⁴ process for a and express in required form	•⁴ $2\sin(x-30)°$	4

Notes:

1. Accept $k(\sin x°\cos a° - \cos x°\sin a°)$ for •¹. Treat $k\sin x°\cos a° - \cos x°\sin a°$ as bad form only if the equations at the •² stage both contain k.
2. Do not penalise the omission of degree signs.
3. $2\sin x°\cos a° - 2\cos x°\sin a°$ or $2(\sin x°\cos a° - \cos x°\sin a°)$ is acceptable for •¹ and •³.
4. In the calculation of $k = 2$, do not penalise the appearance of -1.
5. Accept $k\cos a° = \sqrt{3},\ -k\sin a° = -1$ for •².
6. •² is not available for $k\cos x° = \sqrt{3},\ k\sin x° = 1$, however, •⁴ is still available.
7. •³ is only available for a single value of k, $k > 0$.
8. •³ is not available to candidates who work with $\sqrt{4}$ throughout parts (a) and (b) without simplifying at any stage.
9. •⁴ is not available for a value of a given in radians.
10. Candidates may use any form of the wave equation for •¹, •² and •³, however, •⁴ is only available if the value of a is interpreted in the form $k\sin(x-a)°$
11. Evidence for •⁴ may only appear as a label on the graph in part (b).

Commonly Observed Responses:

Responses with missing information in working:

Candidate A	Candidate B	
•¹ ^ $2\cos a = \sqrt{3}$ $2\sin a = 1$ •² ✓ •³ ✓ $\tan a = \frac{1}{\sqrt{3}},\ a = 30$ $2\sin(x-30)°$ •⁴ ✓	$k\sin x\cos a - k\cos x\sin a$ •¹ ✓ $\cos a = \sqrt{3}$ $\sin a = 1$ •² ✗ $\tan a = \frac{1}{\sqrt{3}}$ $a = 30$ Not consistent with equations at •². $2\sin(x-30)°$ •³ ✓ •⁴ ✗	

Question			Generic scheme	Illustrative scheme	Max mark
14.	(b)		$\bullet^5$ roots identifiable from graph	$\bullet^5$ 30 and 210	
			$\bullet^6$ coordinates of both turning points identifiable from graph	$\bullet^6$ $(120, 2)$ and $(300, -2)$	
			$\bullet^7$ y-intercept and value of y at $x = 360$ identifiable from graph	$\bullet^7$ -1	**3**

Notes:

12. $\bullet^5$, $\bullet^6$and $\bullet^7$ are only available for attempting to draw a "sine" graph with a period of 360°.
13. Ignore any part of a graph drawn outwith $0 \le x \le 360$.
14. Vertical marking is not applicable to $\bullet^5$ and $\bullet^6$.
15. Candidates sketch arrived at in (b) must be consistent with the equation obtained in (a), see also candidates I and J.
16. For any incorrect horizontal translation of the graph of the wave function arrived at in part(a) only $\bullet^6$ is available.

Commonly Observed Responses:

Candidate I	**Candidate J**
(a) $2\sin(x-30)$ correct equation (b) Incorrect translation: Sketch of $2\sin(x+30)$ Only $\bullet^6$ is available	(a) $2\sin(x+30)$ incorrect equation (b) Sketch of $2\sin(x+30)$ All 3 marks are available

 Back to Table

MARKS

15. A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.
The turning point is $(2, 3)$.

Diagram 2 shows part of the graph with equation $y = h(x)$.
The turning point is $(7, 6)$.

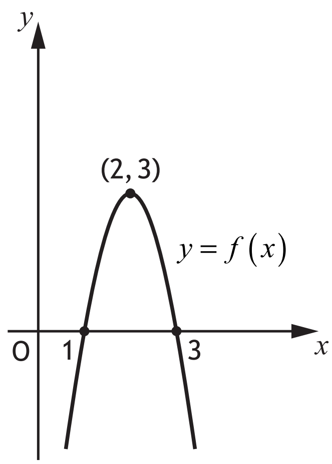


Diagram 1

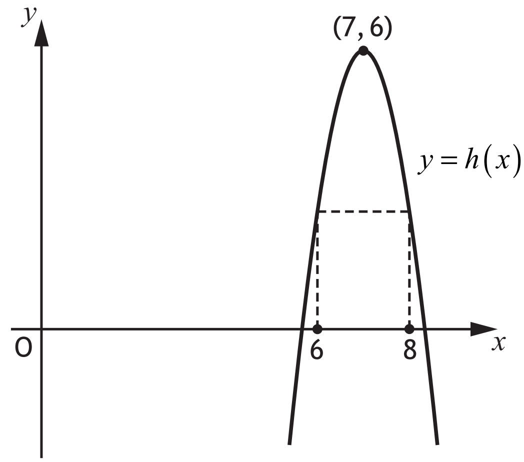


Diagram 2

(a) Given that $h(x) = f(x + a) + b$.

Write down the values of a and b. **2**

(b) It is known that $\int_1^3 f(x)\,dx = 4$.

Determine the value of $\int_6^8 h(x)\,dx$. **1**

(c) Given $f'(1) = 6$, state the value of $h'(8)$. **1**

[END OF QUESTION PAPER]

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•[1] state value of a •[2] state value of b	•[1] -5 •[2] 3	2

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative Scheme	Max Mark
15.	(b)		•[3] state value of integral	•[3] 10	1

Notes:

1. Candidates answer at (b) must be consistent with the value of b obtained in (a).
2. In parts (b) and (c), candidates who have 10 and -6 accompanied by working, the working must be checked to ensure that no errors have occurred prior to the correct answer appearing.

Commonly Observed Responses:

Candidate A

From (a)

$a = -3$ •[1] ✘

$b = 5$ •[2] ✘

$\int h(x)\,dx = 14$ •[3] $\boxed{✓1}$

Question			Generic scheme	Illustrative scheme	Max mark
15.	(c)		•[4] state value of derivative	•[4] -6	1

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

MARKS

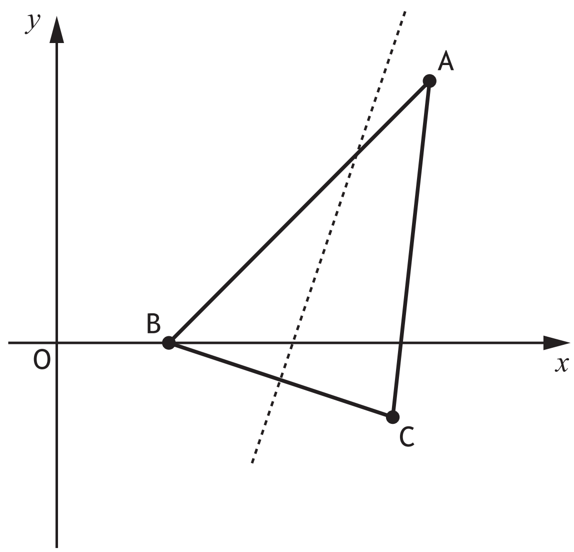
Attempt ALL questions

Total marks — 70

1. Triangle ABC is shown in the diagram below.

The coordinates of B are (3,0) and the coordinates of C are (9,–2).

The broken line is the perpendicular bisector of BC.



(a) Find the equation of the perpendicular bisector of BC. **4**

(b) The line AB makes an angle of 45° with the positive direction of the x-axis.

Find the equation of AB. **2**

(c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC. **2**

2. (a) Show that $(x-1)$ is a factor of $f(x)=2x^3-5x^2+x+2$. **2**

(b) Hence, or otherwise, solve $f(x)=0$. **3**

[Turn over

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] find mid-point of BC	•[1] $(6,-1)$	
			•[2] calculate gradient of BC	•[2] $-\frac{2}{6}$	
			•[3] use property of perpendicular lines	•[3] 3	
			•[4] determine equation of line in a simplified form	•[4] $y=3x-19$	4

Notes:

1. •[4] is only available as a consequence of using a perpendicular gradient **and** a midpoint.
2. The gradient of the perpendicular bisector must appear in simplified form at •[3] or •[4] stage for •[3] to be awarded.
3. At •[4], accept $3x-y-19=0$, $3x-y=19$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
1.	(b)	•[5] use $m=\tan\theta$	•[5] 1	
		•[6] determine equation of AB	•[6] $y=x-3$	2

Notes:

4. At •[6], accept $y-x+3=0$, $y-x=-3$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
1.	(c)	•[7] find x or y coordinate	•[7] $x=8$ or $y=5$	2
		•[8] find remaining coordinate	•[8] $y=5$ or $x=8$	

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		**Method 1** •[1] know to use $x=1$ in synthetic division •[2] complete division, interpret result and state conclusion	**Method 1** •[1] $\begin{array}{r\|rrrr} 1 & 2 & -5 & 1 & 2 \\ \hline & 2 & & & \end{array}$ •[2] $\begin{array}{r\|rrrr} 1 & 2 & -5 & 1 & 2 \\ & & 2 & -3 & -2 \\ \hline & 2 & -3 & -2 & 0 \end{array}$ Remainder $=0$ $\therefore (x-1)$ is a factor	**2**
			Method 2 •[1] know to substitute $x=1$ •[2] complete evaluation, interpret result and state conclusion	**Method 2** •[1] $2(1)^3-5(1)^2+(1)+2$ •[2] $=0$ $\therefore (x-1)$ is a factor	**2**
			Method 3 •[1] start long division and find leading term in quotient •[2] complete division, interpret result and state conclusion	**Method 3** •[1] $\begin{array}{r} 2x^2 \\ (x-1)\overline{)\,2x^3-5x^2+x+2} \end{array}$ •[2] $\begin{array}{rl} & 2x^2-3x-2 \\ (x-1) & \overline{)\,2x^3-5x^2+x+2} \\ & \underline{2x^3-2x^2} \\ & \quad -3x^2+x \\ & \quad \underline{-3x^2+3x} \\ & \qquad -2x+2 \\ & \qquad \underline{-2x+2} \\ & \qquad\quad 0 \end{array}$ remainder $=0$ $\therefore$ $(x-1)$ is a factor	**2**

Question	Generic scheme	Illustrative scheme	Max mark

Notes:

1. Communication at $\bullet^2$ must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded.
2. Accept any of the following for $\bullet^2$:
   - '$f(1)=0$ so $(x-1)$ is a factor'
   - 'since remainder = 0, it is a factor'
   - the 0 from any method linked to the word 'factor' by e.g. 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$'
3. Do not accept any of the following for $\bullet^2$:
   - double underlining the zero or boxing the zero without comment
   - '$x=-1$ is a factor', '$(x+1)$ is a factor', '$(x+1)$ is a root', '$x=1$ is a root', '$(x-1)$ is a root' '$x=-1$ is a root'.
   - the word 'factor' only with no link

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.	(b)		$\bullet^3$ state quadratic factor	$\bullet^3$ $2x^2-3x-2$	
			$\bullet^4$ find remaining factors	$\bullet^4$ $(2x+1)$ and $(x-2)$	
			$\bullet^5$ state solution	$\bullet^5$ $x=-\frac{1}{2}, 1, 2$	3

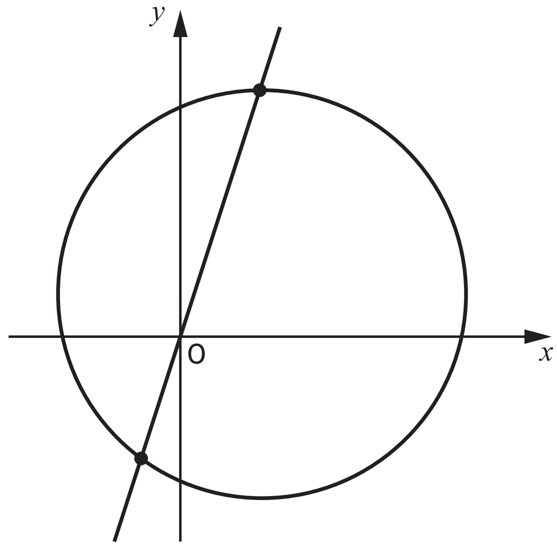
Notes:

4. The appearance of "$=0$" is not required for $\bullet^5$ to be awarded.
5. Candidates who identify a different initial factor and subsequent quadratic factor can gain all available marks.
6. $\bullet^5$ is only available as a result of a valid strategy at $\bullet^3$ and $\bullet^4$.
7. Accept $\left(-\frac{1}{2},0\right)$, $(1,0)$, $(2,0)$ for $\bullet^5$.

Commonly Observed Responses:

MARKS

3. The line $y = 3x$ intersects the circle with equation $(x-2)^2 + (y-1)^2 = 25$.



Find the coordinates of the points of intersection. **5**

4. (a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$. **3**

(b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$. **2**

(c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x. **2**

Question			Generic scheme	Illustrative scheme	Max mark
3.			•¹ substitute for y	•¹ $(x-2)^2+(3x-1)^2=25$ or $x^2-4x+4+(3x)^2-2(3x)+1=25$	
			•² express in standard quadratic form	•² $10x^2-10x-20=0$	
			•³ factorise	•³ $10(x-2)(x+1)=0$	
			•⁴ find x coordinates	•⁴ $x=2$ (•⁴) $x=-1$ (•⁵)	
			•⁵ find y coordinates	•⁵ $y=6$ $y=-3$	5

Notes:

1. At •³ the quadratic must lead to two distinct real roots for •⁴ and •⁵ to be available.
2. •² is only available if '$=0$' appears at •² or •³ stage.
3. If a candidate arrives at an equation which is not a quadratic at •² stage, then •³, •⁴ and •⁵ are not available
4. At •³ do not penalise candidates who fail to extract the common factor or who have divided the quadratic equation by 10.
5. •³ is available for substituting correctly into the quadratic formula.
6. •⁴ and •⁵ may be marked either horizontally or vertically.
7. For candidates who identify **both** solutions by inspection, full marks may be awarded provided they justify that their points lie on **both** the line and the circle. Candidates who identify **both** solutions, but justify only one gain 2 out of 5.

Commonly Observed Responses:

Candidate A		Candidate B
$(x-2)^2+(3x-1)^2=25$	•¹ ✓	Candidates who substitute into the circle equation only •¹ ✓ •² ✓ •³ ✓ •⁴ ✓
$10x^2-10x=20$	•² ✗	Sub $x=2$: $y^2-2y-24=0$, $(y-6)(y+4)=0$, $y=6$ or $\cancel{y=-4}$
$10x(x-1)=20$	•³ $\boxed{✓2}$	Sub $x=-1$: $y^2-2y-15=0$, $(y+3)(y-5)=0$, $y=-3$ or $\cancel{y=5}$
$x=2$ $x=3$	•⁴ ✗	$(2,6)$ $(-1,-3)$ •⁵ ✗
$y=6$ $y=9$	•⁵ $\boxed{✓2}$	

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		**Method 1** •¹ identify common factor •² complete the square •³ process for c and write in required form	**Method 1** •¹ $3(x^2+8x$...... stated or implied by •² •² $3(x+4)^2$...... •³ $3(x+4)^2+2$	3
			Method 2 •¹ expand completed square form •² equate coefficients •³ process for b and c and write in required form	**Method 2** •¹ $ax^2+2abx+ab^2+c$ •² $a=3,\ \ 2ab=24,\ \ ab^2+c=50$ •³ $3(x+4)^2+2$	3

Notes:

1. $3(x+4)^2+2$ with no working gains •¹ and •² only; however, see Candidate G.
2. •³ is only available for a calculation involving both multiplication and subtraction of integers.

Commonly Observed Responses:

Candidate A	Candidate B
$3\left(x^2+8x+\frac{50}{3}\right)$ •¹ ✓ $3\left(x^2+8x+16-16+\frac{50}{3}\right)$ •²^ further working is required	$3x^2+24x+50=3(x+8)^2-64+50$ •¹ ✗ •² ✗ $=3(x+8)^2-14$ •³ $\boxed{✓2}$
Candidate C	**Candidate D**
$ax^2+2abx+ab^2+c$ •¹ ✓ $a=3,\ \ 2ab=24,\ \ b^2+c=50$ •² ✗ $a=3,\ \ b=4,\ \ c=34$ $3(x+4)^2+34$ •³ $\boxed{✓1}$	$3((x^2+24x)+50)$ •¹ ✗ $3((x+12)^2-144)+50$ •² $\boxed{✓1}$ $3(x+12)^2-382$ •³ $\boxed{✓1}$

Question	Generic scheme	Illustrative scheme	Max mark

Candidate E	**Candidate F**
$a(x+b)^2+c=ax^2+2abx+ab^2+c$ •¹ ✓ $a=3,\ \ 2ab=24,\ \ ab^2+c=50$ •² ✓ $b=4,\ c=2$ •³ ✓ •³ is awarded as all working relates to completed square form	$ax^2+2abx+ab^2+c$ •¹ ✓ $a=3,\ \ 2ab=24,\ \ ab^2+c=50$ •² ✓ $b=4,\ c=2$ •³ ✗ •³ is lost as no reference is made to completed square form
Candidate G $3(x+4)^2+2$ Check: $3(x^2+8x+16)+2$ $=3x^2+24x+48+2$ $=3x^2+24x+50$ **Award 3/3**	**Candidate H** $3x^2+24x+50$ $=3(x+4)^2-16+50$ •¹ ✓ •² ✓ $=3(x+4)^2+34$ •³ ✗

Question			Generic scheme	Illustrative scheme	Max mark
4.	(b)		•⁴ differentiate two terms •⁵ complete differentiation	•⁴ $3x^2+24x....$ •⁵ $....+50$	**2**

Notes:

3. •⁴ is awarded for any two of the following three terms: $3x^2$, $+24x$, $+50$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.	(c)		**Method 1** •6 link with (a) and identify sign of $(x+4)^2$ •7 communicate reason **Method 2** •6 identify minimum value of $f'(x)$ •7 communicate reason	**Method 1** •6 $f'(x)=3(x+4)^2+2$ and $(x+4)^2 \geq 0 \ \forall x$ •7 $\therefore 3(x+4)^2+2>0 \Rightarrow$ always strictly increasing **Method 2** •6 eg minimum value =2 or annotated sketch •7 $2>0 \therefore (f'(x)>0) \Rightarrow$ always strictly increasing	2

Notes:

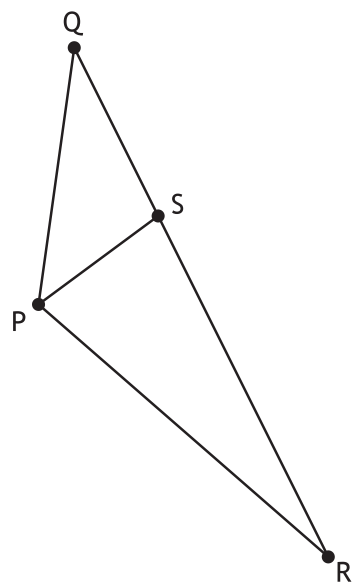
4. Do not penalise $(x+4)^2>0$ or the omission of $f'(x)$ at •6 in Method 1.
5. Responses in part (c) must be consistent with working in parts (a) and (b) for •6 and •7 to be available.
6. Where erroneous working leads to a candidate considering a function which is not always strictly increasing, only •6 is available.
7. At •6 communication should be explicitly in terms of the given function. Do not accept statements such as "(something)$^2 \geq 0$", "something squared ≥ 0". However, •7 is still available.

Commonly Observed Responses:

Candidate I	Candidate J
$f'(x)=3(x+4)^2+2$ $3(x+4)^2+2>0 \Rightarrow$ strictly increasing. Award 1 out of 2	Since $3x^2+24x+50=3(x+4)^2+\frac{166}{50}$ and $(x+4)^2$ is >0 for all x then $3(x+4)^2+\frac{166}{50}>0$ for all x. Hence the curve is strictly increasing for all values of x. •6 ✓ •7 $\boxed{✓1}$

MARKS

5. In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.



(a) Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. **2**

The point S divides QR in the ratio 1:2.

(b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$. **2**

(c) Hence, find the size of angle QPS. **5**

6. Solve $5\sin x - 4 = 2\cos 2x$ for $0 \le x < 2\pi$. **5**

7. (a) Find the x-coordinate of the stationary point on the curve with equation $y = 6x - 2\sqrt{x^3}$. **4**

(b) Hence, determine the greatest and least values of y in the interval $1 \le x \le 9$. **3**

[Turn over

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		$\bullet^1$ identify pathway $\bullet^2$ state $\overrightarrow{PQ}$	$\bullet^1$ $\overrightarrow{PR}+\overrightarrow{RQ}$ stated or implied by $\bullet^2$ $\bullet^2$ $-3\mathbf{i}-4\mathbf{j}+5\mathbf{k}$	**2**

Notes:

1. Award $\bullet^1$ $(9\mathbf{i}+5\mathbf{j}+2\mathbf{k})+(-12\mathbf{i}-9\mathbf{j}+3\mathbf{k})$.
2. Candidates who choose to work with column vectors and leave their answer in the form $\begin{pmatrix}-3\\-4\\5\end{pmatrix}$ cannot gain $\bullet^2$.
3. $\bullet^2$ is not available for simply adding or subtracting vectors within an invalid strategy.
4. Where candidates choose specific points consistent with the given vectors, only $\bullet^1$ and $\bullet^4$ are available. However, should the statement 'without loss of generality' precede the selected points then marks $\bullet^1$, $\bullet^2$, $\bullet^3$ and $\bullet^4$ are all available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)		$\bullet^3$ interpret ratio $\bullet^4$ identify pathway and demonstrate result	$\bullet^3$ $\frac{2}{3}$ or $\frac{1}{3}$ $\bullet^4$ $\overrightarrow{PR}+\frac{2}{3}\overrightarrow{RQ}$ or $\overrightarrow{PQ}+\frac{1}{3}\overrightarrow{QR}$ leading to $\mathbf{i}-\mathbf{j}+4\mathbf{k}$	**2**

Notes:

5. This is a 'show that' question. Candidates who choose to work with column vectors must write their final answer in the required form to gain $\bullet^4$. $\begin{pmatrix}1\\-1\\4\end{pmatrix}$ does not gain $\bullet^4$.
6. Beware of candidates who fudge their working between $\bullet^3$ and $\bullet^4$.

Question			Generic scheme	Illustrative scheme	Max mark
5.	(c)		**Method 1**	**Method 1**	
			•5 evaluate $\overrightarrow{PQ}.\overrightarrow{PS}$	•5 $\overrightarrow{PQ}.\overrightarrow{PS} = 21$	
			•6 evaluate $\lvert\overrightarrow{PQ}\rvert$	•6 $\lvert\overrightarrow{PQ}\rvert = \sqrt{50}$	
			•7 evaluate $\lvert\overrightarrow{PS}\rvert$	•7 $\lvert\overrightarrow{PS}\rvert = \sqrt{18}$	
			•8 use scalar product	•8 $\cos QPS = \dfrac{21}{\sqrt{50}\times\sqrt{18}}$	
			•9 calculate angle	•9 $45\cdot6°$ or $0\cdot795$ radians	5
			Method 2	**Method 2**	
			•5 evaluate $\lvert\overrightarrow{QS}\rvert$	•5 $\lvert\overrightarrow{QS}\rvert = \sqrt{26}$	
			•6 evaluate $\lvert\overrightarrow{PQ}\rvert$	•6 $\lvert\overrightarrow{PQ}\rvert = \sqrt{50}$	
			•7 evaluate $\lvert\overrightarrow{PS}\rvert$	•7 $\lvert\overrightarrow{PS}\rvert = \sqrt{18}$	
			•8 use cosine rule	•8 $\cos QPS = \dfrac{(\sqrt{50})^2+(\sqrt{18})^2-(\sqrt{26})^2}{2\times\sqrt{50}\times\sqrt{18}}$	
			•9 calculate angle	•9 $45\cdot6°$ or $0\cdot795$ radians	5

Notes:

7. For candidates who use $\overrightarrow{PS}$ **not equal to** $\mathbf{i}-\mathbf{j}+4\mathbf{k}$ •5 is not available in Method 1 or •7 in Method 2.
8. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. However, $\sqrt{1^2-1^2+4^2}$ leading to $\sqrt{16}$ indicates an invalid method for calculating the magnitude. No mark can be awarded for any magnitude arrived at using an invalid method.
9. •8 is not available to candidates who simply state the formula $\cos\theta = \dfrac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$.

   However, $\cos\theta = \dfrac{\overrightarrow{PQ}.\overrightarrow{PS}}{\lvert\overrightarrow{PQ}\rvert\times\lvert\overrightarrow{PS}\rvert}$ or $\cos\theta = \dfrac{21}{\sqrt{50}\times\sqrt{18}}$ is acceptable. Similarly for Method 2.
10. Accept answers which round to $46°$ or $0\cdot8$ radians.
11. Do not penalise the omission or incorrect use of units.
12. •9 is only available as a result of using a valid strategy.
13. •9 is only available for a single angle.
14. For a correct answer with no working award 0/5.

Question			Generic scheme	Illustrative scheme	Max mark
6.			•[1] substitute appropriate double angle formula	•[1] $5\sin x - 4 = 2\left(1 - 2\sin^2 x\right)$	
			•[2] express in standard quadratic form	•[2] $4\sin^2 x + 5\sin x - 6 = 0$	
			•[3] factorise	•[3] $(4\sin x - 3)(\sin x + 2)$	
			•[4] solve for $\sin x°$	•[4] •[5] •[4] $\sin x = \frac{3}{4}$, $\sin x = -2$	
			•[5] solve for x	•[5] $x = 0\cdot848,\ 2\cdot29$, $\cancel{\sin x = -2}$	5

Notes:

1. •[1] is not available for simply stating $\cos 2x = 1 - 2\sin^2 x$ with no further working.
2. In the event of $\cos^2 x° - \sin^2 x°$ or $2\cos^2 x° - 1$ being substituted for $\cos 2x$, •[1] cannot be awarded until the equation reduces to a quadratic in $\sin x°$.
3. Substituting $1 - 2\sin^2 A$ or $1 - 2\sin^2 \alpha$ for $\cos 2x$ at •[1] stage should be treated as bad form provided the equation is written in terms of x at •[2] stage. Otherwise, •[1] is not available.
4. '$= 0$' must appear by •[3] stage for •[2] to be awarded. However, for candidates using the quadratic formula to solve the equation, '$= 0$' must appear at •[2] stage for •[2] to be awarded.
5. $5\sin x + 4\sin^2 x - 6 = 0$ does not gain •[2] unless •[3] is awarded.
6. $\sin x = \dfrac{-5 \pm \sqrt{121}}{8}$ gains •[3].
7. Candidates may express the equation obtained at •[2] in the form $4s^2 + 5s - 6 = 0$ or $4x^2 + 5x - 6 = 0$. In these cases, award •[3] for $(4s - 3)(s + 2) = 0$ or $(4x - 3)(x + 2) = 0$. However, •[4] is only available if $\sin x$ appears explicitly at this stage.
8. •[4] and •[5] are only available as a consequence of solving a quadratic equation.
9. •[3], •[4] and •[5] are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$.
10. •[5] is not available to candidates who work in degrees and do not convert their solutions into radian measure.
11. Accept answers which round to $0\cdot85$ and $2\cdot3$ at •[5] eg $\dfrac{49\pi}{180}, \dfrac{131\pi}{180}$.
12. Answers written as decimals should be rounded to no fewer than 2 significant figures.
13. Do not penalise additional solutions at •[5].

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Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] write in differentiable form	•[1] $\ldots - 2x^{\frac{3}{2}}$ stated or implied	
			•[2] differentiate one term	•[2] $\frac{dy}{dx} = 6\ldots$ or $\frac{dy}{dx} = \ldots - 3x^{\frac{1}{2}}\ldots$	
			•[3] complete differentiation and equate to zero	•[3] $\ldots - 3x^{\frac{1}{2}} = 0$ or $6\ldots = 0$	
			•[4] solve for x	•[4] $x = 4$	4

Notes:

1. For candidates who do not differentiate a term involving a fractional index, either •[2] or •[3] is available but not both.
2. •[4] is available only as a consequence of solving an equation involving a fractional power of x.
3. For candidates who integrate one or other of the terms •[4] is unavailable.

Commonly Observed Responses:

Candidate A - differentiating incorrectly		Candidate B - integrating the second term	
$y = 6x - 2x^{\frac{3}{2}}$	•[1] ✓	$y = 6x - 2x^{\frac{3}{2}}$	•[1] ✓
$\frac{dy}{dx} = 6 - 3x^{\frac{5}{2}}$	•[2] ✓	$\frac{dy}{dx} = 6 - \frac{4}{5}x^{\frac{5}{2}}$	•[2] ✓
$6 - 3x^{\frac{5}{2}} = 0$	•[3] ✗	$6 - \frac{4}{5}x^{\frac{5}{2}} = 0$	•[3] ✗
$x = 1{\cdot}32$	•[4] $\boxed{✓1}$	$x = 2{\cdot}24$	•[4] ✗

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7.	(b)		$\bullet^5$ evaluate y at stationary point $\bullet^6$ consider value of y at end points $\bullet^7$ state greatest and least values	$\bullet^5$ 8 $\bullet^6$ 4 and 0 $\bullet^7$ greatest 8, least 0 **stated explicitly**	**3**

Notes:

4. The only valid approach to finding the stationary point is via differentiation. A numerical approach can only gain $\bullet^6$.
5. $\bullet^7$ is not available to candidates who do not consider both end points.
6. Vertical marking is not applicable to $\bullet^6$ and $\bullet^7$.
7. Ignore any nature table which may appear in a candidate's solution; however, the appearance of $(4,8)$ at a nature table is sufficient for $\bullet^5$.
8. Greatest $(4,8)$; least $(9,0)$ does not gain $\bullet^7$.
9. $\bullet^5$ and $\bullet^7$ are not available for evaluating y at a value of x, obtained at $\bullet^4$ stage, which lies outwith the interval $1 \leq x \leq 9$.
10. For candidates who **only** evaluate the derivative, $\bullet^5$,$\bullet^6$ and $\bullet^7$are not available.

Commonly Observed Responses:

MARKS

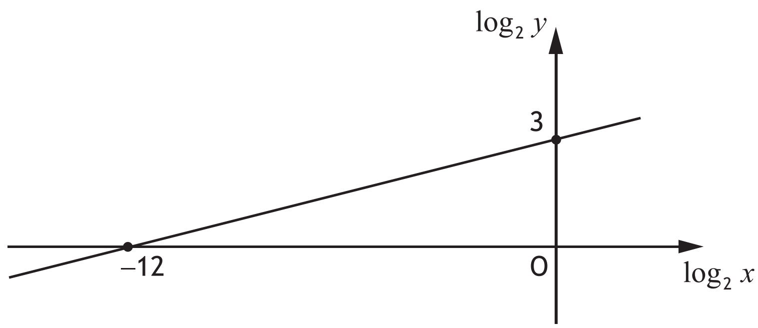
8. Sequences may be generated by recurrence relations of the form $u_{n+1} = k\,u_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$.

(a) Show that $u_2 = 5k^2 - 20k - 20$. **2**

(b) Determine the range of values of k for which $u_2 < u_0$. **4**

9. Two variables, x and y, are connected by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.



Find the values of k and n. **5**

8.	(a)		•[1] find expression for u_1 •[2] find expression for u_2 and express in the correct form	•[1] $5k-20$ •[2] $u_2=k(5k-20)-20$ leading to $u_2=5k^2-20k-20$	**2**

Question			Generic scheme	Illustrative scheme	Max mark
8.	(b)		•³ interpret information	•³ $5k^2 - 20k - 20 < 5$	
			•⁴ express inequality in standard quadratic form	•⁴ $5k^2 - 20k - 25 < 0$	
			•⁵ determine zeros of quadratic expression	•⁵ $-1, 5$	
			•⁶ state range with justification	•⁶ $-1 < k < 5$ with eg sketch or table of signs	4

Notes:

1. Candidates who work with an equation from the outset lose •³ and •⁴. However, •⁵ and •⁶ are still available.
2. At •⁵ do not penalise candidates who fail to extract the common factor or who have divided the quadratic inequation by 5.
3. •⁴ and •⁵ are only available to candidates who arrive at a quadratic expression at •³.
4. At •⁶ accept "$k > -1$ and $k < 5$" or "$k > -1$, $k < 5$" together with the required justification.
5. For a trial and error approach award 0/4.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.			**Method 1**	**Method 1**	
			•1 state linear equation	•1 $\log_2 y = \frac{1}{4}\log_2 x + 3$	
			•2 introduce logs	•2 $\log_2 y = \frac{1}{4}\log_2 x + 3\log_2 2$	
			•3 use laws of logs	•3 $\log_2 y = \log_2 x^{\frac{1}{4}} + \log_2 2^3$	
			•4 use laws of logs	•4 $\log_2 y = \log_2 2^3 x^{\frac{1}{4}}$	
			•5 state k and n	•5 $k = 8,\ n = \frac{1}{4}$ or $y = 8x^{\frac{1}{4}}$	**5**
			Method 2	**Method 2**	
			•1 state linear equation	•1 $\log_2 y = \frac{1}{4}\log_2 x + 3$	
			•2 use laws of logs	•2 $\log_2 y = \log_2 x^{\frac{1}{4}} + 3$	
			•3 use laws of logs	•3 $\log_2 \frac{y}{x^{\frac{1}{4}}} = 3$	
			•4 use laws of logs	•4 $\frac{y}{x^{\frac{1}{4}}} = 2^3$	
			•5 state k and n	•5 $k = 8,\ n = \frac{1}{4}$ or $y = 8x^{\frac{1}{4}}$	**5**

Question			Generic Scheme	Illustrative Scheme	Max Mark
			Method 3	**Method 3** **The equations at •1, •2 and •3 must be stated explicitly.**	
			•1 introduce logs to $y = kx^n$	•1 $\log_2 y = \log_2 kx^n$	
			•2 use laws of logs	•2 $\log_2 y = n\log_2 x + \log_2 k$	
			•3 interpret intercept	•3 $\log_2 k = 3$	
			•4 use laws of logs	•4 $k = 8$	
			•5 interpret gradient	•5 $n = \frac{1}{4}$	5
			Method 4	**Method 4**	
			•1 interpret point on log graph	•1 $\log_2 x = -12$ **and** $\log_2 y = 0$	
			•2 convert from log to exp. form	•2 $x = 2^{-12}$ **and** $y = 2^0$	
			•3 interpret point and convert	•3 $\log_2 x = 0,\ \ \log_2 y = 3$ $x = 1,\ y = 2^3$	
			•4 substitute into $y = kx^n$ and evaluate k	•4 $2^3 = k \times 1^n \Rightarrow k = 8$	
			•5 substitute other point into $y = kx^n$ and evaluate n	•5 $2^0 = 2^3 \times 2^{-12n}$ $\Rightarrow 3 - 12n = 0$ $\Rightarrow n = \frac{1}{4}$	5

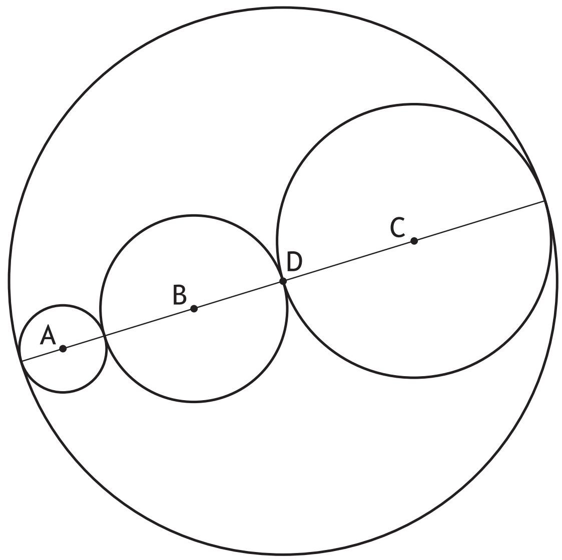
Notes:

1. Markers must not pick and choose between methods. Identify the method which best matches the candidates approach.
2. Treat the omission of base 2 as bad form at •1 and •3 in Method 1, at •1 and •2 for Method 2 and Method 3, and at •1 in Method 4.
3. '$m = \frac{1}{4}$' or 'gradient $= \frac{1}{4}$' does not gain •5 in Method 3.
4. Accept 8 in lieu of 2^3 throughout.
5. In Method 4 candidates may use $(0,3)$ for •1 and •2 followed by $(-12,0)$ for •3.

MARKS

10. (a) Show that the points A(−7, −2), B(2, 1) and C(17, 6) are collinear. **3**

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.



The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D. **4**

11. (a) Show that $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \dfrac{\pi}{2}$. **3**

(b) Hence, differentiate $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x$, where $0 < x < \dfrac{\pi}{2}$. **3**

[END OF QUESTION PAPER]

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		**Method 1** •¹ calculate m_{AB} •² calculate m_{BC} •³ interpret result and state conclusion	**Method 1** •¹ $m_{AB} = \frac{3}{9} = \frac{1}{3}$ see Note 1 •² $m_{BC} = \frac{5}{15} = \frac{1}{3}$ •³ $\ldots \Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	3
			Method 2 •¹ calculate an appropriate vector e.g. $\overrightarrow{AB}$ •² calculate a second vector e.g. $\overrightarrow{BC}$ and compare •³ interpret result and state conclusion	**Method 2** •¹ $\overrightarrow{AB} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ see Note 1 •² $\overrightarrow{BC} = \begin{pmatrix} 15 \\ 5 \end{pmatrix} \therefore \overrightarrow{AB} = \frac{3}{5}\overrightarrow{BC}$ •³ $\ldots \Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	3
			Method 3 •¹ calculate m_{AB} •² find equation of line and substitute point •³ communication	**Method 3** •¹ $m_{AB} = \frac{3}{9} = \frac{1}{3}$ •² eg, $y - 1 = \frac{1}{3}(x - 2)$ leading to $6 - 1 = \frac{1}{3}(17 - 2)$ •³ since C lies on line A, B and C are collinear	

Notes:

1. At •¹ and •² stage, candidates may calculate the gradients/vectors using any pair of points.
2. •³ can only be awarded if a candidate has stated "parallel", "common point" and "collinear".
3. Candidates who state "points A, B and C are parallel" or "m_{AB} and m_{BC} are parallel" do not gain •³.

Question			Generic scheme	Illustrative scheme	Max mark
10.	(b)		•⁴ find radius •⁵ determine an appropriate ratio •⁶ find centre •⁷ state equation of circle	•⁴ $6\sqrt{10}$ •⁵ e.g. 2:3 or $\frac{2}{5}$ (using B and C) or 3:5 or $\frac{8}{5}$ (using A and C) •⁶ $(8,3)$ •⁷ $(x-8)^2+(y-3)^2=360$	4

Notes:

4. Where the correct centre appears without working •⁵ is lost, •⁶ is awarded and •⁷ is still available. Where an incorrect centre or radius **from working** then •⁷ is available. However, if an incorrect centre or an incorrect radius appears ex nihilo •⁷ is not available.
5. Do not accept $(6\sqrt{10})^2$ for •⁷.

Commonly Observed Responses:

Candidate D		Candidate E	
Radius $=6\sqrt{10}$	•⁴ ✓	Radius $=3\sqrt{10}$	•⁴ ✗
Interprets D as midpoint of BC	•⁵ ✗	Interprets D as midpoint of AC	•⁵ ✗
Centre D is $(9\cdot5,\ 3\cdot5)$	•⁶ ✓2	Centre D is $(5,\ 2)$	•⁶ ✓2
$(x-9\cdot5)^2+(y-3\cdot5)^2=360$	•⁷ ✓1	$(x-5)^2+(y-2)^2=90$	•⁷ ✓1

Candidate F		Candidate G	
Radius $=\sqrt{10}$	•⁴ ✗	Radius $=6\sqrt{10}$	•⁴ ✓
Interprets D as midpoint of AC	•⁵ ✗	$\frac{CD}{BD}=\frac{3}{2}$ or simply $\frac{3}{2}$	•⁵ ✓
Centre D is $(5,\ 2)$	•⁶ ✓2	Centre D is $(11,\ 4)$	•⁶ ✗
$(x-5)^2+(y-2)^2=10$	•⁷ ✓2	$(x-11)^2+(y-4)^2=360$	•⁷ ✓1

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		**Method 1** •¹ substitute for $\sin 2x$ •² simplify and factorise •³ substitute for $1-\cos^2 x$ and simplify	**Method 1** •¹ $\frac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above •² $\sin x\left(1-\cos^2 x\right)$ •³ $\sin x\times\sin^2 x$ leading to $\sin^3 x$	3
			Method 2 •¹ substitute for $\sin 2x$ •² simplify and substitute for $\cos^2 x$ •³ expand and simplify	**Method 2** •¹ $\frac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above •² $\sin x-\sin x\left(1-\sin^2 x\right)$ •³ $\sin x-\sin x+\sin^3 x$ leading to $\sin^3 x$	3

Notes:

1. •¹ is not available to candidates who simply quote $\sin 2x = 2\sin x\cos x$ without substituting into the expression given on the LHS. See Candidate B
2. In method 2 where candidates attempt •¹ and •² in the same line of working •¹ may still be awarded if there is an error at •².
3. •³ is not available to candidates who work throughout with A in place of x.
4. Treat multiple attempts which are not scored out as different strategies, and apply General Marking Principle (r).
5. On the appearance of LHS $=0$, the first available mark is lost; however, any further marks are still available.

Commonly Observed Responses:

Candidate A	Candidate B
$\frac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x=\sin^3 x$ •¹ ✓ $\sin x-\sin x\cos^2 x=\sin^3 x$ •² ^ $1-\cos^2 x=\sin^2 x$ •³ ✗ $\sin^2 x=\sin^2 x$ In proving the identity, candidates must work with both sides independently ie in each line of working the LHS must be equivalent to the line above.	$\text{LHS}=\frac{\sin 2x}{2\cos x}-\sin x\cos^2 x$ $\frac{\sin 2x}{2\cos x}=\frac{2\sin x\cos x}{2\cos x}$ $=\sin x$ $\sin x-\sin x\cos^2 x$ •¹ ✓ $\sin x\left(1-\cos^2 x\right)$ •² ✓

Question			Generic scheme	Illustrative scheme	Max mark
11.	(b)		•4 know to differentiate $\sin^3 x$ •5 start to differentiate •6 complete differentiation	•4 $\frac{d}{dx}(\sin^3 x)$ •5 $3\sin^2 x....$ •6 $...\times\cos x$	**3**

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

MARKS

Attempt ALL questions

Total marks – 60

1. Find the equation of the line passing through the point $(-2, 3)$ which is parallel to the line with equation $y + 4x = 7$. **2**

2. Given that $y = 12x^3 + 8\sqrt{x}$, where $x > 0$, find $\frac{dy}{dx}$. **3**

3. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_3 = 6$.

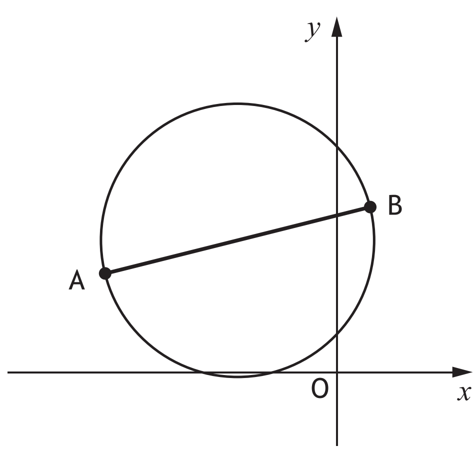
(a) Find the value of u_4. **1**

(b) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(c) Calculate this limit. **2**

4. A and B are the points $(-7, 3)$ and $(1, 5)$.

AB is a diameter of a circle.



Find the equation of this circle. **3**

[Turn over

Specific Marking Instructions for each question

Question			Generic Scheme	Illustrative Scheme	Max Mark
1.			$\bullet^1$ find the gradient $\bullet^2$ state equation	$\bullet^1$ -4 $\bullet^2$ $y+4x=-5$	**2**

Notes:

1. Accept any rearrangement of $y=-4x-5$ for $\bullet^2$.
2. On this occasion accept $y-3=-4\left(x-(-2)\right)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.
3. For any acceptable answer without working, award 2/2.
4. $\bullet^2$ is not available as a consequence of using a perpendicular gradient.
5. For candidates who **explicitly state** $m=4$ leading to $y-3=4\left(x-(-2)\right)$, award 1/2.

   For candidates who state $y-3=4\left(x-(-2)\right)$ with no other working, award 0/2.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
2.			$\bullet^1$ write in differentiable form $\bullet^2$ differentiate first term $\bullet^3$ differentiate second term	$\bullet^1$ $\cdots+8x^{\frac{1}{2}}$ stated or implied by $\bullet^3$ $\bullet^2$ $36x^2$ $\bullet^3$ $4x^{-\frac{1}{2}}$	**3**

Notes:

1. $\bullet^3$ is only available for differentiating a term with a fractional index.
2. Where candidates attempt to integrate throughout, only $\bullet^1$ is available.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
3.	(a)		$\bullet^1$ interpret recurrence relation and calculate u_4	$\bullet^1$ $u_4 = 12$	**1**

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^2$ communicate condition for limit to exist	$\bullet^2$ A limit exists as the recurrence relation is linear and $-1 < \frac{1}{3} < 1$	**1**

Notes:

1. On this occasion for $\bullet^2$ accept:

   any of $-1 < \frac{1}{3} < 1$ or $\left|\frac{1}{3}\right| < 1$ or $0 < \frac{1}{3} < 1$ with no further comment;

   or statements such as:

   "$\frac{1}{3}$ lies between -1 and 1" or "$\frac{1}{3}$ is a proper fraction"

2. $\bullet^2$ is not available for: $-1 \leq \frac{1}{3} \leq 1$ or $\frac{1}{3} < 1$

   or statements such as:

   "It is between -1 and 1" or "$\frac{1}{3}$ is a fraction"

3. Candidates who state $-1 < a < 1$ can only gain $\bullet^2$ if it is **explicitly stated** that $a = \frac{1}{3}$.

Commonly Observed Responses:

Candidate A	**Candidate B**
$a = \frac{1}{3}$ $-1 < a < 1$ so a limit exists. $\bullet^2$ ✓	$u_{n+1} = au_n + b$ $u_{n+1} = \frac{1}{3}u_n + 10$ $-1 < a < 1$ so a limit exists. $\bullet^2$ ^

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(c)		$\bullet^3$ Know how to calculate limit $\bullet^4$ calculate limit	$\bullet^3$ $\dfrac{10}{1-\frac{1}{3}}$ or $L = \frac{1}{3}L + 10$ $\bullet^4$ 15	**2**

Notes:

4. Do not accept $L = \dfrac{b}{1-a}$ with no further working for $\bullet^3$.
5. $\bullet^3$ and $\bullet^4$ are not available to candidates who conjecture that $L = 15$ following the calculation of further terms in the sequence.
6. For $L = 15$ with no working, award 0/2.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.			•[1] find the centre	•[1] $(-3, 4)$ stated or implied by •[3]	**3**
			•[2] calculate the radius	•[2] $\sqrt{17}$	
			•[3] state equation of circle	•[3] $(x+3)^2+(y-4)^2=17$ or equivalent	

Notes:

1. Accept $\frac{\sqrt{68}}{2}$ for •[2].
2. •[3] is not available to candidates who do not simplify $\left(\sqrt{17}\right)^2$ or $\left(\frac{\sqrt{68}}{2}\right)^2$.
3. •[3] is not available to candidates who do not attempt to half the diameter.
4. •[3] is not available to candidates who use either A or B for the centre.
5. •[3] is not available to candidates who substitute a negative value for the radius.
6. •[2]&•[3] are not available to candidates if the diameter or radius appears ex nihilo.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.			•[1] start to integrate	•[1] $\ldots \times \sin(4x+1)$	**2**
			•[2] complete integration	•[2] $2\sin(4x+1)+c$	

Notes:

1. An answer which has not been fully simplified, eg $\frac{8\sin(4x+1)}{4}+c$ or $\frac{4\sin(4x+1)}{2}+c$, does not gain •[2].
2. Where candidates have differentiated throughout, or in part (indicated by the appearance of a negative sign or $\times 4$), see candidates A to F.
3. No marks are available for a line of working containing $\sin(4x+1)^2$ or for any working thereafter.

Commonly Observed Responses:

Candidate A Differentiated throughout: $-32\sin(4x+1)+c$ award 0/2	**Candidate C** Differentiated in part: $32\sin(4x+1)+c$ award 1/2	**Candidate E** Differentiated in part: $-2\sin(4x+1)+c$ award 1/2
Candidate B Differentiated throughout: $-32\sin(4x+1)$ award 0/2	**Candidate D** Differentiated in part: $32\sin(4x+1)$ award 0/2	**Candidate F** Differentiated in part: $-2\sin(4x+1)$ award 0/2

MARKS

5. Find $\int 8\cos(4x+1)\,dx$. **2**

6. Functions f and g are defined on $\mathbb{R}$, the set of real numbers.

The inverse functions f^{-1} and g^{-1} both exist.

(a) Given $f(x) = 3x+5$, find $f^{-1}(x)$. **3**

(b) If $g(2) = 7$, write down the value of $g^{-1}(7)$. **1**

7. Three vectors can be expressed as follows:

$\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$

$\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$

$\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

(a) Find $\overrightarrow{FH}$. **2**

(b) Hence, or otherwise, find $\overrightarrow{FE}$. **2**

8. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact. **5**

5.			●[1] start to integrate ●[2] complete integration	●[1] $\ldots \times \sin(4x+1)$ ●[2] $2\sin(4x+1)+c$	**2**

Notes:

1. An answer which has not been fully simplified, eg $\dfrac{8\sin(4x+1)}{4}+c$ or $\dfrac{4\sin(4x+1)}{2}+c$, does not gain ●[2].
2. Where candidates have differentiated throughout, or in part (indicated by the appearance of a negative sign or $\times 4$), see candidates A to F.
3. No marks are available for a line of working containing $\sin(4x+1)^2$ or for any working thereafter.

Commonly Observed Responses:

Candidate A Differentiated throughout: $-32\sin(4x+1)+c$ award 0/2	**Candidate C** Differentiated in part: $32\sin(4x+1)+c$ award 1/2	**Candidate E** Differentiated in part: $-2\sin(4x+1)+c$ award 1/2
Candidate B Differentiated throughout: $-32\sin(4x+1)$ award 0/2	**Candidate D** Differentiated in part: $32\sin(4x+1)$ award 0/2	**Candidate F** Differentiated in part: $-2\sin(4x+1)$ award 0/2

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.	(a)			**Method 1:**	**3**
			•¹ equate composite function to x	•¹ $f\left(f^{-1}(x)\right)=x$	
			•² write $f\left(f^{-1}(x)\right)$ in terms of $f^{-1}(x)$	•² $3f^{-1}(x)+5=x$	
			•³ state inverse function	•³ $f^{-1}(x)=\dfrac{x-5}{3}$	
				Method 2:	**3**
			•¹ write as $y=3x+5$ and start to rearrange	•¹ $y-5=3x$	
			•² complete rearrangement	•² $x=\dfrac{y-5}{3}$	
			•³ state inverse function	•³ $f^{-1}(x)=\dfrac{x-5}{3}$	
				Method 3	**3**
			•¹ interchange variables	•¹ $x=3y+5$	
			•² complete rearrangement	•² $\dfrac{x-5}{3}=y$	
			•³ state inverse function	•³ $f^{-1}(x)=\dfrac{x-5}{3}$	

Notes:

1. $y=\dfrac{x-5}{3}$ does not gain •³.

2. At •³ stage, accept f^{-1} expressed in terms of any dummy variable eg $f^{-1}(y)=\dfrac{y-5}{3}$.

3. $f^{-1}(x)=\dfrac{x-5}{3}$ with no working gains 3/3.

Commonly Observed Responses:

Candidate A

$$\times 3 \quad +5$$

$$x \rightarrow 3x \rightarrow 3x+5=f(x)$$

$$\div 3 \quad -5$$

•¹ ✓

$\dfrac{x-5}{3}$ •² ✓

$f^{-1}(x)=\dfrac{x-5}{3}$ •³ ✓

•¹ awarded for knowing to perform inverse operations in reverse order.

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		●[1] correct value	●[1] 2	**1**

Notes:

Commonly Observed Responses:

Candidate B

$g(x) = 3x + 1$

$g(2) = 3 \times 2 + 1 = 7$

$g^{-1}(x) = \dfrac{x-1}{3}$

$g^{-1}(7) = \dfrac{7-1}{3} = 2$ ●[4] ✗

If the candidate had followed this by stating that this would be true for all functions g for which $g(2) = 7$ and g^{-1} exists then ●[4] would be awarded.

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		•1 identify pathway •2 state $\overrightarrow{FH}$	•1 $\overrightarrow{FG}+\overrightarrow{GH}$ •2 $\mathbf{i}+3\mathbf{j}-4\mathbf{k}$	2

Notes:

1. Award •1 for $(-2\mathbf{i}-6\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+9\mathbf{j}-7\mathbf{k})$.
2. For $\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ without working, award both •1 and •2.
3. Accept, throughout the question, solutions written as column vectors.
4. •2 is not available for adding or subtracting vectors within an invalid strategy.
5. Where candidates choose specific points consistent with the given vectors only •1 and •4 are available. However, should the statement 'without loss of generality' precede the selected points then all 4 marks are available.

Commonly Observed Responses:

Candidate A

$\overrightarrow{FH}=\overrightarrow{FG}+\overrightarrow{EH}$ •1 ✗

$$\begin{pmatrix}-2\\-6\\3\end{pmatrix}+\begin{pmatrix}2\\3\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\-3\\4\end{pmatrix}$$ •2 $\boxed{\checkmark 2}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•1 identify pathway •2 $\overrightarrow{FE}$	•1 $\overrightarrow{FH}+\overrightarrow{HE}$ or equivalent •2 $-\mathbf{i}-5\mathbf{k}$	2

Notes:

6. Award •3 for $(\mathbf{i}+3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+3\mathbf{j}+\mathbf{k})$
   or $(\mathbf{i}+3\mathbf{j}-4\mathbf{k})+(-2\mathbf{i}-3\mathbf{j}-\mathbf{k})$
   or $(-2\mathbf{i}-6\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+9\mathbf{j}-7\mathbf{k})-(2\mathbf{i}+3\mathbf{j}+\mathbf{k})$
   or $(-2\mathbf{i}-6\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+9\mathbf{j}-7\mathbf{k})+(-2\mathbf{i}-3\mathbf{j}-\mathbf{k})$.
7. For $-\mathbf{i}-5\mathbf{k}$ without working, award 0/2.
8. •4 is not available for simply adding or subtracting vectors. There must be evidence of a valid strategy at •3.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.			●[1] substitute for y	●[1] $x^2+(3x-5)^2+2x-4(3x-5)-5\ldots$	5
			Method 1 & 2	**Method 1**	
			●[2] express in standard quadratic form	●[2] $10x^2-40x+40$ $\}=0$	
			●[3] factorise or use discriminant	●[3] $10(x-2)^2$ $\}=0$	
			●[4] interpret result to demonstrate tangency	●[4] only one solution implies tangency (or repeated factor implies tangency)	
			●[5] find coordinates	●[5] $x=2,\ y=1$	
				Method 2	
				●[2] $10x^2-40x+40=0$ **stated explicitly**	
				●[3] $(-40)^2-4\times10\times40$ or $(-4)^2-4\times1\times4$	
				●[4] $b^2-4ac=0$ so line is a tangent	
				●[5] $x=2,\ y=1$	
			Method 3	**Method 3**	
			●[1] make inference and state m_{rad}	●[1] If $y=3x-5$ is a tangent, $m_{rad}=\frac{-1}{3}$	
			●[2] find the centre and the equation of the radius	●[2] $(-1,2)$ and $3y=-x+5$	
			●[3] solve simultaneous equations	●[3] $\left.\begin{array}{l}3y=-x+5\\ y=3x-5\end{array}\right\}\rightarrow(2,1)$	
			●[4] verify location of point of intersection	●[4] check $(2,1)$ lies on the circle.	
			●[5] communicates result	●[5] $\therefore$ the line is a tangent to the circle	

Notes:

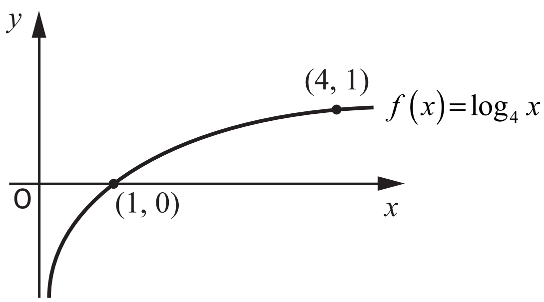
1. In Method 1 "$=0$" must appear at ●[2] or ●[3] stage for ●[2] to be awarded.
2. Award ●[3] and ●[4] for correct use of quadratic formula to get equal (repeated) roots $\Rightarrow$ line is a tangent.

MARKS

9. (a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$. **4**

(b) Hence determine the range of values of x for which the function f is strictly increasing. **2**

10. The diagram below shows the graph of the function $f(x) = \log_4 x$, where $x > 0$.



The inverse function, f^{-1}, exists.

On the diagram in your answer booklet, sketch the graph of the inverse function. **2**

11. (a) A and C are the points $(1, 3, -2)$ and $(4, -3, 4)$ respectively.

Point B divides AC in the ratio $1 : 2$.

Find the coordinates of B. **2**

(b) $k\overrightarrow{\mathrm{AC}}$ is a vector of magnitude 1, where $k > 0$.

Determine the value of k. **3**

[Turn over

Question			Generic Scheme	Illustrative Scheme	Max Mark
Commonly Observed Responses:					
Candidate A $x^2+(3x-5)^2+2x-4(3x-5)-5=0$ $\bullet^1$ ✓ $10x^2-40x+40=0$ $\bullet^2$ ✓ $b^2-4ac=(-40)^2-4\times10\times40=0\Rightarrow$ tgt $\bullet^3$ ✓				**Candidate B** $x^2+(3x-5)^2+2x-4(3x-5)-5=0$ $\bullet^1$ ✓ $10x^2-40x+40$ $\bullet^2$ ∧ $b^2-4ac=(-40)^2-4\times10\times40=0\Rightarrow$ tgt $\bullet^3$ $\boxed{\checkmark 1}$	
Candidate C $x^2+(3x-5)^2+2x-4(3x-5)-5=0$ $\bullet^1$ ✓ $x^2+\underline{9x^2+25}+2x-12x+20-5=0$ $10x^2-10x+40=0$ $\bullet^2$ ✗ $b^2-4ac=(-10)^2-4\times10\times40=-1500\Rightarrow$ no real roots so line is not a tangent $\bullet^3$ $\boxed{\checkmark 1}$ $\bullet^4$ and $\bullet^5$ are unavailable.				**Candidate D** $x^2+(3x-5)^2+2x-4(3x-5)-5=0$ $\bullet^1$ ✓ $10x^2-40x+40=0$ $\bullet^2$ ✓ $10(x-2)^2$ $\bullet^3$ ✓ Repeated root $\Rightarrow$ Only one point of contact. $\bullet^4$ ✓	
9	(a)		$\bullet^1$ know to and differentiate one term $\bullet^2$ complete differentiation and equate to zero $\bullet^3$ factorise derivative $\bullet^4$ process for x	$\bullet^1$ eg $f'(x)=3x^2...$ $\bullet^2$ $3x^2+6x-24=0$ $\bullet^3$ $3(x+4)(x-2)$ $\bullet^4$ -4 and 2	**4**
Notes:					
1. $\bullet^2$ is only available if $"=0"$ appears at $\bullet^2$ or $\bullet^3$ stage. 2. $\bullet^3$ is available for substituting correctly in the quadratic formula. 3. At $\bullet^3$ do not penalise candidates who fail to extract the common factor or who have divided the quadratic equation by 3. 4. $\bullet^3$ and $\bullet^4$ are not available to candidates who arrive at a linear expression at $\bullet^2$.					
Commonly Observed Responses:					

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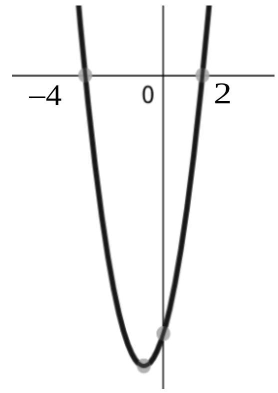
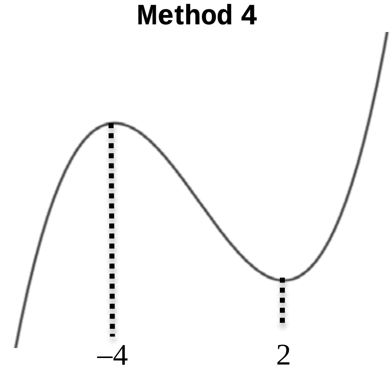
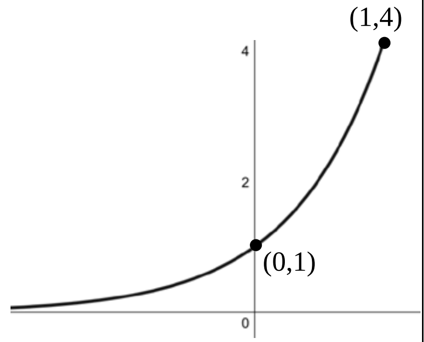
Question		Generic Scheme	Illustrative Scheme	Max Mark
	(b)	●5 know how to identify where curve is increasing	●5 **Method 1**  **Method 2** $3x^2+6x-24>0$ **Method 3** Table of signs for a derivative – see the additional page for acceptable responses. **Method 4** 	2
		●6 state range	●6 $x<-4$ and $x>2$	

Notes:

5. For $x<-4$ and $x>2$ without working award 0/2.
6. $2<x<-4$ does not gain ●6.

Commonly Observed Responses:

Back to Table

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.			●[1] graph reflected in $y = x$ ●[2] correct annotation	●[1]  ●[2] $(0,1)$ and $(1,4)$	**2**

Notes:

1. For ●[1] accept any graph of the correct shape and orientation which crosses the $y-$axis. The graph must not cross the $x-$axis.
2. Both $(0,1)$ and $(1,4)$ must be marked and labelled on the graph for ●[2] to be awarded.
3. ●[2] is only available where the candidate has attempted to reflect the given curve in the line $y = x$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.	(a)		•¹ interpret ratio •² determine coordinates	•¹ $\frac{1}{3}$ •² $(2,1,0)$	**2**

Notes:

1. •¹ may be implied by •² or be evidenced by their working.
2. For $(3,-1,2)$ award 1/2.
3. For $(2,1,0)$ without working award 2/2.
4. $\begin{pmatrix}2\\1\\0\end{pmatrix}$ gains 1/2.
5. $\begin{pmatrix}3\\-1\\2\end{pmatrix}$ gains 0/2.

Commonly Observed Responses:

Candidate A	**Candidate B**
$\overrightarrow{BC}=\frac{1}{3}\overrightarrow{AC}$ •¹ ✗ $(3,-1,2)$ •² $\boxed{✓1}$	$\frac{\overrightarrow{AB}}{\overrightarrow{BC}}=\frac{1}{2}$ $2\overrightarrow{AB}=\overrightarrow{BC}$ $2(\mathbf{b}-\mathbf{a})=\mathbf{c}-\mathbf{b}$ $3\mathbf{b}=\mathbf{c}+2\mathbf{a}$ •¹ ✓ $3\mathbf{b}=\begin{pmatrix}6\\3\\0\end{pmatrix}$ $\mathbf{b}=\begin{pmatrix}2\\1\\0\end{pmatrix}$ $B(2,1,0)$ •² ✓

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		●¹ find $\overrightarrow{AC}$	●¹ $\overrightarrow{AC}=\begin{pmatrix}3\\-6\\6\end{pmatrix}$	3
			●² find $\lvert\overrightarrow{AC}\rvert$	●² 9	
			●³ determine k	●³ $\frac{1}{9}$	

Notes:

6. Evidence for ●³ may appear in part (a).
7. ●³ may be implied at ●⁴ stage by :
   - $\sqrt{3^2+(-6)^2+6^2}$
   - $\sqrt{3^2-6^2+6^2}=9$
   - $\sqrt{3^2+-6^2+6^2}=9$.
8. $\sqrt{81}$ must be simplified at the ●⁴ or ●⁵ stage for ●⁴ to be awarded.
9. ●⁵ can only be awarded as a consequence of a valid strategy at ●⁴. k must be >0 .

Commonly Observed Responses:

Candidate A		Candidate B		Candidate C	
$\lvert\overrightarrow{AC}\rvert=\sqrt{81}$	●⁴ ✓	$\lvert\overrightarrow{AC}\rvert=\sqrt{81}$	●⁴ ✓2	$\lvert\overrightarrow{AC}\rvert=\sqrt{81}$	●⁴ ✓2
$\frac{1}{9}$	●⁵ ✓	$\frac{1}{\sqrt{81}}$	●⁵ ✓1		●⁵ ^

ALTERNATIVE STRATEGY

Where candidates use the distance formulae to determine the distance from A to C, award ●³ for $AC=\sqrt{3^2+6^2+6^2}$.

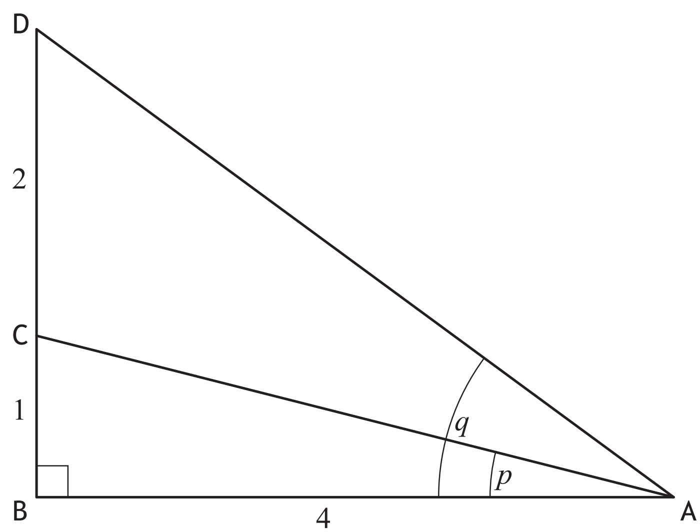
MARKS

12. The functions f and g are defined on $\mathbb{R}$, the set of real numbers by
$f(x) = 2x^2 - 4x + 5$ and $g(x) = 3 - x$.

(a) Given $h(x) = f(g(x))$, show that $h(x) = 2x^2 - 8x + 11$. **2**

(b) Express $h(x)$ in the form $p(x+q)^2 + r$. **3**

13. Triangle ABD is right-angled at B with angles BAC $= p$ and BAD $= q$ and lengths as shown in the diagram below.



Show that the exact value of $\cos(q-p)$ is $\dfrac{19\sqrt{17}}{85}$. **5**

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.	(a)		•¹ interpret notation •² demonstrate result	•¹ $2(3-x)^2-4(3-x)+5$ •² $18-12x+2x^2-12+4x+5$ leading to $2x^2-8x+11$	**2**

Notes:

1. At •² there must be a relevant intermediate step between •¹ and the final answer for •² to be awarded.
2. $f(3-x)$ alone is not sufficient to gain •¹.
3. Beware of candidates who fudge their working between •¹ and •².

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)			**Method 1**	**3**
			•¹ identify common factor	•¹ $2\left[x^2-4x\ldots\right.$ stated or implied by •²	
			•² start to complete the square	•² $2(x-2)^2\ldots$	
			•³ write in required form	•³ $2(x-2)^2+3$	
				Method 2	
			•¹ expand completed square form	•¹ $px^2+2pqx+pq^2+r$	
			•² equate coefficients	•² $p=2,\ 2pq=-8,\ pq^2+r=11$	
			•³ process for q and r and write in required form	•³ $2(x-2)^2+3$	

Notes:

4. At •⁵ $2(x+(-2))^2+3$ must be simplified to $2(x-2)^2+3$.
5. $2(x-2)^2+3$ with no working gains •⁵ only; however, see Candidate G.
6. Where a candidate has used the function they arrived at in part (a) as $h(x)$, •³ is not available. However, •⁴ and •⁵ can still be gained for dealing with an expression of equivalent difficulty.
7. •⁵ is only available for a calculation involving both the multiplication and addition of integers.

Question	Generic Scheme	Illustrative Scheme	Max Mark
13.	●[1] calculate lengths AC and AD	●[1] $AC=\sqrt{17}$ and $AD=5$ stated or implied by ●[3]	**5**
	●[2] select appropriate formula and express in terms of p and q	●[2] $\cos q\cos p+\sin q\sin p$ stated or implied by ●[4]	
	●[3] calculate two of $\cos p,\ \cos q,\ \sin p,\ \sin q$	●[3] $\cos p=\frac{4}{\sqrt{17}}$, $\cos q=\frac{4}{5}$ $\sin p=\frac{1}{\sqrt{17}}$, $\sin q=\frac{3}{5}$	
	●[4] calculate other two and substitute into formula	●[4] $\frac{4}{5}\times\frac{4}{\sqrt{17}}+\frac{3}{5}\times\frac{1}{\sqrt{17}}$	
	●[5] arrange into required form	●[5] $\frac{19}{5\sqrt{17}}\times\frac{\sqrt{17}}{\sqrt{17}}=\frac{19\sqrt{17}}{85}$ or $\frac{19}{5\sqrt{17}}=\frac{19\sqrt{17}}{5\times17}=\frac{19\sqrt{17}}{85}$	

Notes:

1. For any attempt to use $\cos(q-p)=\cos q-\cos p$, only ●[1] and ●[3] are available.
2. At the ●[3] and ●[4] stages, do not penalise the use of fractions greater than 1 resulting from an error at ●[1]. ●[5] will be lost.
3. Candidates who write $\cos\left(\frac{4}{5}\right)\times\cos\left(\frac{4}{\sqrt{17}}\right)+\sin\left(\frac{3}{5}\right)\times\sin\left(\frac{1}{\sqrt{17}}\right)$ gain ●[1], ●[2] and ●[3].

   ●[4] and ●[5] are unavailable.
4. Clear evidence of multiplying by $\frac{\sqrt{17}}{\sqrt{17}}$ must be seen between ●[4] and ●[5] for ●[5] to be awarded.
5. ●[4] implies ●[1], ●[2] and ●[3].

Commonly Observed Responses:

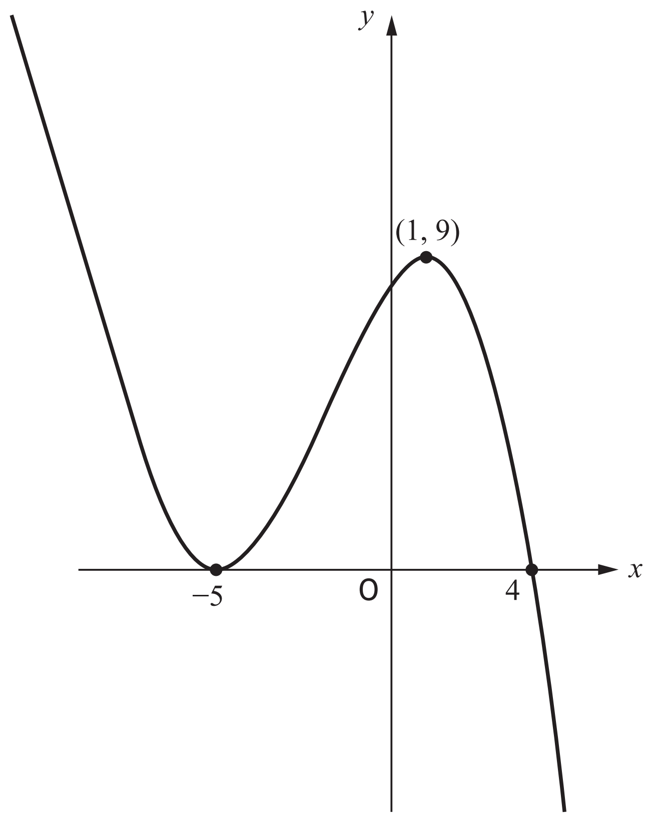
Candidate A		Candidate B	
$\frac{4}{5}\times\frac{4}{\sqrt{17}}+\frac{3}{5}\times\frac{1}{\sqrt{17}}$	●[4] ✓	$AC=\sqrt{17}$ and $AD=\sqrt{21}$	●[1] ✗
$\frac{19}{5\sqrt{17}}\times\sqrt{17}$		$\cos q\cos p+\sin q\sin p$	●[2] ✓
$\frac{19\sqrt{17}}{85}$	●[5] ✗	$\cos p=\frac{4}{\sqrt{17}}$ $\sin p=\frac{1}{\sqrt{17}}$	●[3] ✓
		$\frac{\sqrt{17}}{\sqrt{21}}\times\frac{4}{\sqrt{17}}+\frac{2}{\sqrt{21}}\times\frac{1}{\sqrt{17}}$	●[4] ✗
		=... ●[5] not available	

MARKS

14. (a) Evaluate $\log_5 25$. **1**

(b) Hence solve $\log_4 x + \log_4 (x-6) = \log_5 25$, where $x > 6$. **5**

15. The diagram below shows the graph with equation $y = f(x)$, where $f(x) = k(x-a)(x-b)^2$.



(a) Find the values of a, b and k. **3**

(b) For the function $g(x) = f(x) - d$, where d is positive, determine the range of values of d for which $g(x)$ has exactly one real root. **1**

[END OF QUESTION PAPER]

Back to Table

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.	(a)		•¹ state value	•¹ 2	**1**

Notes:

1. Evidence for •¹ may not appear until part (b).

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•² use result of part (a)	•² $\log_4 x + \log_4 (x-6) = 2$	**5**
			•³ use laws of logarithms	•³ $\log_4 x(x-6) = 2$	
			•⁴ use laws of logarithms	•⁴ $x(x-6) = 4^2$	
			•⁵ write in standard quadratic form	•⁵ $x^2 - 6x - 16 = 0$	
			•⁶ solve for x and identify appropriate solution	•⁶ 8	

Notes:

2. •³& •⁴ can only be awarded for use of laws of logarithms applied to algebraic expressions of equivalent difficulty.
3. •⁴ is not available for $x(x-6) = 2^4$; however candidates may still gain •⁵& •⁶.
4. •⁶ is only available for solving a polynomial of degree 2 or higher.
5. •⁶ is not available for responses which retain invalid solutions.

Commonly Observed Responses:

Candidate A			**Candidate B**			**Candidate C**		
$\log_5 25 = 5$	•¹	✗	$\log_5 25 = 2$	•¹	✓	$\log_5 25 = 2$	•¹	✓
$\log_4 x(x-6) = 5$	•²	$\boxed{\checkmark 1}$	$\log_4 x(x-6) = 2$	•²	✓	$\log_4 x(x-6) = 2$	•²	✓
	•³	$\boxed{\checkmark 1}$		•³	✓		•³	✓
$x(x-6) = 4^5$	•⁴	$\boxed{\checkmark 1}$	$x(x-6) = 8$	•⁴	✗	$x(x-6) = 8$	•⁴	✗
$x^2 - 6x - 1024 = 0$	•⁵	$\boxed{\checkmark 1}$	$x^2 - 6x - 8 = 0$	•⁵	$\boxed{\checkmark 1}$	$x^2 - 6x + 8 = 0$	•⁵	✗
35.14...	•⁶	$\boxed{\checkmark 1}$	7.12...	•⁶	$\boxed{\checkmark 1}$	$x = 2, 4$	•⁶	✗
						or		
						$x = \cancel{2}, \cancel{4}$	•⁶	✗

 Back to Table

Question			Generic Scheme	Illustrative Scheme	Max Mark
15.	(a)		$\bullet^1$ value of a $\bullet^2$ value of b $\bullet^3$ calculate k	$\bullet^1$ $a=4$ $\bullet^2$ $b=-5$ $\bullet^3$ $k=-\frac{1}{12}$	**3**

Notes:

1. Evidence for the values of a and b may first appear in an expression for $f(x)$. Where marks have been awarded for a and b in an expression for $f(x)$ ignore any values attributed to a and b in subsequent working.

Commonly Observed Responses:

Candidate A

Both roots interchanged

$a=-5$ $\bullet^1$ ✗

$b=4$ $\bullet^2$ $\boxed{\checkmark 1}$

$k=\frac{1}{6}$ $\bullet^3$ $\boxed{\checkmark 1}$

Candidate B

$a=4$ $\bullet^1$ ✓

$b=5$ $\bullet^2$ ✗

$k=-\frac{3}{16}$ $\bullet^3$ $\boxed{\checkmark 1}$

Candidate C

Using $(1,9)$

$a=-4$ $\bullet^1$ ✗

$b=5$ $\bullet^2$ $\boxed{\checkmark 1}$

$k=\frac{9}{80}$ $\bullet^3$ $\boxed{\checkmark 1}$

Candidate D - BEWARE

Using $(0,9)$

$a=-5$ $\bullet^1$ ✗

$b=4$ $\bullet^2$ $\boxed{\checkmark 1}$

$k=\frac{9}{80}$ $\bullet^3$ ✗

Summary for expressions of $f(x)$ for $\bullet^1$ and $\bullet^2$:

signs correct, brackets correct

$f(x)=(x-4)(x+5)^2$ $\bullet^1$ ✓ $\bullet^2$ ✓

signs incorrect, brackets correct

$f(x)=(x+4)(x-5)^2$ $\bullet^1$ ✗ $\bullet^2$ $\boxed{\checkmark 1}$

signs correct, brackets incorrect

$f(x)=(x+5)(x-4)^2$ $\bullet^1$ ✗ $\bullet^2$ $\boxed{\checkmark 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^1$ state range of values	$\bullet^1$ $d>9$	**1**

Notes:

Commonly Observed Responses:

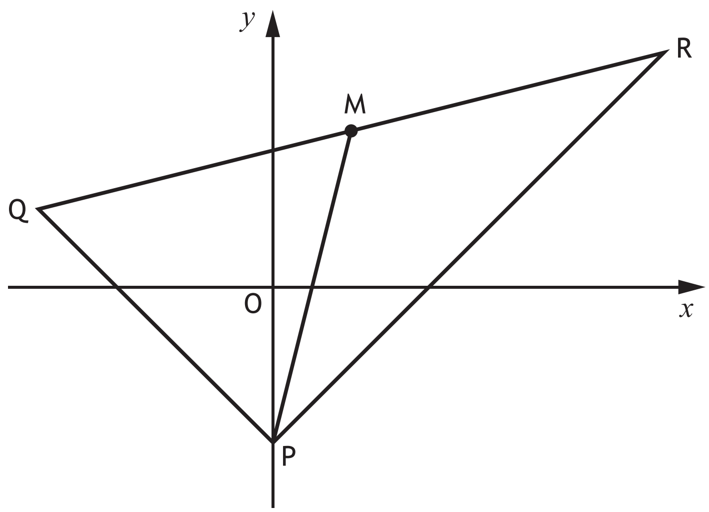
[END OF MARKING INSTRUCTIONS]

MARKS

Attempt ALL questions

Total marks — 70

1. PQR is a triangle with vertices P$(0,-4)$, Q$(-6,2)$ and R$(10,6)$.



(a) (i) State the coordinates of M, the midpoint of QR. **1**

(ii) Hence find the equation of PM, the median through P. **2**

(b) Find the equation of the line, *L*, passing through M and perpendicular to PR. **3**

(c) Show that line *L* passes through the midpoint of PR. **3**

2. Find the range of values for p such that $x^2 - 2x + 3 - p = 0$ has no real roots. **3**

[Turn over

Specific Marking Instructions for each question

Question			Generic Scheme	Illustrative Scheme	Max Mark
1.	(a)	i	●¹ state the midpoint M	●¹ $(2,4)$	**1**
		ii	●² calculate gradient of median ●³ determine equation of median	●² 4 ●³ $y = 4x - 4$	**2**

Notes:

1. ●³ is not available as a consequence of using a perpendicular gradient.
2. Accept any rearrangement of $y = 4x - 4$ for ●³.
3. On this occasion, accept $y - 4 = 4(x-2)$ or $y-(-4) = 4(x-0)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.
4. ●³ is only available as a consequence of using points M and P, or any other point which lies on PM, for example the midpoint $(1,0)$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		●¹ calculate gradient of PR ●² use property of perpendicular lines ●³ determine equation of line	●¹ 1 ●² -1 stated or implied by ●⁶ ●³ $y = -x + 6$	**3**

Notes:

5. ●⁶ is only available as a consequence of using M and a perpendicular gradient.
6. Candidates who use a gradient perpendicular to QR cannot gain ●⁴ but ●⁵ and ●⁶ are still available. See Candidate A.
7. Beware of candidates who use the coordinates of P and Q to arrive at $m = -1$. See Candidate B.
8. On this occasion, accept $y - 4 = -1(x-2)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.

Question			Generic Scheme	Illustrative Scheme	Max Mark
Commonly Observed Responses:					
Candidate A $m_{QR} = \frac{1}{4}$ ●4 ✗ $m_{perp} = -4$ ●5 ✓1 $y = -4x + 12$ ●6 ✓1				**Candidate B - BEWARE** $m_{PQ} = \frac{2-(-4)}{-6-0}$ ●4 ^ $= -1$ ●5 ✗ $y - 4 = -1(x-2)$ ●6 ✓2 $y = -x + 6$ Note: ●4 ●5 and ●6 may still be available for any candidate that demonstrates that PQ is also perpendicular to PR.	
	(c)		●1 find the midpoint of PR ●2 substitute x-coordinate into equation of L. ●3 verify y-coordinate and communicate conclusion	**Method 1** ●1 $(5,1)$ ●2 $y = -5 + 6 \ (1 = -x + 6)$ ●3 $y = 1 \ (x = 5)$ $\therefore$ L passes through the midpoint of PR	3
			●7 find the midpoint of PR ●8 substitute x and y coordinates into the equation of L ●9 verify the point satisfies the equation and communicate conclusion	**Method 2** ●7 $x + y = 6$ sub $(5,1)$ ●8 $5 + 1 = 6$ ●9 $\therefore$ point $(5,1)$ satisfies equation.	
			●7 find the midpoint of PR ●8 find equation of PR ●9 use simultaneous equations and communicate conclusion	**Method 3** ●7 $(5,1)$ ●8 $y = x - 4$ ●9 $y = 1, \ x = 5$ $\therefore$ L passes through the midpoint of PR	

Back to Table

Question			Generic Scheme	Illustrative Scheme	Max Mark
				Method 4	
			●[7] find the midpoint of PR	●[7] $(5,1)$	
			●[8] find equation of perpendicular bisector of PR	●[8] $y-1=-1(x-5)\to y=-x+6$	
			●[9] communicate conclusion	●[9] The equation of the perpendicular bisector is the same as L therefore L passes through the midpoint of PR.	

Notes:

9. A relevant statement is required for ●[9] to be awarded.
10. Erroneous working accompanied by a statement such as "L does not pass through the midpoint." does NOT gain ●[9].
11. Beware of candidates substituting $(1,5)$ instead of $(5,1)$
12. On this occasion, for Method 3, at ●[8] accept $y-1=1(x-5)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.

Commonly Observed Responses:

Candidate C

$(5,1)$ mid-point ●[7] ✓

$y+x=6$

Sub$(5,1)$ ●[8] ×

$5+1=6$

$\therefore$ point $(5,1)$ satisfies equation. ●[9] ×

Candidate has substituted 5 for y and 1 for x.

Question			Generic Scheme	Illustrative Scheme	Max Mark
2.			•¹ use the discriminant	•¹ $(-2)^2-4(1)(3-p)$	**3**
			•² simplify and apply the condition for no real roots	•² $-8+4p<0$	
			•³ state range	•³ $p<2$	

Notes:

1. At the •¹ stage, treat $(-2)^2-4(1)3-p$ and $-2^2-4(1)(3-p)$ as bad form only if the candidate deals with the 'bad form' **term** correctly in the inequation at •².
2. If candidates have the condition 'discriminant = 0', then •² and •³ are unavailable.
3. If candidates have the condition 'discriminant $>$ 0', 'discriminant $\geq$ 0' or 'discriminant $\leq$ 0' then •² is lost, but •³ is available **provided the discriminant has been simplified correctly at •²**.
4. If a candidate works with an equation, then •² and •³ are unavailable. However, see Candidate D.

Commonly Observed Responses:

Candidate A

$(-2)^2-4(1)3-p$	•¹	✓
$-8+4p<0$	•²	✓
$p<2$	•³	✓

Candidate B

$(-2)^2-4(1)(3-p)$	•¹	✓
$-8-4p<0$	•²	×
$p>-2$	•³	$\boxed{✓1}$

Candidate C

$(-2)^2-4(1)3-p$	•¹	×
$-8-p<0$	•²	$\boxed{✓2}$ eased
$p>-8$	•³	$\boxed{✓2}$ eased

Candidate D - Special Case

$b^2-4ac<0$		
$(-2)^2-4(1)(3-p)=0$	•¹	✓
$-8+4p=0$		
$p=2$	•²	✓
$p<2$	•³	$\boxed{✓2}$

•2 is awarded since the condition (first line), its application (final line) and the simplification of the discriminant all appear.

Candidate E

$-2^2-4(1)(3-p)$	•¹	✓
$-8+4p<0$	•²	✓
$p<2$	•³	✓

Candidate F

$-2^2-4(1)(3-p)$	•¹	×
$-16+4p<0$	•²	$\boxed{✓2}$
$p<4$	•³	$\boxed{✓1}$

Candidate G

$-2^2-4(1)(3-p)=0$	•¹	✓
$-8+4p=0$	•²	×
$p=2$	•³	×

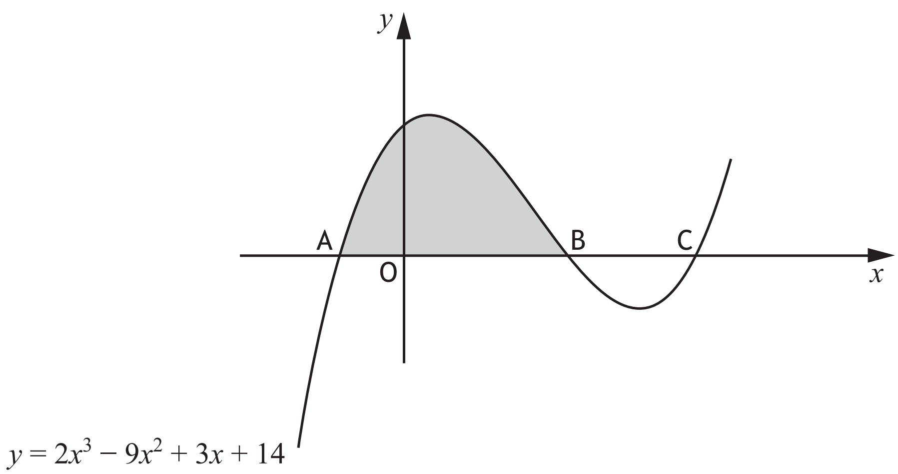
MARKS

3. (a) (i) Show that $(x+1)$ is a factor of $2x^3 - 9x^2 + 3x + 14$. **2**

(ii) Hence solve the equation $2x^3 - 9x^2 + 3x + 14 = 0$. **3**

(b) The diagram below shows the graph with equation $y = 2x^3 - 9x^2 + 3x + 14$.

The curve cuts the x-axis at A, B and C.



(i) Write down the coordinates of the points A and B. **1**

(ii) Hence calculate the shaded area in the diagram. **4**

4. Circles C_1 and C_2 have equations $(x+5)^2 + (y-6)^2 = 9$ and $x^2 + y^2 - 6x - 16 = 0$ respectively.

(a) Write down the centres and radii of C_1 and C_2. **4**

(b) Show that C_1 and C_2 do not intersect. **3**

Question			Generic Scheme	Illustrative Scheme	Max Mark
3.	(a)	i		**Method 1**	2
			●[1] know to substitute $x=-1$	●[1] $2(-1)^3-9\times(-1)^2+3\times(-1)+14$	
			●[2] complete evaluation, interpret result and state conclusion	●[2] $=0\ \therefore\ (x+1)$ is a factor	
				Method 2	
			●[1] know to use $x=-1$ in synthetic division	●[1] $\begin{array}{r\|rrrr} -1 & 2 & -9 & 3 & 14 \\ & & -2 & & \\ \hline & 2 & -11 & & \end{array}$	
			●[2] complete division, interpret result and state conclusion	●[2] $\begin{array}{r\|rrrr} -1 & 2 & -9 & 3 & 14 \\ & & -2 & 11 & -14 \\ \hline & 2 & -11 & 14 & 0 \end{array}$ remainder $=0\ \therefore\ (x+1)$ is a factor	
				Method 3	
			●[1] start long division and find leading term in quotient	●[1] $\begin{array}{r\|l} & 2x^2 \\ \hline (x+1) & 2x^3-9x^2+3x+14 \end{array}$	
			●[2] complete division, interpret result and state conclusion	●[2] $\begin{array}{r\|l} & 2x^2-11x+14 \\ \hline (x+1) & 2x^3-9x^2+3x+14 \\ & \underline{2x^3+2x^2} \\ & \quad -11x^2+3x \\ & \quad \underline{-11x^2-11x} \\ & \qquad 14x+14 \\ & \qquad \underline{14x+14} \\ & \qquad\qquad 0 \end{array}$ remainder $=0\ \therefore\ (x+1)$ is a factor	

Back to Table

Question			Generic Scheme	Illustrative Scheme	Max Mark
Notes:					

1. Communication at $\bullet^2$ must be consistent with working at that stage ie a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded.
2. Accept any of the following for $\bullet^2$:
   - '$f(-1)=0$ so $(x+1)$ is a factor'
   - 'since remainder = 0, it is a factor'
   - the 0 from any method linked to the word 'factor' by eg 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$'
3. Do not accept any of the following for $\bullet^2$:
   - double underlining the zero or boxing the zero without comment
   - '$x=1$ is a factor', '$(x-1)$ is a factor', '$(x-1)$ is a root', '$x=-1$ is a root', '$(x+1)$ is a root'
   - the word 'factor' only with no link

Commonly Observed Responses:

		ii	$\bullet^3$ state quadratic factor	$\bullet^3$ $2x^2-11x+14$	3
			$\bullet^4$ find remaining linear factors or substitute into quadratic formula	$\bullet^4$ $....(2x-7)(x-2)$ or $\dfrac{11\pm\sqrt{(-11)^2-4\times2\times14}}{2\times2}$	
			$\bullet^5$ state solution	$\bullet^5$ $x=-1,\ 2,\ 3{\cdot}5$	

Notes:

4. On this occasion, the appearance of "$=0$" is not required for $\bullet^5$ to be awarded.
5. Be aware that the solution, $x=-1,\ 2,\ 3{\cdot}5$, may not appear until part (b).

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)	(i)	•[6] state coordinates	•[6] $(-1,0)$ and $(2,0)$	1
Notes:					
6. '-1 and 2' does not gain •[6] 7. $x=-1, y=0$ **and** $x=2, y=0$ gains •[6]					
Commonly Observed Responses:					
		(ii)	•[7] know to integrate with respect to x	•[7] $\int\left(2x^3-9x^2+3x+14\right)dx$	4
			•[8] integrate	•[8] $\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{3x^2}{2}+14x$	
			•[9] interpret limits and substitute	•[9] $\left(\frac{2\times2^4}{4}-\frac{9\times2^3}{3}+\frac{3\times2^2}{2}+14\times2\right)$ $-\left(\frac{2\times(-1)^4}{4}-\frac{9\times(-1)^3}{3}+\frac{3\times(-1)^2}{2}+14\times(-1)\right)$	
			•[10] evaluate integral	•[10] 27	
			Candidate A $\int\left(2x^3-9x^2+3x+14\right)dx$ •[7] ✓ $\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{3x^2}{2}+14x$ •[8] ✓ 27 •[9] ^ •[10] ✓1		
			Candidate B $\int\left(2x^3-9x^2+3x+14\right)dx$ •[7] ✓ $\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{3x^2}{2}+14x$ •[8] ✓ $\left(\frac{2\times(-1)^4}{4}-\frac{9\times(-1)^3}{3}+\frac{3\times(-1)^2}{2}+14\times(-1)\right)-\left(\frac{2\times2^4}{4}-\frac{9\times2^3}{3}+\frac{3\times2^2}{2}+14\times2\right)$ •[9] × -27, hence area is 27 •[10] ✓1 **However**, $-27=27$ does not gain •[10].		

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.	(a)		●[1] centre of C_1	●[1] $(-5,6)$	**4**
			●[2] radius of C_1	●[2] 3	
			●[3] centre of C_2	●[3] $(3,0)$	
			●[4] radius of C_2	●[4] 5	

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		●[5] calculate the distance between the centres	●[1] 10	**3**
			●[6] calculate the sum of the radii	●[2] 8	
			●[7] interpret significance of calculations	●[3] $8 < 10 \therefore$ the circles do not intersect	

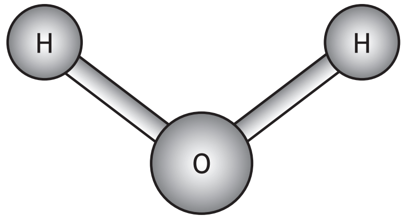
Notes:

1. For ●[7] to be awarded a comparison must appear.
2. Candidates who write '$r_1 + r_2 < \mathrm{D}$', or similar, must have identified the value of $r_1 + r_2$ and the value of D somewhere in their solution for ●[7] to be awarded.
3. Where earlier errors lead to the candidate dealing with non-integer values, do not penalise inaccuracies in rounding unless they lead to an inconsistent conclusion.

Commonly Observed Responses:

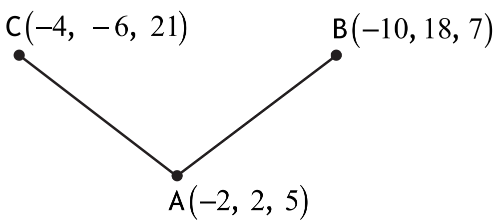
MARKS

5. The picture shows a model of a water molecule.



Relative to suitable coordinate axes, the oxygen atom is positioned at point A$(-2,\ 2,\ 5)$.

The two hydrogen atoms are positioned at points B$(-10, 18, 7)$ and C$(-4,\ -6,\ 21)$ as shown in the diagram below.



(a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. **2**

(b) Hence, or otherwise, find the size of angle BAC. **4**

6. Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula

$$B(t) = 200e^{0\cdot107t},$$

where t represents the number of hours since the study began.

(a) State the number of bacteria present at the start of the study. **1**

(b) Calculate the time taken for the number of bacteria to double. **4**

[Turn over

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.	(a)		●[1] find $\overrightarrow{AB}$	●[1] $\begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}$	**2**
			●[2] find $\overrightarrow{AC}$	●[2] $\begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$	

Notes:

1. For candidates who find **both** $\overrightarrow{BA}$ **and** $\overrightarrow{CA}$ correctly, only ●[2] is available (repeated error).
2. Accept vectors written horizontally.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		**Method 1**	**Method 1**	**4**
			●[1] evaluate $\overrightarrow{AB}.\overrightarrow{AC}$	●[1] $\overrightarrow{AB}.\overrightarrow{AC} = 16 - 128 + 32 = -80$	
			●[2] evaluate $\lvert\overrightarrow{AB}\rvert$ and $\lvert\overrightarrow{AC}\rvert$	●[2] $\lvert\overrightarrow{AB}\rvert = \lvert\overrightarrow{AC}\rvert = 18$	
			●[3] use scalar product	●[3] $\cos BAC = \dfrac{-80}{18 \times 18}$	
			●[4] calculate angle	●[4] $104 \cdot 3°$ or $1 \cdot 82$ radians	
			Method 2	**Method 2**	
			●[3] calculate length of BC	●[3] $BC = \sqrt{808}$	
			●[4] calculate lengths of AB and AC	●[4] $AB = AC = 18$	
			●[5] use cosine rule	●[5] $\cos BAC = \dfrac{18^2 + 18^2 - \sqrt{808}^2}{2 \times 18 \times 18}$	
			●[6] calculate angle	●[6] $104 \cdot 3°$ or $1 \cdot 82$ radians	

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.	(a)		•1 state the number	•1 200	**1**

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•2 interpret context and form equation •3 knowing to use logarithms appropriately. •4 simplify •5 evaluate t	•2 $2 = e^{0\cdot107t}$ •3 $\ln 2 = \ln\left(e^{0\cdot107t}\right)$ •4 $\ln 2 = 0\cdot107t$ •5 $t = 6\cdot478...$	**4**

Notes:

1. Accept $400 = 200e^{0\cdot107t}$ or equivalent for •2
2. Any base may be used at the •3 stage.
3. •3 may be assumed by •4.
4. Accept $t = 6\cdot5$.
5. At •5 ignore incorrect units. However, see Candidates B and C.
6. The calculation at •5 must involve the evaluation of a logarithm within a valid strategy for •5 to be awarded.
7. Candidates who take an iterative approach to arrive at $t = 6\cdot5$ gain •2 only.
   However, if, in the iterations, $B(t)$ is evaluated for $t = 6\cdot45$ and $t = 6\cdot55$ then award 4/4.

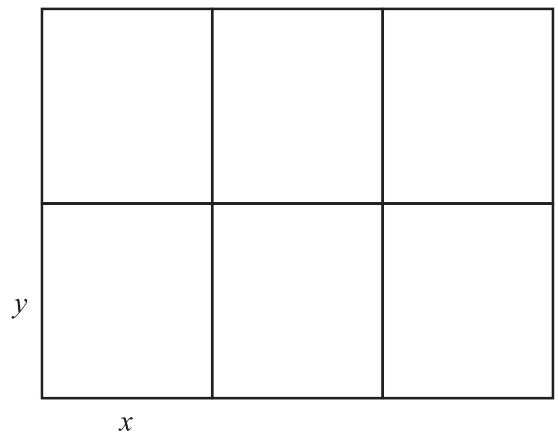
Commonly Observed Responses:

Candidate A $2 = e^{0\cdot107t}$ •2 ✓ $\log_{10} 2 = \log_{10}\left(e^{0\cdot107t}\right)$ •3 ✓ $\log_{10} 2 = 0\cdot107t\log_{10}e$ •4 ✓ $t = 6\cdot478\ldots$ •5 ✓	**Candidate B** $t = 6\cdot48$ hours •5 ✓ $t = 6$ hours 48 minutes
Candidate C $\ln(2) = 0\cdot107t$ •4 ✓ $t = 6$ hours 48 minutes •5 ✗	**Candidate D** $400 = 200e^{0\cdot107t}$ •2 ✓ $e^{0\cdot107t} = 2$ •3 ^ $t = 6\cdot48$ hours $\boxed{✓1}$ •4 $\boxed{✓1}$ •5

MARKS

7. A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.



(a) The area of land being set aside is $108\,\text{m}^2$.

Show that the total length of fencing, L metres, is given by

$$L(x) = 9x + \frac{144}{x}\ .$$ **3**

(b) Find the value of x that minimises the length of fencing required. **6**

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		•¹ expression for length in terms of x and y •² obtain an expression for y •³ demonstrate result	•¹ $9x+8y$ •² $y=\frac{108}{6x}$ •³ $\mathrm{L}(x)=9x+8\left(\frac{108}{6x}\right)$ leading to $\mathrm{L}(x)=9x+\frac{144}{x}$	3
Notes:					
1. The substitution for y at •³ must be clearly shown for •³ to be available. 2. For candidates who omit some, or all, of the internal fencing, only •² is available.					
Commonly Observed Responses:					
	(b)		•⁴ know to and start to differentiate •⁵ complete differentiation •⁶ set derivative equal to 0 •⁷ obtain for x •⁸ verify nature of stationary point •⁹ interpret and communicate result	•⁴ $\mathrm{L}'(x)=9\ldots$ •⁵ $L'(x)=9-\frac{144}{x^2}$ •⁶ $9-\frac{144}{x^2}=0$ •⁷ $x=4$ •⁸ Table of signs for a derivative – see the additional page. •⁹ Minimum at $x=4$ **or** •⁸ $\mathrm{L}''(x)=\frac{288}{x^3}$ •⁹ $\mathrm{L}''(4)>0$ $\therefore$ minimum Do not accept $\frac{d^2y}{dx^2}=\ldots$	6

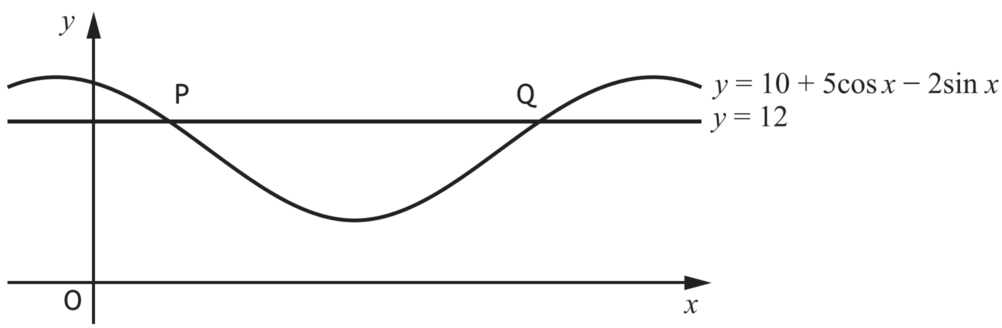
MARKS

8. (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x + a)$,

where $k > 0$ and $0 < a < 2\pi$. **4**

(b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.



Find the x-coordinates of P and Q. **4**

9. For a function f, defined on a suitable domain, it is known that:

- $f'(x) = \dfrac{2x+1}{\sqrt{x}}$

- $f(9) = 40$

Express $f(x)$ in terms of x. **4**

[Turn over for next question

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.	(a)		•¹ use compound angle formula •² compare coefficients •³ process for k •⁴ process for a and express in required form	•¹ $k\cos x\cos a - k\sin x\sin a$ **stated explicitly** •² $k\cos a = 5, k\sin a = 2$ **stated explicitly** •³ $k = \sqrt{29}$ •⁴ $\sqrt{29}\cos(x + 0\cdot38)$	4

Notes:

1. Treat $k\cos x\cos a - \sin x\sin a$ as bad form only if the equations at the •² stage both contain k.
2. $\sqrt{29}\cos x\cos a - \sqrt{29}\sin x\sin a$ or $\sqrt{29}(\cos x\cos a - \sin x\sin a)$ is acceptable for •¹ and •³.
3. Accept $k\cos a = 5, -k\sin a = -2$ for •².
4. •² is not available for $k\cos x = 5, k\sin x = 2$, however, •⁴ is still available.
5. •³ is only available for a single value of $k, k > 0$.
6. Candidates who work in degrees and do not convert to radian measure do not gain •⁴.
7. Candidates may use any form of the wave equation for •¹, •² and •³, however, •⁴ is only available if the value of a is interpreted in the form $k\cos(x + a)$.
8. Accept any answer for a that rounds to $0\cdot38$.
9. Evidence for •⁴ may not appear until part (b).

Commonly Observed Responses:

Responses with missing information in working:

Candidate A	**Candidate B**
•¹ ^ $\sqrt{29}\cos a = 5$ •² ✓ $\sqrt{29}\sin a = 2$ •³ ✓ $\tan a = \frac{2}{5}, \ a = 0\cdot3805...$ $\sqrt{29}\cos(x + 0\cdot38)$ •⁴ ✓	$k\cos x\cos a - k\sin x\sin a$ •¹ ✓ $\cos a = 5$ $\sin a = 2$ •² ✗ •³ ✓ $\tan a = \frac{2}{5}$ $a = 0\cdot38$ Not consistent with equations at •². $\sqrt{29}\cos(x + 0\cdot38)$ •⁴ ✗

Responses with the correct expansion of $k\cos(x + a)$ but errors for either •² or •⁴.

Candidate C	**Candidate D**	**Candidate E**
$k\cos a = 5, \ k\sin a = 2$ •² ✓ $\tan a = \frac{5}{2}$ •⁴ ✗ $a = 1\cdot19$	$k\cos a = 2, \ k\sin a = 5$ •² ✗ $\tan a = \frac{5}{2}, \ a = 1\cdot19$ $\sqrt{29}\cos(x + 1\cdot19)$ •⁴ $\boxed{✓1}$	$k\cos a = 5, \ k\sin a = -2$ •² ✗ $\tan a = \frac{-2}{5}$ $\sqrt{29}\cos(x + 5\cdot90)$ •⁴ $\boxed{✓1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
Responses with the incorrect labelling; $k\cos A\cos B - k\sin A\sin B$ from formula list.					

Candidate F	**Candidate G**	**Candidate H**
$k\cos A\cos B - k\sin A\sin B$ $\bullet^1$ ✗	$k\cos A\cos B - k\sin A\sin B$ $\bullet^1$ ✗	$k\cos A\cos B - k\sin A\sin B$ $\bullet^1$ ✗
$k\cos a = 5,\ k\sin a = 2$ $\bullet^2$ ✓	$k\cos x = 5,\ k\sin x = 2$ $\bullet^2$ ✗	$k\cos B = 5,\ k\sin B = 2$ $\bullet^2$ $\boxed{✓1}$
$\tan a = \frac{2}{5},\ a = 0\cdot3805...$	$\tan x = \frac{2}{5},\ x = 0\cdot3805...$	$\tan B = \frac{2}{5},\ B = 0\cdot3805...$
$\sqrt{29}\cos(x + 0\cdot38)$ $\bullet^3$ ✓ $\bullet^4$ ✓	$\sqrt{29}\cos(x + 0\cdot38)$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓1}$	$\sqrt{29}\cos(x + 0\cdot38)$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^5$ equate to 12 and simplify constant terms	$\bullet^5$ $5\cos x - 2\sin x = 2$ or $5\cos x - 2\sin x - 2 = 0$	4
			$\bullet^6$ use result of part (a) and rearrange	$\bullet^6$ $\cos(x + 0\cdot3805\ldots) = \frac{2}{\sqrt{29}}$	
			$\bullet^7$ solve for $x + a$	$\bullet^7$ (col. $\bullet^7$): $1\cdot1902\ldots$, (col. $\bullet^8$): $5\cdot0928\ldots$	
			$\bullet^8$ solve for x	$\bullet^8$ (col. $\bullet^7$): $0\cdot8097\ldots$, (col. $\bullet^8$): $4\cdot712\ldots$	

Notes:

10. The values of x may be given in radians or degrees.
11. Do not penalise candidates who attribute the values of x to the wrong points.
12. Accept any answers, in degrees or radians, that round correct to one decimal place.
13. $\bullet^4$ is unavailable for candidates who give their answer in degrees in part (a) and in part (b). $\bullet^4$ is unavailable for candidates who give their answer in degrees in part (a) and radians in part (b). $\bullet^8$ is unavailable for candidates who give their answer in radians in part (a) and degrees in part (b).

Conversion Table:

Degrees	Radians
$21\cdot8$	$0\cdot3805...$
$46\cdot4$	$0\cdot8097...$
$68\cdot2$	$1\cdot190...$
270	$4\cdot712...$ or $\frac{3\pi}{2}$
$291\cdot8$	$5\cdot0928...$

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
9.			•¹ write in integrable form	•¹ $2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$	4
			•² integrate one term	•² $\frac{4}{3}x^{\frac{3}{2}}$ or $2x^{\frac{1}{2}}$	
			•³ complete integration	•³ $2x^{\frac{1}{2}} + c$ or $\frac{4}{3}x^{\frac{3}{2}} + c$	
			•⁴ state expression for $f(x)$	•⁴ $f(x) = \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 2$	

Notes:

1. For candidates who do not attempt to write $f'(x)$ as the sum of two integrable terms, award 0/4.
2. $\bullet^2$ and $\bullet^3$ are only available for integrating terms involving fractional indices.
3. The term integrated at $\bullet^3$ must have an index of opposite sign to that of the term integrated at $\bullet^2$.
4. For candidates who differentiate one term, only $\bullet^1$ and $\bullet^2$are available.
5. For candidates who differentiate both terms, only $\bullet^1$ is available.
6. For $\bullet^4$. accept ‘ $f(x) = \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c,\ c = -2$ ’
7. Candidates must simplify coefficients in <u>**their**</u> final line of working for the last mark available for that line of working to be awarded.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$f'(x) = 2x + x^{-\frac{1}{2}}$	$\bullet^1$ ✗	$f'(x) = 2x + x^{-\frac{1}{2}}$	$\bullet^1$ ✗
$x^2 + 2x^{\frac{1}{2}} + c$	$\bullet^2$ ✓ $\bullet^3$ $\boxed{✓2}$	$x^2 + 2x^{\frac{1}{2}}$	$\bullet^2$ ✓ $\bullet^3$ ✗
$f(x) = x^2 + 2x^{\frac{1}{2}} - 47$	$\bullet^4$ $\boxed{✓1}$	$f(x) = x^2 + 2x^{\frac{1}{2}}$	$\bullet^4$ ^
Candidate C		**Candidate D**	
$f'(x) = 2x^{\frac{1}{2}} + 1$	$\bullet^1$ ✗	$f'(x) = \dfrac{2x+1}{x^{\frac{1}{2}}}$	$\bullet^1$ ^
$\frac{4}{3}x^{\frac{3}{2}} + x + c$	$\bullet^2$ ✓ $\bullet^3$ $\boxed{✓2}$	$\dfrac{x^2 + x}{2x^{\frac{3}{2}}} + c$	$\bullet^2$ ✗ $\bullet^3$ ✗ $\bullet^4$ ✗ See Note 1
$f(x) = \frac{4}{3}x^{\frac{3}{2}} + x - 5$	$\bullet^4$ $\boxed{✓1}$	$f(x) = \dfrac{x^2 + x}{2x^{\frac{3}{2}}} + \dfrac{115}{3}$	

MARKS

10. (a) Given that $y = \left(x^2 + 7\right)^{\frac{1}{2}}$, find $\dfrac{dy}{dx}$. **2**

(b) Hence find $\displaystyle\int \frac{4x}{\sqrt{x^2 + 7}}\, dx$. **1**

11. (a) Show that $\sin 2x \tan x = 1 - \cos 2x$, where $\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$. **4**

(b) Given that $f\left(x\right) = \sin 2x \tan x$, find $f'(x)$. **2**

[END OF QUESTION PAPER]

Question			Generic Scheme	Illustrative Scheme	Max Mark
Candidate E					
$f'(x)=2x^{\frac{1}{2}}+x^{-\frac{1}{2}}$			•1 ✓		
$=\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+c$			•2 ✓		
			•3 $\boxed{✓2}$ •4 ^		
10.	(a)		•1 Start to differentiate •2 Complete differentiation	•1 $\frac{1}{2}\left(x^2+7\right)^{-\frac{1}{2}}$..... •2 ....$\times\ 2x$	**2**
Notes:					
1. On this occasion there is no requirement to simplify coefficients.					
Commonly Observed Responses:					
	(b)		•3 link to (a) and integrate	•3 $4\left(x^2+7\right)^{\frac{1}{2}}\ (+c)$	**1**
Notes:					
2. A candidate's answer at •3 must be consistent with earlier working.					
Commonly Observed Responses:					

Candidate A

$\int 4x(x^2+7)^{\frac{-1}{2}}dx$

$=\frac{4x(x^2+7)^{\frac{1}{2}}}{\frac{1}{2}\times 2x}+c$

$=\frac{4x(x^2+7)^{\frac{1}{2}}}{x}+c$

$=4(x^2+7)^{\frac{1}{2}}+c$ •3 ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.	(a)		•¹ substitute for $\sin 2x$ and $\tan x$	•¹ $(2\sin x\cos x)\times\dfrac{\sin x}{\cos x}$	**4**
			•² simplify	•² $2\sin^2 x$	
			•³ use an appropriate substitution	•³ $2\left(1-\cos^2 x\right)$ or $1-\left(1-2\sin^2 x\right)$	
			•⁴ simplify and communicate result	•⁴ $1-\cos 2x = 1-\cos 2x$ or $2\sin^2 x = 2\sin^2 x$ $\therefore$ Identity shown	

Notes:

1. •¹ is not available to candidates who simply quote $\sin 2x = 2\sin x\cos x$ and $\tan x = \dfrac{\sin x}{\cos x}$ without substituting into the identity.
2. •⁴ is not available to candidates who work throughout with A in place of x.
3. •³ is not available to candidates who simply quote $\cos 2x = 1-2\sin^2 x$ without substituting into the identity.
4. On this occasion, at •⁴ do not penalise the omission of 'LHS = RHS' or a similar statement.

Commonly Observed Responses:

Candidate A

$\sin 2x\tan x = 1-\cos 2x$

$2\sin x\cos x\times\dfrac{\sin x}{\cos x} = 1-\cos 2x$ •¹ ✓

$2\sin^2 x = 1-\cos 2x$ •² ✓

$2\sin^2 x - 1 = -\cos 2x$ •³ ✗ •⁴ ✗

$-(1-2\sin^2 x) = -\cos 2x$

$-\cos 2x = -\cos 2x$

In proving the identity, candidates must work with both sides independently. ie in each line of working the LHS must be equivalent to the left hand side of the line above.

Candidate B

$\sin 2x\tan x = 1-\cos 2x$ •¹ ^ •² ^

$\sin 2x\tan x = 1-(1-2\sin^2 x)$

$\sin 2x\tan x = 2\sin^2 x$ •³ ✓

$\tan x = \dfrac{2\sin^2 x}{2\sin x\cos x}$ •⁴ ✗

$\tan x = \tan x$

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•5 link to (a) and substitute •6 differentiate	•5 $f(x)=1-\cos 2x$ or $f(x)=2\sin^2 x$ •6 $f'(x)=2\sin 2x$ or $f'(x)=4\sin x\cos x$	2

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

MARKS

Attempt ALL questions

Total marks – 60

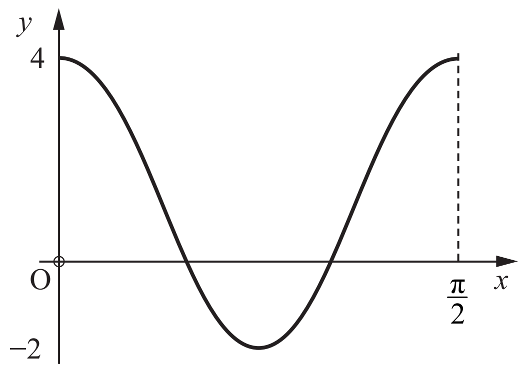
1. Vectors $\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ are perpendicular.

Determine the value of t. **2**

2. Find the equation of the tangent to the curve $y = 2x^3 + 3$ at the point where $x = -2$. **4**

3. Show that $(x + 3)$ is a factor of $x^3 - 3x^2 - 10x + 24$ and hence factorise $x^3 - 3x^2 - 10x + 24$ fully. **4**

4. The diagram shows part of the graph of the function $y = p\cos qx + r$.



Write down the values of p, q and r. **3**

5. A function g is defined on $\mathbb{R}$, the set of real numbers, by $g(x) = 6 - 2x$.

(a) Determine an expression for $g^{-1}(x)$. **2**

(b) Write down an expression for $g(g^{-1}(x))$. **1**

6. Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$. **3**

7. A function f is defined on a suitable domain by $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$.

Find $f'(4)$. **4**

[Turn over

Back to Table

Detailed Marking Instructions for each question

Question		Generic Scheme	Illustrative Scheme	Max Mark
1.				
		$\bullet^1$ equate scalar product to zero $\bullet^2$ state value of t	$\bullet^1$ $-24+2t+6=0$ $\bullet^2$ $t=9$	**2**
Notes:				
Commonly Observed Responses:				
Candidate A $-24+2t+6=-1$ $\bullet^1$ ✗ $t=\frac{17}{2}$ or $8\frac{1}{2}$ $\bullet^2$ $\boxed{✓1}$				
2.				
		$\bullet^1$ know to and differentiate $\bullet^2$ evaluate $\frac{dy}{dx}$ $\bullet^3$ evaluate y-coordinate $\bullet^4$ state equation of tangent	$\bullet^1$ $6x^2$ $\bullet^2$ 24 $\bullet^3$ -13 $\bullet^4$ $y=24x+35$	**4**
Notes:				
1. $\bullet^4$ is only available if an attempt has been made to find the gradient from differentiation. 2. At mark $\bullet^4$ accept $y+13=24(x+2)$, $y-24x=35$ or any other rearrangement of the equation.				
Commonly Observed Responses:				

<table>
<tr><th>Question</th><th>Generic Scheme</th><th>Illustrative Scheme</th><th>Max Mark</th></tr>
<tr><td>3.</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>Method 1</td><td>4</td></tr>
<tr><td></td><td>•1 know to use $x=-3$</td><td>•1 $(-3)^3-3(-3)^2-10(-3)+24$</td><td></td></tr>
<tr><td></td><td>•2 interpret result and state conclusion</td><td>•2 $=0\,\therefore (x+3)$ is a factor.</td><td></td></tr>
<tr><td></td><td></td><td>Method 2</td><td></td></tr>
<tr><td></td><td></td><td>•1 $$\begin{array}{r|rrrr} -3 & 1 & -3 & -10 & 24 \\ & & -3 & & \\ \hline & 1 & & & \end{array}$$</td><td></td></tr>
<tr><td></td><td></td><td>•2 $$\begin{array}{r|rrrr} -3 & 1 & -3 & -10 & 24 \\ & & -3 & 18 & -24 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$ remainder $=0\,\therefore (x+3)$ is a factor.</td><td></td></tr>
<tr><td></td><td></td><td>Method 3</td><td></td></tr>
<tr><td></td><td></td><td>•1 $$\begin{array}{r l} & x^2 \\ x+3 \big) & \overline{x^3-3x^2-10x+24} \\ & x^3+3x^2 \end{array}$$</td><td></td></tr>
<tr><td></td><td></td><td>•2 $=0\,\therefore (x+3)$ is a factor.</td><td></td></tr>
<tr><td></td><td>•3 state quadratic factor</td><td>•3 x^2-6x+8 stated or implied by •4</td><td></td></tr>
<tr><td></td><td>•4 factorise completely</td><td>•4 $(x+3)(x-4)(x-2)$</td><td></td></tr>
</table>

Notes:

1. Communication at •2 must be consistent with working at that stage ie a candidate's working must arrive legitimately at 0 before •2 is awarded.
2. Accept any of the following for •2:
   ' $f(-3)=0$ so $(x+3)$ is a factor'
   'since remainder is 0, it is a factor'
   the 0 from the table linked to the word 'factor' by eg 'so', 'hence', '$\therefore$ ', '$\rightarrow$ ', '$\Rightarrow$ '
3. Do not accept any of the following for •2:
   double underlining the zero or boxing the zero without comment
   ' $x=3$ is a factor', '$(x-3)$ is a factor', ' $x=-3$ is a root', '$(x-3)$ is a root', "$(x+3)$ is a root"
   the word 'factor' **only,** with no link
4. At •4 the expression may be written in any order.
5. An incorrect quadratic correctly factorised may gain •4
6. Where the quadratic factor obtained is irreducible, candidates must clearly demonstrate that $b^2-4ac<0$ to gain •4
7. $=0$ must appear at •1 or •2 for •2 to be awarded.
8. For candidates who do not arrive at 0 at the •2 stage •2•3•4 not available.
9. Do not penalise candidates who attempt to solve a cubic equation. However, within this working there may be evidence of the correct factorisation of the cubic.

Back to Table

Commonly Observed Responses:

Candidate A		Candidate B	
$\begin{array}{r\|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$ $\Rightarrow x-2$ is a factor	$\bullet^2$ ✓1 $\bullet^3$ ✓	$\begin{array}{r\|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$ $\Rightarrow x-2$ is a factor	$\bullet^1$ ✓2 $\bullet^2$ ✓1
$(x-2)(x^2-x-12)$	$\bullet^4$ ✓		
$(x-2)(x-4)(x+3) \Rightarrow x+3$ is a factor	$\bullet^1$ ✓		

4.				
		$\bullet^1$ state the value of p	$\bullet^1$ $p=3$	3
		$\bullet^2$ state the value of q	$\bullet^2$ $q=4$	
		$\bullet^3$ state the value of r	$\bullet^3$ $r=1$	

Notes:

1. These are the only acceptable responses for p, q and r.

Commonly Observed Responses:

5(a).				
		$\bullet^1$ let $y=6-2x$ and rearrange.	$\bullet^1$ $x=\dfrac{6-y}{2}$ or $y=\dfrac{6-x}{2}$	2
		$\bullet^2$ state expression.	$\bullet^2$ $g^{-1}(x)=\dfrac{6-x}{2}$ or $3-\dfrac{x}{2}$ or $\dfrac{x-6}{-2}$	
		Method 2	**Method 2**	
		$\bullet^3$ equates composite function to x	$g(g^{-1}(x))=x$ this gains $\bullet^3$	
		$\bullet^1$ start to rearrange.	$6-2g^{-1}(x)=x$	
		$\bullet^2$ state expression.	$g^{-1}(x)=\dfrac{6-x}{2}$ or $3-\dfrac{x}{2}$ or $\dfrac{x-6}{-2}$	

Notes:

1. At $\bullet^1$ accept any equivalent expression with any 2 distinct variables.

Commonly Observed Responses:

5(b).				
		$\bullet^3$ state expression	$\bullet^3$ x	1

Notes:

2. Candidates using method 2 may be awarded $\bullet^3$ at line one.
3. For candidates who attempt to find the composite function $g\left(g^{-1}(x)\right)$, accept $6-2\left(\dfrac{6-x}{2}\right)$ for $\bullet^3$.
4. In this case $\bullet^3$ may be awarded as follow through where an incorrect $g^{-1}(x)$ is found at $\bullet^2$, provided it includes the variable x.

Commonly Observed Responses:

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Question	Generic Scheme	Illustrative Scheme	Max Mark
6.			
	•¹ use laws of logs	•¹ $\log_6 27^{\frac{1}{3}}$	**3**
	•² use laws of logs	•² $\log_6\left(12\times 27^{\frac{1}{3}}\right)$	
	³ evaluate log	•³ 2	

Notes:

Commonly Observed Responses:

Candidate A

$\log_6 12+\log_6 9$ •¹ ✗

$\log_6(12\times 9)$ •² ✓1

$\log_6 108$ •³ ✓2

Candidate B

$\frac{1}{3}\log_6(12\times 27)$

$\frac{1}{3}\log_6 324$

$\log_6 324^{\frac{1}{3}}$

Award 1 out of 3 ^,^ ✓1

Question	Generic Scheme	Illustrative Scheme	Max Mark
7.			
	•¹ write in differentiable form	•¹ $3x^{\frac{3}{2}}-2x^{-1}$	**4**
	•² differentiate first term	•² $\frac{9}{2}x^{\frac{1}{2}}+\dots$	
	•³ differentiate second term	•³ $\dots+2x^{-2}$	
	•⁴ evaluate derivative at $x=4$	•⁴ $9\frac{1}{8}$	

Notes:

1. •² must involve a fractional index.
2. •³ must involve a negative index.
3. •⁴ is only available as a consequence of substituting into a 'derivative' containing a fractional or negative index.
4. If no attempt has been made to expand the bracket at •¹ then •² & •³ are not available. •⁴ is still available as follow through.

Commonly Observed Responses:

Candidate A

$$f(x)=3x^{\frac{1}{2}}-2x^{-\frac{1}{4}}$$

$$f'(x)=\frac{3}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{5}{4}}$$

$$=\frac{3}{2\sqrt{x}}+\frac{1}{2\sqrt[4]{x^5}}$$

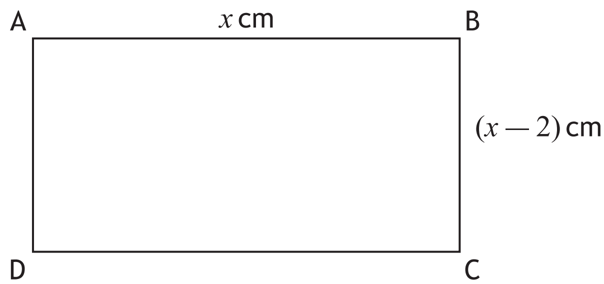
$$f'(4)=\frac{3}{2\sqrt{4}}+\frac{1}{2\sqrt[4]{4^5}}$$

$$=\frac{3}{4}+\frac{1}{8\sqrt{2}}$$

•¹ ✗
•² ✓1
•³ ✓1
•⁴ ✓1

MARKS

8. ABCD is a rectangle with sides of lengths x centimetres and $(x - 2)$ centimetres, as shown.



If the area of ABCD is less than $15\,\text{cm}^2$, determine the range of possible values of x. **4**

9. A, B and C are points such that AB is parallel to the line with equation $y + \sqrt{3}x = 0$ and BC makes an angle of $150°$ with the positive direction of the x-axis.

Are the points A, B and C collinear? **3**

10. Given that $\tan 2x = \frac{3}{4}$, $0 < x < \frac{\pi}{4}$, find the exact value of

(a) $\cos 2x$ **1**

(b) $\cos x$. **2**

11. T$(-2, -5)$ lies on the circumference of the circle with equation

$$(x + 8)^2 + (y + 2)^2 = 45.$$

(a) Find the equation of the tangent to the circle passing through T. **4**

(b) This tangent is also a tangent to a parabola with equation $y = -2x^2 + px + 1 - p$, where $p > 3$.

Determine the value of p. **6**

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Question	Generic Scheme	Illustrative Scheme	Max Mark
8.			
	•[1] interpret information	•[1] $x(x-2)<15$	**4**
	•[2] express in standard quadratic form	•[2] $x^2-2x-15<0$	
	•[3] factorise	•[3] $(x-5)(x+3)<0$	
	•[4] state range	•[4] $2<x<5$	

Notes:

Commonly Observed Responses:

Candidate A $x(x-2)=15$ •[1] ✗ $x^2-2x-15=0$ •[2] $\boxed{✓ 2}$ $x=-3,\ 5$ •[3] $\boxed{✓ 1}$ •[4] ^	**Candidate B - Mistaking perimeter for area** $4x-4<15$ $x<\frac{19}{4}$ **Award 1/4**
Candidate C $x^2-2x<15$ $x>2$ **Award 1/4**	**Candidate D** $x^2-2x<15$ $x>2$ $x<5$ **Inequalities not linked by 'and'** **Award 2/4**
Candidate E $x^2-2x<15$ $x>2$ **and** $x<5$ **Inequalities linked by 'and'** **Award 4/4**	

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Question		Generic Scheme	Illustrative Scheme	Max Mark
9.				
		$\bullet^1$ find gradient of AB	$\bullet^1$ $m_{AB} = -\sqrt{3}$	**3**
		$\bullet^2$ calculate gradient of BC	$\bullet^2$ $m_{BC} = -\frac{1}{\sqrt{3}}$	
		$\bullet^3$ interpret results and state conclusion	$\bullet^3$ $m_{AB} \neq m_{BC} \Rightarrow$ points are not collinear.	
			Method 2	
			$\bullet^1$ $m_{AB} = -\sqrt{3}$	
			$\bullet^2$ AB makes 120° with positive direction of the $x-$axis .	
			$\bullet^3$ $120 \neq 150$ so points are not collinear.	

Notes:

1. The statement made at $\bullet^3$ must be consistent with the gradients or angles found for $\bullet^1$ and $\bullet^2$.

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
10(a).				
		$\bullet^1$ state value of $\cos 2x$	$\bullet^1$ $\frac{4}{5}$	**1**

Notes:

Commonly Observed Responses:

Candidate A		**Candidate B**	
$\cos 2x = \frac{3}{5}$	$\bullet^1$ ✗	$\cos 2x = 4$	$\bullet^1$ ✗
$2\cos^2 x - 1 = ...$	$\bullet^2$ $\boxed{✓1}$	$2\cos^2 x - 1 = 4$	$\bullet^2$ $\boxed{✓1}$
$\cos x = \frac{2}{\sqrt{5}}$	$\bullet^3$ $\boxed{✓1}$	$\cos^2 x = \frac{5}{2}$	
		$\cos x = \sqrt{\frac{5}{2}}$	$\bullet^3$ ✗ invalid answer

Question		Generic Scheme	Illustrative Scheme	Max Mark
10(b).				
		$\bullet^2$ use double angle formula	$\bullet^2$ $2\cos^2 x - 1 = ...$	**2**
		$\bullet^3$ evaluate $\cos x$	$\bullet^3$ $\frac{3}{\sqrt{10}}$	

Notes:

1. Ignore the inclusion of $-\frac{3}{\sqrt{10}}$.
2. At $\bullet^2$ the double angle formula must be equated to the candidates answer to part (a).

Commonly Observed Responses:

Back to Table

Question		Generic Scheme	Illustrative Scheme	Max Mark
11(a).				
		•¹ state coordinates of centre	•¹ $(-8,-2)$	**4**
		•² find gradient of radius	•² $-\frac{1}{2}$	
		•³ state perpendicular gradient	•³ 2	
		•⁴ determine equation of tangent	•⁴ $y=2x-1$	

Notes:

1. •⁴ is only available as a consequence of trying to find and use a perpendicular gradient.

2. At mark •⁴ accept $y+5=2(x+2)$, $y-2x=-1$, $y-2x+1=0$ or any other rearrangement of the equation.

Commonly Observed Responses:

Back to Table

Question	Generic Scheme	Illustrative Scheme	Max Mark
11(b).			
	Method 1	**Method 1**	**6**
	•5 arrange equation of tangent in appropriate form and equate y_{tangent} to y_{parabola}	•5 $2x-1=-2x^2+px+1-p$	
	•6 rearrange and equate to 0	•6 $2x^2+(2-p)x+p-2=0$	
	•7 know to use discriminant and identify $a,\ b,$ and c	•7 $(2-p)^2-4\times 2\times(p-2)$	
	•8 simplify and equate to 0	•8 $p^2-12p+20=0$	
	•9 start to solve	•9 $(p-10)(p-2)=0$	
	•10 state value of p	•10 $p=10$	
	Method 2	**Method 2**	
	•5 arrange equation of tangent in appropriate form and equate y_{tangent} to y_{parabola}	•5 $2x-1=-2x^2+px+1-p$	
	•6 find $\frac{dy}{dx}$ for parabola	•6 $\frac{dy}{dx}=-4x+p$	
	•7 equate to gradient of line and rearrange for p	•7 $2=-4x+p$ $p=2+4x$	
	•8 substitute and arrange in standard form	•8 $0=2x^2-4x$	
	•9 factorise and solve for x	•9 $0=2x(x-2)$ $x=0,\ x=4$	
	•10 state value of p	•10 $p=10$	

Notes:

1. At •6 accept $2x^2+2x-px+p-2=0$.
2. At •7 accept $a=2,\ b=(2-p)$, and $c=(p-2)$.

Commonly Observed Responses:

Just using the parabola

$a=-2 \quad b=p \quad c=1-p$

$b^2-4ac=p^2-4\times(-2)(1-p)$

$=p^2-8p+8=0$

$p=4\pm\sqrt{8}$

$p=4+\sqrt{8}$ as $p>3$

•5 ^
•6 ^
•7 ✓1
•8 ✓2
•9 ✓1
•10 ✓1

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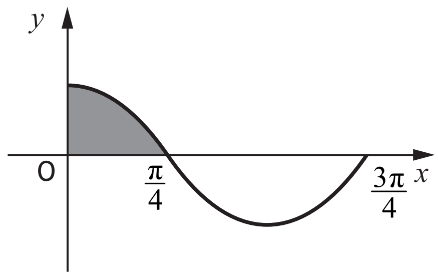
MARKS

12. The diagram shows part of the graph of $y = a\cos bx$.

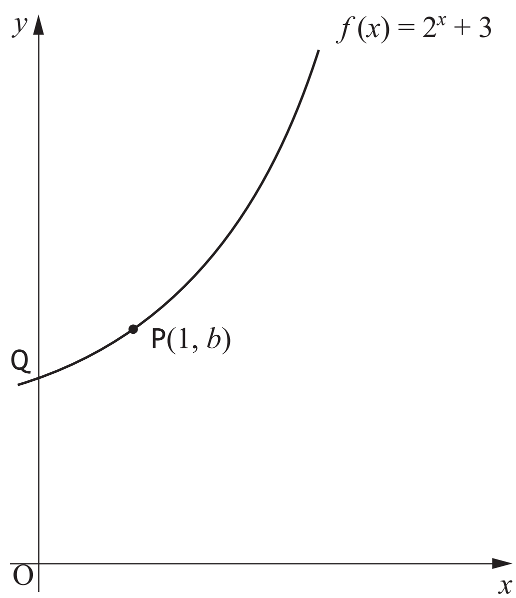
The shaded area is $\frac{1}{2}$ unit^2.



What is the value of $\int_0^{\frac{3\pi}{4}} (a\cos bx)\,dx$? **2**

13. The function $f(x) = 2^x + 3$ is defined on $\mathbb{R}$, the set of real numbers.

The graph with equation $y = f(x)$ passes through the point P$(1, b)$ and cuts the y-axis at Q as shown in the diagram.



(a) What is the value of b? **1**

(b) (i) Copy the above diagram.

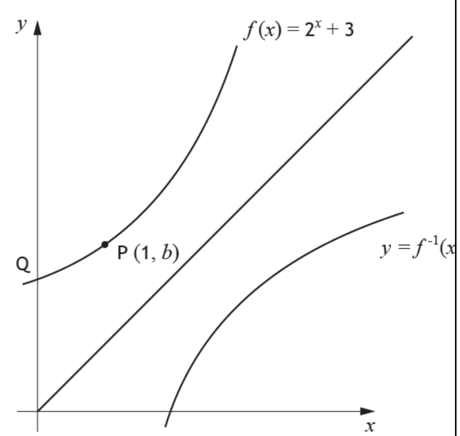
On the same diagram, sketch the graph with equation $y = f^{-1}(x)$. **1**

(ii) Write down the coordinates of the images of P and Q. **3**

(c) R $(3,11)$ also lies on the graph with equation $y = f(x)$.

Find the coordinates of the image of R on the graph with equation $y = 4 - f(x+1)$. **2**

[Turn over

Question	Generic Scheme	Illustrative Scheme	Max Mark
12.			
	•¹ interpret integral below $x-$axis •² evaluate	•¹ -1 (accept area below $x-$axis $=1$) •² $-\frac{1}{2}$	**2**
Notes:			
1. For candidates who calculate the area as $\frac{3}{2}$ award 1 out of 2.			
Commonly Observed Responses:			
13(a)			
	•¹ calculate b	•¹ 5	**1**
Notes:			
Commonly Observed Responses:			
13 (b)(i)			
	• ² reflecting in the line $y=x$	•² 	**1**
Notes:			
1. If the reflected graph cuts the $y-$axis, •² is not awarded.			
Commonly Observed Responses:			

Question	Generic Scheme	Illustrative Scheme	Max Mark
13(b)(ii)			
	•³ calculate y intercept •⁴ state coordinates of image of Q •⁵ state coordinates of image of P	•³ 4 •⁴ $(4,\ 0)$ see note 2 •⁵ $(5,\ 1)$	**3**
Notes:			
2. •⁴ can only be awarded if $(4,0)$ is clearly identified either by their labelling or by their diagram. 3. •³ is awarded for the appearance of 4, or $(4,0)$ or $(0,4)$. 4. •⁵ is awarded for the appearance of $(5,1)$. Ignore any labelling attached to this point.			
Commonly Observed Responses:			
Candidate A $y = f(x)$ reflected in $x-$axis 4 •³ ✓ (0,-4) •⁴ $\boxed{\checkmark 2}$ (1,-5) •⁵ $\boxed{\checkmark 1}$	**Candidate B** $y = f(x)$ reflected in $y-$axis 4 •³ ✓ (0,4) •⁴ $\boxed{\checkmark 2}$ (-1,5) •⁵ $\boxed{\checkmark 2}$		
13(c)			
	•⁶ state x coordinate of R •⁷ state y coordinate of R	•⁶ $x = 2$ •⁷ $y = -7$	**2**
Notes:			
Commonly Observed Responses:			
14.			
	•¹ identify length of radius •² determine value of k	$y-$axis tangent to circle | Circle passes through origin •¹ $r = 6$ | $r = \sqrt{61}$ •² $k = 25$ | $k = 0$	**2**

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MARKS

14. The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points.

What is the value of k? **2**

15. The rate of change of the temperature, T °C of a mug of coffee is given by

$$\frac{dT}{dt} = \frac{1}{25}t - k \ , \ 0 \leq t \leq \ 50$$

- t is the elapsed time, in minutes, after the coffee is poured into the mug
- k is a constant
- initially, the temperature of the coffee is 100 °C
- 10 minutes later the temperature has fallen to 82 °C.

Express T in terms of t. **6**

[END OF QUESTION PAPER]

14.				
		$\bullet^1$ identify length of radius $\bullet^2$ determine value of k	$y-$axis tangent to circle | Circle passes through origin $\bullet^1$ $r=6$ | $r=\sqrt{61}$ $\bullet^2$ $k=25$ | $k=0$	**2**

Question	Generic Scheme	Illustrative Scheme	Max Mark
15.			
	•¹ know to integrate	•¹ $\int$	**6**
	•² integrate a term	•² $\frac{1}{50}t^2 \ldots$ or $\ldots - kt$	
	•³ complete integration	•³ $\ldots - kt$ or $\frac{1}{50}t^2 \ldots$	
	•⁴ find constant of integration	•⁴ $c = 100$	
	•⁵ find value of k	•⁵ $k = 2$	
	•⁶ state expression for T	•⁶ $T = \frac{1}{50}t^2 - 2t + 100$	

Notes:

1. Accept unsimplified expressions at •² and •³ stage.
2. •⁴, •⁵ and •⁶ are not available for candidates who have not considered the constant of integration.
3. •¹ may be implied by •².

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

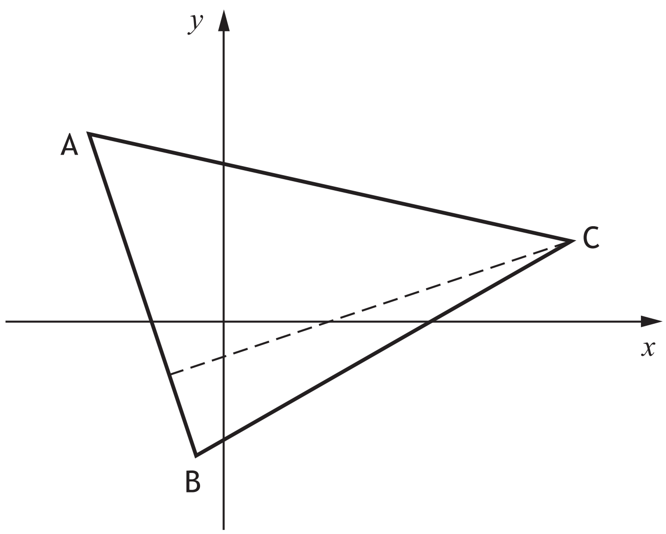
MARKS

Attempt ALL questions

Total marks – 70

1. The vertices of triangle ABC are A$(-5, 7)$, B$(-1, -5)$ and C$(13, 3)$ as shown in the diagram.

The broken line represents the altitude from C.



(a) Show that the equation of the altitude from C is $x - 3y = 4$. **4**

(b) Find the equation of the median from B. **3**

(c) Find the coordinates of the point of intersection of the altitude from C and the median from B. **2**

2. Functions f and g are defined on suitable domains by

$f(x) = 10 + x$ and $g(x) = (1 + x)(3 - x) + 2$.

(a) Find an expression for $f(g(x))$. **2**

(b) Express $f(g(x))$ in the form $p(x + q)^2 + r$. **3**

(c) Another function h is given by $h(x) = \dfrac{1}{f(g(x))}$.

What values of x cannot be in the domain of h? **2**

[Turn over

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Paper 2

Question	Generic Scheme	Illustrative Scheme	Max Mark
1(a)			
$\bullet^1$ calculate gradient of AB		$\bullet^1$ $m_{AB} = -3$	
$\bullet^2$ use property of perpendicular lines		$\bullet^2$ $m_{alt} = \frac{1}{3}$	
$\bullet^3$ substitute into general equation of a line		$\bullet^3$ $y-3=\frac{1}{3}(x-13)$	
$\bullet^4$ demonstrate result		$\bullet^4$ $\ldots \Rightarrow x-3y=4$	**4**

Notes:

1. $\bullet^3$ is only available as a consequence of trying to find and use a perpendicular gradient.
2. $\bullet^4$ is only available if there is/are appropriate intermediate lines of working between $\bullet^3$ and $\bullet^4$.
3. The ONLY acceptable variations for the final equation for the line in $\bullet^4$ are $4 = x-3y,\ -3y+x=4,\ 4=-3y+x$.

Commonly Observed Responses:

Candidate A	**Candidate B**
$m_{AB} = \frac{-1-(-5)}{-5-7} = \frac{4}{-12} = -\frac{1}{3}$ $\bullet^1$ ✗	For $\bullet^4$
$m_{alt} = 3$ $\bullet^2$ $\boxed{✓1}$	$y-3=\frac{1}{3}x-\frac{13}{3}$
$y-3=3(x-13)$ $\bullet^3$ $\boxed{✓1}$	$y=\frac{1}{3}x-\frac{4}{3}$
$\bullet^4$ ✗	$3y = x-4$ - not acceptable
$\bullet^4$ is not available	$3y-x=-4$ - not acceptable
	$x-3y=4$ ✓

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Question	Generic Scheme	Illustrative Scheme	Max Mark
1(b)			
	•5 calculate midpoint of AC	•5 $M_{AC}=(4,5)$	
	•6 calculate gradient of median	•6 $m_{BM}=2$	
	•7 determine equation of median	•7 $y=2x-3$	**3**

Notes:

4. •6 and •7 are not available to candidates who do not use a midpoint.
5. •7 is only available as a consequence of using a non-perpendicular gradient and a midpoint.
6. Candidates who find either the median through A or the median through C or a side of the triangle gain 1 mark out of 3.
7. At •7 accept $y-(-5)=2(x-(-1))$, $y-5=2(x-4)$, $y-2x+3=0$ or any other rearrangement of the equation.

Commonly Observed Responses:

Median through A	**Median through C**
$M_{BC}=(6,-1)$	$M_{AB}=(-3,1)$
$m_{AM}=\dfrac{-8}{11}$	$m_{CM}=\dfrac{1}{8}$
$y+1=\dfrac{-8}{11}(x-6)$ or $y-7=\dfrac{-8}{11}(x+5)$	$y-3=\dfrac{1}{8}(x-13)$ or $y-1=\dfrac{1}{8}(x+3)$
Award 1/3	**Award 1/3**

Question	Generic Scheme	Illustrative Scheme	Max Mark
1(c)			
	•8 calculate x or y coordinate	•8 $x=1$ or $y=-1$	
	•9 calculate remaining coordinate of the point of intersection	•9 $y=-1$ or $x=1$	**2**

Notes:

8. If the candidate's 'median' is either a vertical or horizontal line then award 1 out of 2 if both coordinates are correct, otherwise award 0.

Commonly Observed Responses:

For candidates who find the altitude through B in part (b)	**Candidate A**
$x=-\dfrac{1}{5}$ •8 ✓1	(b) $y-5=2(x-4)$ •7 ✓
$y=-\dfrac{7}{5}$ •9 ✓1	$y=2x-13$ - error
	(c) $x-3y=4$ •8 ×
	$y=2x-13$ •9 ✓1
	Leading to $x=7$ and $y=1$

Question	Generic Scheme	Illustrative Scheme	Max Mark
2 (a)			
	$\bullet^1$ interpret notation $\bullet^2$ state a correct expression	$\bullet^1$ $f((1+x)(3-x)+2)$ stated or implied by $\bullet^2$ $\bullet^2$ $10+(1+x)(3-x)+2$ stated or implied by $\bullet^3$	2

Notes:

1. $\bullet^1$ is not available for $g\big(f(x)\big)=g(10+x)$ but $\bullet^2$ may be awarded for $(1+10+x)\big(3-(10+x)\big)+2$.

Commonly Observed Responses:

Candidate A	Candidate B / Candidate C
(a) $f(g(x))='g(f(x))'$ $=(1+10+x)(3-(10+x))+2$ $\quad\bullet^1$ × $\quad\bullet^2$ $\boxed{✓1}$ **(b)** $=-75-18x-x^2$ or $-x^2-18x-75$ $\quad\bullet^3$ $\boxed{✓1}$ $=-(x^2+18x$ $\quad\bullet^4$ $\boxed{✓1}$ $=-(x+9)^2$ $\quad\bullet^5$ $\boxed{✓1}$ $=-(x+9)^2+6$ **(c)** $-(x+9)^2+6=0$ $\quad\bullet^6$ $\boxed{✓1}$ $x=-9+\sqrt{6}$ or $-9-\sqrt{6}$ $\quad\bullet^7$ $\boxed{✓1}$	**Candidate B** $f(g(x))$ $\quad\bullet^1$ ^ $=10((1+x)-(3-x))+2$ $\quad\bullet^2$ × **Candidate C** $f(g(x))$ $\quad\bullet^1$ ^ $=10((1+x)(3-x)+2)$ $\quad\bullet^2$ ×

Question	Generic Scheme	Illustrative Scheme	Max Mark
2 (b)			
	$\bullet^3$ write $f(g(x))$ in quadratic form	$\bullet^3$ $15+2x-x^2$ or $-x^2+2x+15$	
	Method 1	**Method 1**	
	$\bullet^4$ identify common factor	$\bullet^4$ $-1(x^2-2x$ stated or implied by $\bullet^5$	
	$\bullet^5$ complete the square	$\bullet^5$ $-1(x-1)^2+16$	
	Method 2	**Method 2**	
	$\bullet^4$ expand completed square form and equate coefficients	$\bullet^4$ $px^2+2pqx+pq^2+r$ and $p=-1$,	
	$\bullet^5$ process for q and r and write in required form	$\bullet^5$ $q=-1$ and $r=16$ Note if $p=1$ $\bullet^5$ is not available	3

Notes:

2. Accept $16-(x-1)^2$ or $-\left[(x-1)^2-16\right]$ at $\bullet^5$.

Commonly Observed Responses:

Candidate A	**Candidate B**	**Candidate C**
$-(x^2-2x-15)$ $\bullet^4$ ✓ $-(x^2-2x+1-1-15)$ $-(x-1)^2-16$ $\bullet^5$ ×	$15+2x-x^2$ $\bullet^3$ ✓ $x^2-2x-15$ $\bullet^4$ × $px^2+2pqx+pq^2+r$ and $p=1$ $q=-1$ $r=-16$ $\bullet^5$ $\boxed{✓2}$ eased	$-x^2+2x+15$ $\bullet^3$ ✓ $-(x+1)^2\ldots$ $\bullet^4$ × $-(x+1)^2+14$ $\bullet^5$ ×
Candidate D	**Candidate E**	**Candidate F**
$15+2x-x^2$ $\bullet^3$ ✓ $x^2-2x-15$ $\bullet^4$ × $(x-1)^2-16$ $\bullet^5$ $\boxed{✓2}$ eased Eased, unitary coefficient of x^2 (lower level skill)	$15+2x-x^2$ $\bullet^3$ ✓ $x^2-2x-15$ $\bullet^4$ ✓ $(x-1)^2-16$ so $15+2x-x^2=-(x-1)^2+16$ $\bullet^5$ ✓	$-x^2+2x+15$ $\bullet^3$ ✓ $-(x+1)^2\ldots$ $\bullet^4$ × $-(x+1)^2+16$ $\bullet^5$ $\boxed{✓1}$

2(c)

$\bullet^6$ identify critical condition $\bullet^7$ identify critical values	$\bullet^6$ $-1(x-1)^2+16=0$ or $f((g(x))=0$ $\bullet^7$ 5 and -3	**2**

Notes:

3. Any communication indicating that the denominator cannot be zero gains $\bullet^6$.

4. Accept $x=5$ and $x=-3$ or $x\neq5$ and $x\neq-3$ at $\bullet^7$.

5. If $x=5$ and $x=-3$ appear without working award 1/2.

Commonly Observed Responses:

Candidate A	**Candidate B**
$\dfrac{1}{-(x-1)^2+16}$ $\bullet^6$ ✓ $x\neq5$ $\bullet^7$ ^	$\dfrac{1}{f(g(x))}$ $f(g(x))>0$ $\bullet^6$ × $x=-3,\ x=5$ $\bullet^7$ ✓ $-3<x \quad x<5$

3(a)

$\bullet^1$ determine the value of the required term	$\bullet^1$ $22\frac{3}{4}$ or $\frac{91}{4}$ or $22\cdot75$	**1**

Notes:

1. Do not penalise the inclusion of incorrect units.
2. Accept rounded and unsimplified answers following evidence of correct substitution.

Commonly Observed Responses:

MARKS

3. A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3} f_n + 32, \qquad f_1 = 32$
- $t_{n+1} = \frac{3}{4} t_n + 13, \qquad t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

(a) Calculate t_2, the height of the toad at the end of the second day. **1**

(b) Determine whether or not either of them will eventually escape from the well. **5**

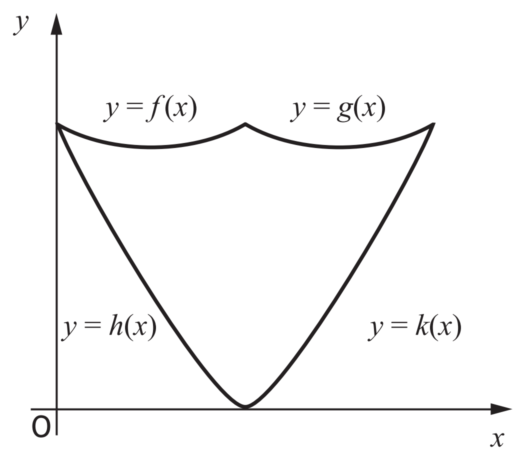
3(a)		
•[1] determine the value of the required term	•[1] $22\frac{3}{4}$ or $\frac{91}{4}$ or $22\cdot75$	**1**
Notes:		
1. Do not penalise the inclusion of incorrect units. 2. Accept rounded and unsimplified answers following evidence of correct substitution.		
Commonly Observed Responses:		

Question	Generic Scheme	Illustrative Scheme	Max Mark
3 (b)			
	Method 1 **(Considering both limits)**	**Method 1**	
	•2 know how to calculate limit	•2 $\frac{32}{1-\frac{1}{3}}$ or $L = \frac{1}{3}L + 32$	
	•3 know how to calculate limit	•3 $\frac{13}{1-\frac{3}{4}}$ or $L = \frac{3}{4}L + 13$	
	•4 calculate limit	•4 48	
	•5 calculate limit	•5 52	
	•6 interpret limits and state conclusion	•6 $52 > 50$ $\therefore$ toad will escape	
	Method 2 **(Frog first then numerical for toad)**	**Method 2**	
	•2 know how to calculate limit	•2 $\frac{32}{1-\frac{1}{3}}$ or $L = \frac{1}{3}L + 32$	
	•3 calculate limit	•3 48	
	•4 determine the value of the highest term less than 50	•4 $49\cdot803...$	
	•5 determine the value of the lowest term greater than 50	•5 $50\cdot352...$	
	•6 interpret information and state conclusion	•6 $50\cdot352 > 50$ $\therefore$ toad will escape	
	Method 3 **(Numerical method for toad only)**	**Method 3**	
	•2 continues numerical strategy	•2 numerical strategy	
	•3 exact value	•3 $30\cdot0625$	
	•4 determine the value of the highest term less than 50	•4 $49\cdot803...$	
	•5 determine the value of the lowest term greater than 50	•5 $50\cdot352...$	
	•6 interpret information and state conclusion	•6 $50\cdot352 > 50$ $\therefore$ toad will escape	
	Method 4 **(Limit method for toad only)**	**Method 4**	
	•2 & •3 know how to calculate limit	•2 & •3 $\frac{13}{1-\frac{3}{4}}$ or $L = \frac{3}{4}L + 13$	
	•4 & •5 calculate limit •	•4 & •5 52	
	•6 interpret limit and state conclusion	•6 $52 > 50$ $\therefore$ toad will escape	**5**

MARKS

4. A wall plaque is to be made to commemorate the 150th anniversary of the publication of "*Alice's Adventures in Wonderland*".

The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.



- $f(x) = \frac{1}{4}x^2 - \frac{1}{2}x + 3$
- $g(x) = \frac{1}{4}x^2 - \frac{3}{2}x + 5$
- $h(x) = \frac{3}{8}x^2 - \frac{9}{4}x + 3$
- $k(x) = \frac{3}{8}x^2 - \frac{3}{4}x$

(a) Find the x-coordinate of the point of intersection of the graphs with equations $y = f(x)$ and $y = g(x)$. **2**

The graphs of the functions $f(x)$ and $h(x)$ intersect on the y-axis.

The plaque has a vertical line of symmetry.

(b) Calculate the area of the wall plaque. **7**

[Turn over

Question	Generic Scheme	Illustrative Scheme	Max Mark
4 (a)			
	$\bullet^1$ know to equate $f(x)$ and $g(x)$ $\bullet^2$ solve for x	$\bullet^1$ $\frac{1}{4}x^2 - \frac{1}{2}x + 3 = \frac{1}{4}x^2 - \frac{3}{2}x + 5$ $\bullet^2$ $x = 2$	2

Notes:

1. $\bullet^1$ and $\bullet^2$ are not available to candidates who: (i) equate zeros, (ii) give answer only without working, (iii) arrive at $x = 2$ with erroneous working.

Commonly Observed Responses:

Candidate A	Candidate B
$y = \frac{1}{4}x^2 - \frac{1}{2}x + 3$ $y = \frac{1}{4}x^2 - \frac{3}{2}x + 5$ $\bullet^1$ ✓ subtract to get $0 = x - 2$ $x = 2$ $\bullet^2$ ✓	$\frac{1}{4}x^2 - \frac{1}{2}x = -3$ $\frac{1}{4}x^2 - \frac{3}{2}x = -5$ $\bullet^1$ × $x = 2$ $\bullet^2$ × *In this case the candidate has equated zeros*
Candidate C $f(x) = \frac{1}{4}x^2 - \frac{1}{2}x + 3$ $g(x) = \frac{1}{4}x^2 - \frac{3}{2}x + 5$ $f'(x) = \frac{1}{2}x - \frac{1}{2}$ $g'(x) = \frac{1}{2}x - \frac{3}{2}$ . . $x = 1$ $x = 3$ $\bullet^1$ ✓ $\therefore x = 2$ $\bullet^2$ ✓	

Question	Generic Scheme	Illustrative Scheme	Max Mark
4 (b)			
	•[3] know to integrate	•[3] $\int$	
	•[4] interpret limits	•[4] $\int_0^2$	
	•[5] use 'upper – lower'	•[5] $\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3) - (\frac{3}{8}x^2 - \frac{9}{4}x + 3)\, dx$	
	•[6] integrate	•[6] $-\frac{1}{24}x^3 + \frac{7}{8}x^2$ accept unsimplified integral	
	•[7] substitute limits	•[7] $\left(-\frac{1}{24}\times 2^3 + \frac{7}{8}\times 2^2\right) - 0$	
	•[8] evaluate area between $f(x)$ and $h(x)$	•[8] $\frac{19}{6}$	
	•[9] state total area	•[9] $\frac{19}{3}$	7

Notes:

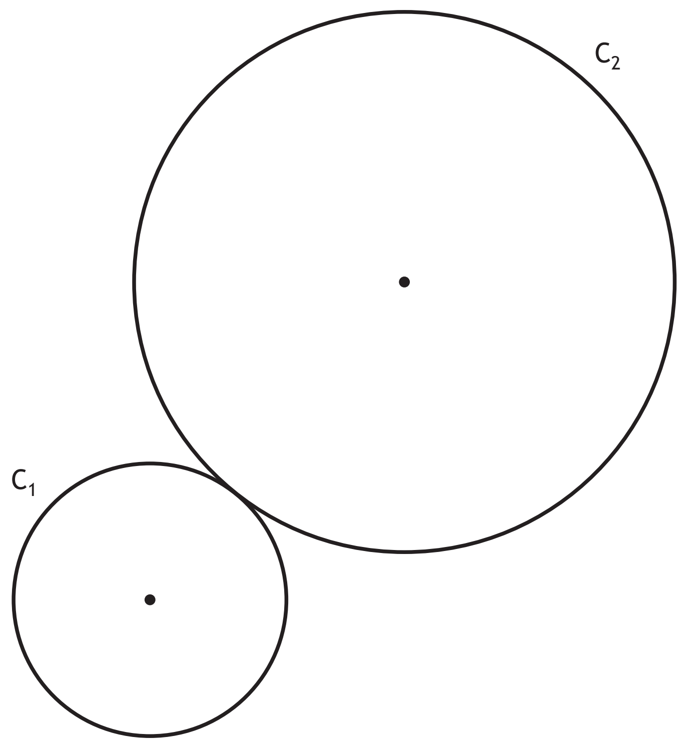
2. If limits $x = 0$ and $x = 2$ appear ex nihilo award •[4].
4. If a candidate differentiates at •[6] then •[6], •[7] and •[8] are not available. However, •[9] is still available.
5. Candidates who substitute at •[7], without attempting to integrate at •[6], cannot gain •[6], •[7] or •[8]. However, •[9] is still available.
6. Evidence for •[8] may be implied by •[9].
7. •[9] is a strategy mark and should be awarded for correctly multiplying their solution at •[8], or for any other valid strategy applied to previous working.
8. For •[5] both a term containing a variable and the constant term must be dealt with correctly.
9. In cases where •[5] is not awarded, •[6] may be gained for integrating an expression of equivalent difficulty ie a polynomial of at least degree two. •[6] is unavailable for the integration of a linear expression.
10. •[8] must be as a consequence of substituting into a term where the power of x is not equalto 1 or 0.

MARKS

5. Circle C_1 has equation $x^2 + y^2 + 6x + 10y + 9 = 0$.

The centre of circle C_2 is $(9, 11)$.

Circles C_1 and C_2 touch externally.



(a) Determine the radius of C_2. **4**

A third circle, C_3, is drawn such that:

- both C_1 and C_2 touch C_3 internally
- the centres of C_1, C_2 and C_3 are collinear.

(b) Determine the equation of C_3. **4**

Question	Generic Scheme	Illustrative Scheme	Max Mark
5(a)			
	●[1] state centre of C_1 ●[2] state radius of C_1 ●[3] calculate distance between centres of C_1 and C_2 ●[4] calculate radius of C_2	●[1] $(-3,-5)$ ●[2] 5 ●[3] 20 ●[4] 15	**4**

Notes:

1. For ●[4] to be awarded radius of C_2 must be greater than the radius of C_1.
2. Beware of candidates who arrive at the correct solution by finding the point of contact by an invalid strategy.
3. ●[4] is for $\text{Distance}_{c1c2} - r_{c1}$ but only if the answer obtained is greater than r_{c1}.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
5 (b)			
	•5 find ratio in which centre of C_3 divides line joining centres of C_1 and C_2 •6 determine centre of C_3 •7 calculate radius of C_3 •8 state equation of C_3	•5 $3:1$ •6 $(6,7)$ •7 $r=20$ (answer must be consistent with distance between centres) •8 $(x-6)^2+(y-7)^2=400$	**4**

Notes:

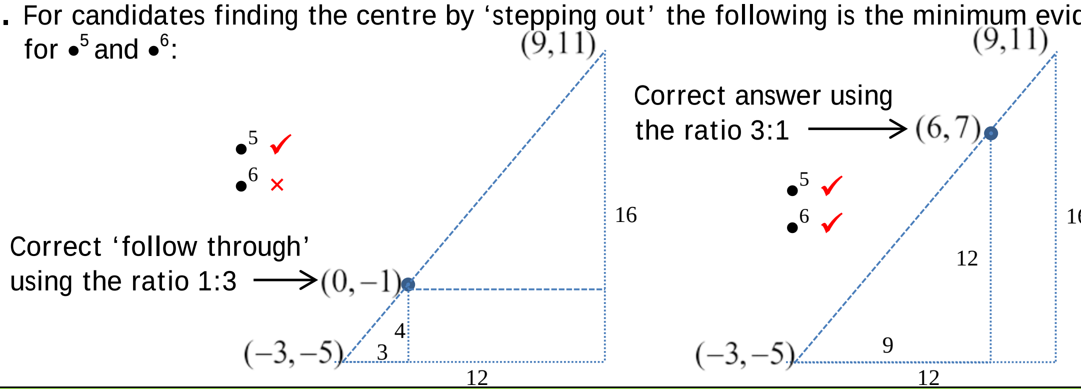
4. For •5 accept ratios $\pm3:\pm1$, $\pm1:\pm3$, $\mp3:\pm1$, $\mp1:\pm3$ (or the appearance of $\frac{3}{4}$).

5. •7 is for $r_{c2}+r_{c1}$.

6. Where candidates arrive at an incorrect centre or radius from working then •8 is available. However •8 is not available if either centre or radius appear ex nihilo (see note 5).

7. Do not accept 20^2 for •8.

8. For candidates finding the centre by 'stepping out' the following is the minimum evidence for •5 and •6:



Commonly Observed Responses:

Candidate A

using the mid-point of centres: •5 ×

centre $C_3=(3,3)$ •6 $\boxed{✓ 2}$

radius of $C_3=20$ •7 ✓

$(x-3)^2+(y-3)^2=400$ •8 $\boxed{✓ 1}$

Candidate B

$C_1=(-3,-5)$ $\xleftrightarrow{1:3}$ $C_2(9,11)$ $r=20$

$C_3=\frac{1}{4}\begin{pmatrix}0\\-4\end{pmatrix}$ •5 ✓note 4

$C_3=(0,-1)$ •6 $\boxed{✓ 2}$

•7 ✓

$x^2+(y+1)^2=400$ •8 $\boxed{✓ 1}$

Candidate C - touches C_1 internally only

•5 ×

•6 centre $C_3=(3,3)$ ×

•7 radius of C_3 = radius of $C_2=15$ $\boxed{✓ 1}$

•8 $(x-3)^2+(y-3)^2=225$ $\boxed{✓ 1}$

Candidate D - touches C_2 internally only

•5 ×

•6 centre $C_3=(3,3)$ ×

•7 radius of C_3 = radius of $C_1=5$ $\boxed{✓ 1}$

•8 $(x-3)^2+(y-3)^2=25$ $\boxed{✓ 1}$

Candidate E - centre C_3 collinear with C_1, C_2

•5 ×

•6 e.g. centre $C_3=(21,27)$ ×

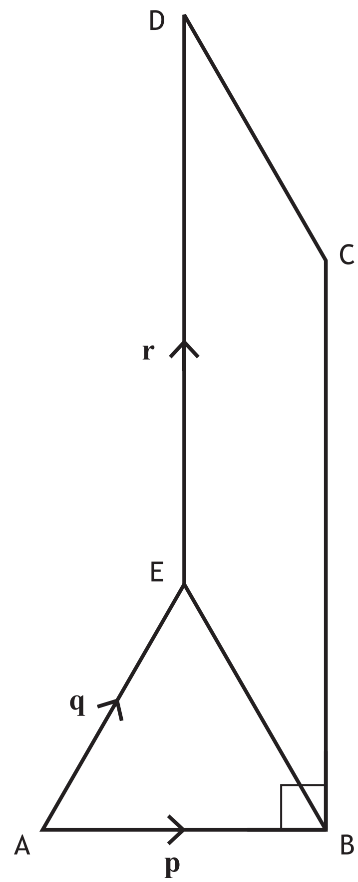
•7 radius of $C_3=45$ (touch C_1 internally only) $\boxed{✓ 1}$

•8 $(x-21)^2+(y-27)^2=2025$ $\boxed{✓ 1}$

MARKS

6. Vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are represented on the diagram as shown.

- BCDE is a parallelogram
- ABE is an equilateral triangle
- $|\mathbf{p}| = 3$
- Angle ABC $= 90°$



(a) Evaluate $\mathbf{p}.(\mathbf{q}+\mathbf{r})$. **3**

(b) Express $\overrightarrow{EC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$. **1**

(c) Given that $\overrightarrow{AE}.\overrightarrow{EC} = 9\sqrt{3}-\frac{9}{2}$, find $|\mathbf{r}|$. **3**

[Turn over

Question	Generic Scheme	Illustrative Scheme	Max Mark
6 (a)			
	$\bullet^1$ Expands	$\bullet^1$ $\mathbf{p.q}+\mathbf{p.r}$	
	$\bullet^2$ Evaluate $\mathbf{p.q}$	$\bullet^2$ $4\frac{1}{2}$	
	$\bullet^3$ Completes evaluation	$\bullet^3$ $...+0=4\frac{1}{2}$	**3**
Notes:			
1. For $\mathbf{p}.(\mathbf{q}+\mathbf{r})=\mathbf{pq}+\mathbf{pr}$ with no other working $\bullet^1$ is not available.			
Commonly Observed Responses:			
6 (b)			
	$\bullet^4$ correct expression	$\bullet^4$ $-\mathbf{q}+\mathbf{p}+\mathbf{r}$ or equivalent	**1**
6 (c)			
	$\bullet^5$ correct substitution	$\bullet^5$ $-\mathbf{q.q}+\mathbf{q.p}+\mathbf{q.r}$	
	$\bullet^6$ start evaluation	$\bullet^6$ $-9+....+3\lvert\mathbf{r}\rvert\cos 30^\circ=9\sqrt{3}-\frac{9}{2}$	
	$\bullet^7$ find expression for $\lvert\mathbf{r}\rvert$	$\bullet^7$ $\lvert\mathbf{r}\rvert=\frac{3\sqrt{3}}{\cos 30}$	**3**
Notes:			
2. Award $\bullet^5$ for $-\mathbf{q}^2+\mathbf{q.p}+\mathbf{q.r}$			
Commonly Observed Responses:			

Candidate A		**Candidate B**	
$-\mathbf{q.q}+\mathbf{q.p}+\mathbf{q.r}=9\sqrt{3}-\frac{9}{2}$	$\bullet^5$ ✓	$-\mathbf{q.q}+\mathbf{q.p}+\mathbf{q.r}=9\sqrt{3}-\frac{9}{2}$	$\bullet^5$ ✓
$-9+\frac{9}{2}+3\lvert\mathbf{r}\rvert\cos 150^\circ=9\sqrt{3}-\frac{9}{2}$	$\bullet^6$ ✗	$-9+\frac{9}{2}+3\lvert\mathbf{r}\rvert\cos 30^\circ=9\sqrt{3}-\frac{9}{2}$	$\bullet^6$ ✓
$\lvert\mathbf{r}\rvert=\frac{3\sqrt{3}}{\cos 150}$	$\bullet^7$ $\boxed{✓1}$	$\lvert\mathbf{r}\rvert=6$	$\bullet^7$ ✓

MARKS

7. (a) Find $\int (3\cos 2x + 1)\,dx$. **2**

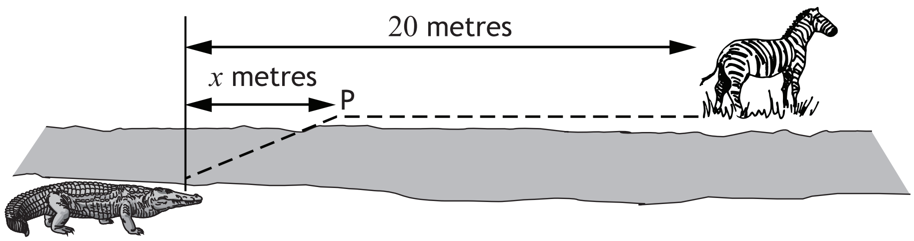
(b) Show that $3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x$. **2**

(c) Hence, or otherwise, find $\int (\sin^2 x - 2\cos^2 x)\,dx$. **2**

8. A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river.

Crocodiles travel at different speeds on land and in water.

The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.



The time taken, T, measured in tenths of a second, is given by

$$T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$$

(a) (i) Calculate the time taken if the crocodile does not travel on land. **1**

(ii) Calculate the time taken if the crocodile swims the shortest distance possible. **1**

(b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time. **8**

Question	Generic Scheme	Illustrative Scheme	Max Mark
7 (a)			
$\bullet^1$ integrate a term $\bullet^2$ complete integration with constant		$\bullet^1$ $\frac{3}{2}\sin 2x$ OR x $\bullet^2$ $x+c$ ¦ $\frac{3}{2}\sin 2x+c$	2

Notes:

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
7 (b)			
$\bullet^3$ substitute for cos 2*x* $\bullet^4$ substitute for 1 and complete		$\bullet^3$ $3\left(\cos^2 x-\sin^2 x\right)\ldots$ or $\ldots(\sin^2 x+\cos^2 x)$ $\bullet^4$ $\ldots\left(\sin^2 x+\cos^2 x\right)=4\cos^2 x-2\sin^2 x$	2

Notes:

1. Any valid substitution for $\cos 2x$ is acceptable for $\bullet^3$.
2. Candidates who show that $4\cos^2 x-2\sin^2 x=3\cos 2x+1$ may gain both marks.
3. Candidates who quote the formula for $\cos 2x$ in terms of A but do not use in the context of the question cannot gain $\bullet^3$.

Commonly Observed Responses:

Candidate A

$3\cos 2x+1=(2\cos^2 x-1)+(2\cos^2 x-1)+(1-2\sin^2 x)+1$ $\bullet^3$ ✓

$=4\cos^2 x-2\sin^2 x$ $\bullet^4$ ✓

Candidate B

$4\cos^2 x-2\sin^2 x=2(\cos 2x+1)-(1-\cos 2x)$ $\bullet^3$ ✓

$=3\cos 2x+1$ $\bullet^4$ ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
7 (c)			
$\bullet^5$ interpret link $\bullet^6$ state result		$\bullet^5$ $-\frac{1}{2}\int\ldots$ $\bullet^6$ $-\frac{3}{4}\sin 2x-\frac{1}{2}x+c$	2

Notes:

Commonly Observed Responses:

Candidate A

$\int \sin^2 x-2\cos^2 x\,dx$

$=\int(3\cos 2x+1)\,dx$ $\bullet^5$ ✗

$\frac{3}{2}\sin 2x+x+c$ $\bullet^6$ ✗

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Question	Generic Scheme	Illustrative Scheme	Max Mark
8 (a) (i)			
	•[1] calculate T when $x = 20$	•[1] 10·4 or 104	1
8 (a) (ii)			
	•[2] calculate T when $x = 0$	•[2] 11 or 110	1

Notes:

1. Accept correct answers with no units.
2. Accept $5\sqrt{436}$ or $10\sqrt{109}$ or equivalent for $T(20)$.
3. For correct substitution alone, with no calculation •[1] and •[2] are not available.
4. For candidates who calculate T when $x = 0$ at •[1] then •[2] is available as follow through for calculating T when $x = 20$ in part(ii).

Commonly Observed Responses:

a)	**(i)**	$10 \cdot 4$	•[1] ✓ **See note 1**
	(ii)	110	•[2] ✓
b)	leading to 9·8 seconds		•[10] × **See note 7**

Question	Generic Scheme	Illustrative Scheme	Max Mark
8 (b)			
	•³ write function in differential form	•³ $5(36+x^2)^{\frac{1}{2}}+\ ...$	
	•⁴ start differentiation of first term	•⁴ $5\times\frac{1}{2}(\ )^{-\frac{1}{2}}\ldots$	
	•⁵ complete differentiation of first term	•⁵ $\ldots\ldots\ldots\ldots\times 2x\ldots\ldots\ldots$	
	•⁶ complete differentiation and set candidate's derivative $=0$	•⁶ $5x(36+x^2)^{-\frac{1}{2}}-4=0$	
	•⁷ start to solve	•⁷ $5x=4(36+x^2)^{\frac{1}{2}}$ or $\dfrac{5x}{(36+x^2)^{\frac{1}{2}}}=4$	
	•⁸ know to square both sides	•⁸ $25x^2=16(36+x^2)$ or $\dfrac{25x^2}{(36+x^2)}=16$	
	•⁹ find value of x	•⁹ $x=8$	
	•¹⁰ calculate minimum time	•¹⁰ T = 9·8 or 98 no units required	8

Notes:

5. Incorrect expansion of $(\ldots)^{\frac{1}{2}}$ at stage •³ only •⁶ and •¹⁰ are available as follow through.

6. Incorrect expansion of $(\ldots)^{-\frac{1}{2}}$ at stage •⁷ only •¹⁰ is available as follow through.

7. Where candidates have omitted units, then •¹⁰ is only available if the implied units are consistent throughout their solution.

8. •¹⁰ is only available as a follow through for a value which is less than the values obtained for the two extremes.

Commonly Observed Responses:

MARKS

9. The blades of a wind turbine are turning at a steady rate.

The height, h metres, of the tip of one of the blades above the ground at time, t seconds, is given by the formula

$$h = 36\sin(1\cdot5t) - 15\cos(1\cdot5t) + 65.$$

Express $36\sin(1\cdot5t) - 15\cos(1\cdot5t)$ in the form

$k\sin(1\cdot5t - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$,

and hence find the **two** values of t for which the tip of this blade is at a height of 100 metres above the ground during the first turn. **8**

[END OF QUESTION PAPER]

Question	Generic Scheme	Illustrative Scheme	Max Mark
9.			
	•¹ use compound angle formula	•¹ $k\sin 1\cdot5t\cos a - k\cos 1\cdot5t\sin a$	
	•² compare coefficients	•² $k\cos a = 36$, $k\sin a = 15$ **stated explicitly**	
	•³ process for k	•³ $k = 39$	
	•⁴ process for a	•⁴ $a = 0{\cdot}39479\ldots\text{rad}$ or $22\cdot6^\circ$	
	•⁵ equates expression for h to 100	•⁵ $39\sin\left(1\cdot5t - 0\cdot39479...\right) + 65 = 100$	
	•⁶ write in standard format and attempt to solve	•⁶ $\sin\left(1\cdot5t - 0\cdot39479...\right) = \frac{35}{39}$ $\Rightarrow 1\cdot5t - 0\cdot39479... = \sin^{-1}\left(\frac{35}{39}\right)$	
	•⁷ solve equation for $1\cdot5t$	•⁷ $1\cdot5t = 1\cdot508$ and $2\cdot422$ (•⁷ for $1\cdot508$, •⁸ for $2\cdot422$)	
	•⁸ process solutions for t	•⁸ $t = 1\cdot006$ and $1\cdot615$	**8**

Notes:

1. Treat $k\sin 1\cdot5t\cos a - \cos 1\cdot5t\sin a$ as bad form only if the equations at the •² stage both contain *k.*
2. $39\sin 1\cdot5t\cos a - 39\cos 1\cdot5t\sin a$ or $39\left(\sin 1\cdot5t\cos a - \cos 1\cdot5t\sin a\right)$ is acceptable for •¹ and •³.
3. Accept $k\cos a = 36$ and $-k\sin a = -15$ for •².
4. •² is not available for $k\cos 1\cdot5t = 36$ and $k\sin 1\cdot5t = 15$, however, •⁴ is still available.
5. •³ is only available for a single value of k, $k > 0$.
6. •⁴ is only available for a single value of a.
7. The angle at •⁴ must be consistent with the equations at •² even when this leads to an angle outwith the required range.
8. Candidates who identify and use any form of the wave equation may gain •¹, •² and •³, however, •⁴ is only available if the value of a is interpreted for the form $k\sin(1\cdot5t - a)$.
9. Candidates who work consistently in degrees cannot gain •⁸.
10. Do not penalise additional solutions at •⁸.
11. On this occasion accept any answers which round to $1\cdot0$ and $1\cdot6$ (2 significant figures required).

MARKS

Attempt ALL questions

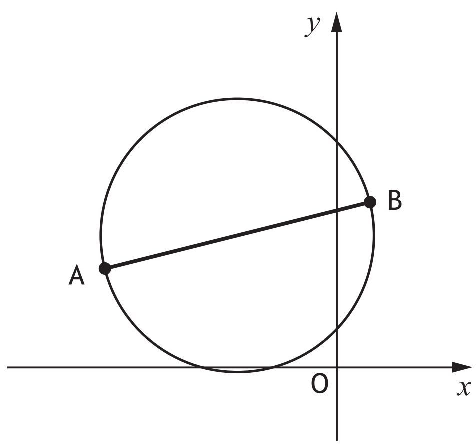
Total marks — 70

1. A curve has equation $y = x^2 - 4x + 7$.

Find the equation of the tangent to this curve at the point where $x = 5$. **4**

2. A and B are the points $(-7, 3)$ and $(1, 5)$.

AB is a diameter of a circle.



Find the equation of this circle. **3**

3. Line l_1 has equation $\sqrt{3}y - x = 0$.

(a) Line l_2 is perpendicular to l_1. Find the gradient of l_2. **2**

(b) Calculate the angle l_2 makes with the positive direction of the x-axis. **2**

Marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	•[1] differentiate •[2] calculate gradient •[3] find the value of y •[4] find equation of tangent	•[1] $2x-4$ •[2] 6 •[3] 12 •[4] $y=6x-18$	**4**
2.	•[1] find the centre •[2] calculate the radius •[3] state equation of circle	•[1] $(-3,4)$ •[2] $\sqrt{17}$ •[3] $(x+3)^2+(y-4)^2=17$ or equivalent	**3**
3. (a)	•[1] find gradient l_1 •[2] state gradient l_2	•[1] $\frac{1}{\sqrt{3}}$ •[2] $-\sqrt{3}$	**2**
3. (b)	•[3] using $m=\tan\theta$ •[4] calculating angle	•[3] $\tan\theta=-\sqrt{3}$ •[4] $\theta=\frac{2\pi}{3}$ or $120°$	**2**
4.	•[1] complete integration •[2] substitute limits •[3] evaluate	•[1] $-\frac{1}{6}x^{-1}$ •[2] $\left(-\frac{1}{6\times 2}\right)-\left(-\frac{1}{6\times 1}\right)$ •[3] $\frac{1}{12}$	**3**

MARKS

4. Evaluate $\int_{1}^{2} \frac{1}{6}x^{-2}\, dx$. **3**

5. The points $\mathrm{A}(0, 9, 7)$, $\mathrm{B}(5, -1, 2)$, $\mathrm{C}(4, 1, 3)$ and $\mathrm{D}(x, -2, 2)$ are such that $\overrightarrow{\mathrm{AB}}$ is perpendicular to $\overrightarrow{\mathrm{CD}}$.

Determine the value of x. **4**

6. Determine the range of values of p such that the equation $x^2 + (p+1)x + 9 = 0$ has no real roots. **4**

7. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact. **5**

4.	$\bullet^1$ complete integration	$\bullet^1$ $-\frac{1}{6}x^{-1}$	**3**
	$\bullet^2$ substitute limits	$\bullet^2$ $\left(-\frac{1}{6\times 2}\right)-\left(-\frac{1}{6\times 1}\right)$	
	$\bullet^3$ evaluate	$\bullet^3$ $\frac{1}{12}$	

Question	Generic scheme	Illustrative scheme	Max mark
5.			4
	●1 find $\overrightarrow{CD}$	●1 $\begin{pmatrix} x-4 \\ -3 \\ -1 \end{pmatrix}$	
	●2 find $\overrightarrow{AB}$	●2 $\begin{pmatrix} 5 \\ -10 \\ -5 \end{pmatrix}$	
	●3 equate scalar product to zero	●3 $5(x-4)+(-10)(-3)+(-5)(-1)=0$	
	●4 calculate value of x	●4 $x=-3$	
6.			4
	●1 substitute into discriminant	●1 $(p+1)^2-4\times1\times9$	
	●2 apply condition for no real roots	●2 $\ldots<0$	
	●3 determine zeroes of quadratic expression	●3 $-7, 5$	
	●4 state range with justification	●4 $-7<p<5$ with eg sketch or table of signs	
7.			5
	●1 substitute for y in equation of circle	●1 $x^2+(3x-5)^2+2x-4(3x-5)-5=0$	
	●2 express in standard quadratic form	●2 $10x^2-40x+40=0$	
	●3 demonstrate tangency	●3 $10(x-2)^2=0$ only one solution implies tangency	
	●4 find x-coordinate	●4 $x=2$	
	●5 find y-coordinate	●5 $y=1$	

MARKS

8. For the polynomial, $x^3 - 4x^2 + ax + b$

- $x - 1$ is a factor
- -12 is the remainder when it is divided by $x - 2$

(a) Determine the values of a and b. **5**

(b) Hence solve $x^3 - 4x^2 + ax + b = 0$. **3**

9. A sequence is generated by the recurrence relation $u_{n+1} = m u_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. **2**

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(ii) Calculate this limit. **2**

10. (a) Evaluate $\log_5 25$. **1**

(b) Hence solve $\log_4 x + \log_4 (x - 6) = \log_5 25$, where $x > 6$. **5**

11. Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$. **3**

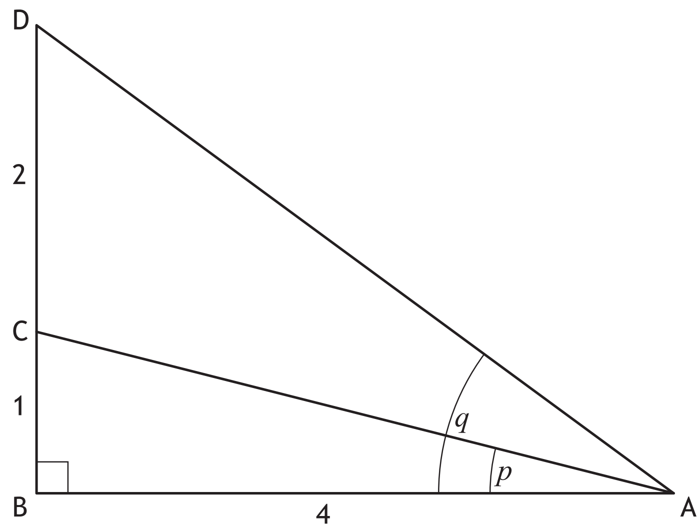
Question	Generic scheme	Illustrative scheme	Max mark
8. (a)	•[1] use appropriate strategy •[2] obtain an expression for a and b •[3] obtain a second expression for a and b •[4] find the value of a or b •[5] find the second value	•[1] $(1)^3 - 4(1)^2 + a(1) + b = 0$ •[2] $a + b = 3$ •[3] $2a + b = -4$ •[4] $a = -7$ or $b = 10$ •[5] $b = 10$ or $a = -7$	**5**
8. (b)	•[6] obtain quadratic factor •[7] complete factorisation •[8] state solutions	•[6] $\left(x^2 - 3x - 10\right)$ •[7] $(x-1)(x-5)(x+2)$ •[8] $x = 1$, $x = 5$, $x = -2$	**3**
9. (a)	•[1] interpret information •[2] solve to find m	•[1] $13 = 28m + 6$ •[2] $m = \frac{1}{4}$	**2**
9. (b) (i)	•[3] state condition	•[3] a limit exists as $-1 < \frac{1}{4} < 1$	**1**
9. (b) (ii)	•[4] know how to calculate limit •[5] calculate limit	•[4] $L = \frac{1}{4}L + 6$ •[5] $L = 8$	**2**

Back to Table

10. (a)	$\bullet^1$ state value	$\bullet^1$ 2	**1**
10. (b)	$\bullet^1$ use laws of logarithms $\bullet^2$ link to part (a) $\bullet^3$ use laws of logarithms $\bullet^4$ write in standard quadratic form $\bullet^5$ solve for x and identify appropriate solution	$\bullet^1$ $\log_4 x(x-6)$ $\bullet^2$ $\log_4 x(x-6)=2$ $\bullet^3$ $x(x-6)=4^2$ $\bullet^4$ $x^2-6x-16=0$ $\bullet^5$ 8	**5**
11.	$\bullet^1$ start to differentiate $\bullet^2$ complete differentiation $\bullet^3$ evaluate derivative	$\bullet^1$ $3\times 4\sin^2 x\ldots$ $\bullet^2$ $\ldots\times\cos x$ $\bullet^3$ $\dfrac{-3\sqrt{3}}{2}$	**3**

MARKS

12. Triangle ABD is right-angled at B with angles BAC $= p$ and BAD $= q$ and lengths as shown in the diagram below.



Show that the exact value of $\cos(q-p)$ is $\frac{19\sqrt{17}}{85}$. **5**

13. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. **4**

14. (a) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. **5**

(b) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$. **2**

12.			**5**
	$\bullet^1$ calculate lengths AC and AD	$\bullet^1$ $AC = \sqrt{17}$ and $AD = 5$ stated or implied by $\bullet^3$	
	$\bullet^2$ select appropriate formula and express in terms of p and q	$\bullet^2$ $\cos q \cos p + \sin q \sin p$ stated or implied by $\bullet^4$	
	$\bullet^3$ calculate two of $\cos p,\ \cos q,\ \sin p,\ \sin q$	$\bullet^3$ $\cos p = \frac{4}{\sqrt{17}}$, $\cos q = \frac{4}{5}$ $\sin p = \frac{1}{\sqrt{17}}$, $\sin q = \frac{3}{5}$	
	$\bullet^4$ calculate other two and substitute into formula	$\bullet^4$ $\frac{4}{5}\times\frac{4}{\sqrt{17}}+\frac{3}{5}\times\frac{1}{\sqrt{17}}$	
	$\bullet^5$ arrange into required form	$\bullet^5$ $\frac{19}{5\sqrt{17}}\times\frac{\sqrt{17}}{\sqrt{17}}=\frac{19\sqrt{17}}{85}$ or $\frac{19}{5\sqrt{17}}=\frac{19\sqrt{17}}{5\times 17}=\frac{19\sqrt{17}}{85}$	

Question	Generic scheme	Illustrative scheme	Max mark
13.	●[1] know to and start to integrate ●[2] complete integration ●[3] substitute for x and y ●[4] state expression for y	●[1] eg $y=\frac{4}{2}x^2...$ ●[2] $y=\frac{4}{2}x^2-\frac{6}{3}x^3+c$ ●[3] $9=2(-1)^2-2(-1)^3+c$ ●[4] $y=2x^2-2x^3+5$	4
14. (a)	●[1] use double angle formula ●[2] express as a quadratic in $\cos x°$ ●[3] start to solve ●[4] reduce to equations in $\cos x°$ only ●[5] process solutions in given domain	**Method 1:** Using factorisation ●[1] $2\cos^2 x°-1...$ **stated or implied by ●[2]** ●[2] $2\cos^2 x°-3\cos x°+1=0$ ●[3] $(2\cos x°-1)(\cos x°-1)$ $=0$ must appear at either of these lines to gain ●[2] **Method 2:** Using quadratic formula ●[1] $2\cos^2 x°-1...$ **stated or implied by ●[2]** ●[2] $2\cos^2 x°-3\cos x°+1=0$ **stated explicitly** ●[3] $\frac{-(-3)\pm\sqrt{(-3)^2-4\times2\times1}}{2\times2}$ **In both methods:** ●[4] $\cos x°=\frac{1}{2}$ and $\cos x°=1$ ●[5] 0, 60, 300 Candidates who include 360 lose ●[5]. **or** ●[4] $\cos x=1$ and $x=0$ ●[5] $\cos x°=\frac{1}{2}$ and $x=60$ or 300 Candidates who include 360 lose ●[5].	5
14. (b)	●[6] interpret relationship with (a) ●[7] state valid values	●[6] $2x=0$ and 60 and 300 ●[7] 0, 30, 150, 180, 210 and 330	2

MARKS

15. Functions f and g are defined on suitable domains by $f(x) = x^3 - 1$ and $g(x) = 3x + 1$.

(a) Find an expression for $k(x)$, where $k(x) = g(f(x))$. **2**

(b) If $h(k(x)) = x$, find an expression for $h(x)$. **3**

[END OF SPECIMEN QUESTION PAPER]

Question	Generic scheme	Illustrative scheme	Max mark
15. (a)	•[1] interpret notation •[2] complete process	•[1] $g\left(x^3-1\right)$ •[2] $3x^3-2$	**2**
15. (b)	•[3] start to rearrange for x •[4] rearrange •[5] state expression for $h(x)$	•[3] $3x^3=y+2$ •[4] $x=\sqrt[3]{\dfrac{y+2}{3}}$ •[5] $h(x)=\sqrt[3]{\dfrac{x+2}{3}}$	**3**

[END OF SPECIMEN MARKING INSTRUCTIONS]

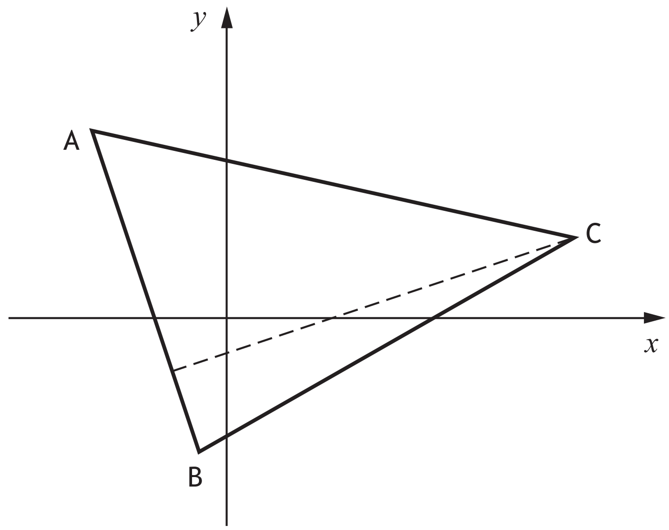
MARKS

Attempt ALL questions

Total marks — 80

1. The vertices of triangle ABC are A(−5, 7), B(−1, −5) and C(13, 3) as shown in the diagram.

The broken line represents the altitude from C.



(a) Find the equation of the altitude from C. **3**

(b) Find the equation of the median from B. **3**

(c) Find the coordinates of the point of intersection of the altitude from C and the median from B. **2**

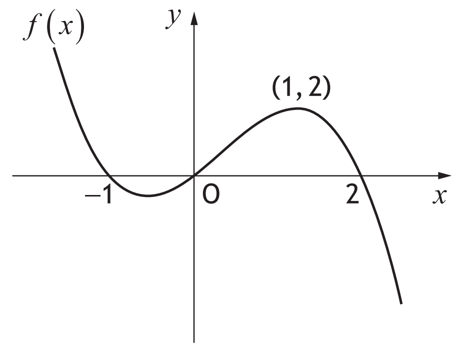
2. Find $\int \frac{4x^3+1}{x^2}\,dx,\ x \neq 0$. **4**

1. (a)	•[1] calculate gradient of AB •[2] use property of perpendicular lines •[3] determine equation of altitude	•[1] $m_{AB} = -3$ •[2] $m_{alt} = \frac{1}{3}$ •[3] $x - 3y = 4$	**3**
1. (b)	•[4] calculate midpoint of AC •[5] calculate gradient of median •[6] determine equation of median	•[4] $(4,5)$ •[5] $m_{BM} = 2$ •[6] $y = 2x - 3$	**3**
1. (c)	•[7] find x or y coordinate •[8] find remaining coordinate	•[7] $x = 1$ or $y = -1$ •[8] $y = -1$ or $x = 1$	**2**
2.	•[1] write in integrable form •[2] integrate one term •[3] integrate other term •[4] complete integration and simplify	•[1] $4x + x^{-2}$ •[2] eg $\frac{4}{2}x^2 + \ldots$ •[3] $\ldots\frac{x^{-1}}{-1}$ •[4] $2x^2 - x^{-1} + c$	**4**

MARKS

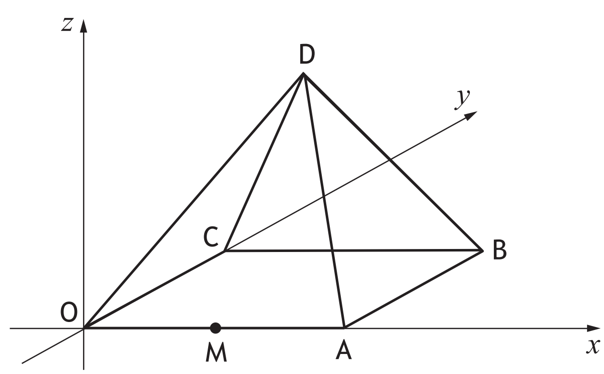
3. The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$.

The curve passes through $(-1, 0)$, $(0, 0)$, $(1, 2)$ and $(2, 0)$.



Find the values of a, b and k. **3**

4. D,OABC is a square-based pyramid as shown.



- O is the origin and OA = 4 units.
- M is the mid-point of OA.
- $\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

(a) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

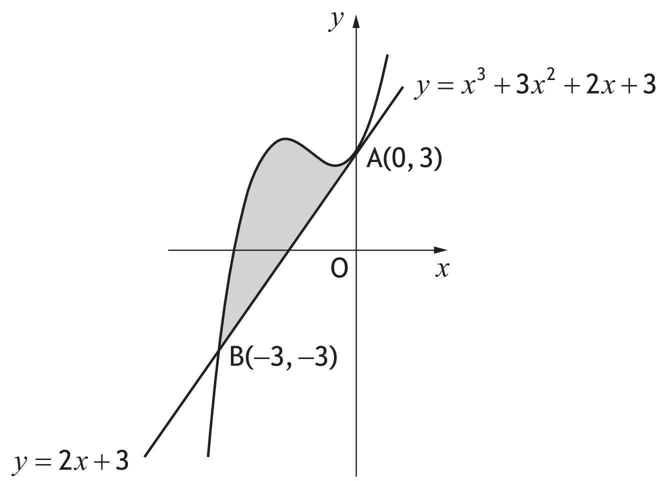
(b) Find the size of angle BDM. **5**

3.	$\bullet^1$ value of a $\bullet^2$ value of b $\bullet^3$ calculate k	$\bullet^1$ 1 $\bullet^2$ -2 $\bullet^3$ -1	**3**

Question	Generic scheme	Illustrative scheme	Max mark
4. (a)	•[1] state components of $\overrightarrow{DB}$	•[1] $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$	3
	•[2] state coordinates of M	•[2] $(2,0,0)$ stated or implied by •[3]	
	•[3] state components of $\overrightarrow{DM}$	•[3] $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$	
4. (b)	•[4] evaluate $\overrightarrow{DB}.\overrightarrow{DM}$	•[4] 32	5
	•[5] evaluate $\lvert\overrightarrow{DB}\rvert$	•[5] $\sqrt{44}$	
	•[6] evaluate $\lvert\overrightarrow{DM}\rvert$	•[6] $\sqrt{40}$	
	•[7] use scalar product	•[7] $\cos BDM = \dfrac{32}{\sqrt{44}\sqrt{40}}$	
	•[8] calculate angle	•[8] $40 \cdot 3°$ or 0.703 rads	

MARKS

5. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown.



The line meets the curve again at B(–3, –3).

Find the area enclosed by the line and the curve. **5**

6. (a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$. **3**

(b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$. **2**

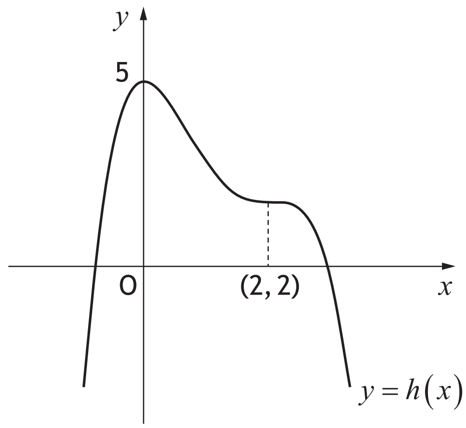
(c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x. **2**

Question	Generic scheme	Illustrative scheme	Max mark
5.	●[1] know to integrate and interpret limits ●[2] use 'upper — lower' ●[3] integrate ●[4] substitute limits ●[5] evaluate area	●[1] $\int_{-3}^{0} \ldots dx$ ●[2] $\int_{-3}^{0} \left(x^3+3x^2+2x+3\right)-\left(2x+3\right)dx$ ●[3] $\frac{1}{4}x^4+x^3$ ●[4] $0-\left(\frac{1}{4}(-3)^4+(-3)^3\right)$ ●[5] $\frac{27}{4}$ units^2	**5**

Question		Generic scheme	Illustrative scheme	Max mark
6.	(a)	**Method 1**	**Method 1**	3
		•[1] identify common factor	•[1] $3(x^2+8x......$ stated or implied by •[2]	
		•[2] complete the square	•[2] $3(x+4)^2......$	
		•[3] process for c and write in required form	•[3] $3(x+4)^2+2$	
		Method 2	**Method 2**	3
		•[1] expand completed square form	•[1] $ax^2+2abx+ab^2+c$	
		•[2] equate coefficients	•[2] $a=3,\ \ 2ab=24,\ \ ab^2+c=50$	
		•[3] process for b and c and write in required form	•[3] $3(x+4)^2+2$	
6.	(b)	•[4] differentiate two terms	•[4] $3x^2+24x....$	2
		•[5] complete differentiation	•[5] $....+50$	
6.	(c)	**Method 1**	**Method 1**	2
		•[6] link with (a) and identify sign of $(x+4)^2$	•[6] $f'(x)=3(x+4)^2+2$ and $(x+4)^2\geq 0\ \forall\ x$	
		•[7] communicate reason	•[7] $\therefore 3(x+4)^2+2>0\ \Rightarrow$ always strictly increasing	
		Method 2	**Method 2**	2
		•[6] identify minimum value of $f'(x)$	•[6] eg minimum value $=2$ or annotated sketch	
		•[7] communicate reason	•[7] $2>0\ \therefore\left(f'(x)>0\right)\Rightarrow$ always strictly increasing	

MARKS

7. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at (0, 5) and (2, 2).



On separate diagrams sketch the graphs of:

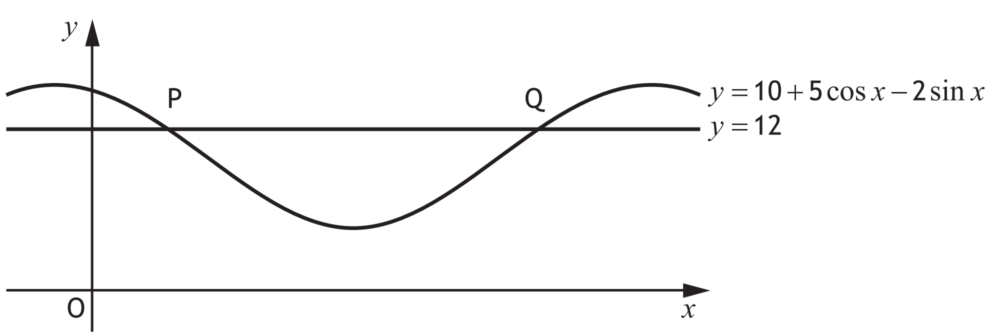
(a) $y = 2 - h(x)$. **2**

(b) $y = h'(x)$. **3**

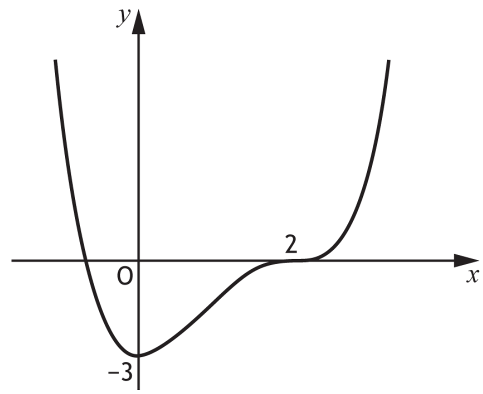
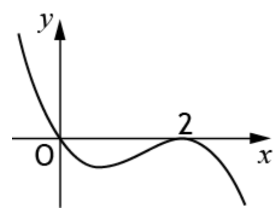
8. (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x + a)$, where $k > 0$ and $0 < a < 2\pi$. **4**

(b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.



Find the x-coordinates of P and Q. **4**

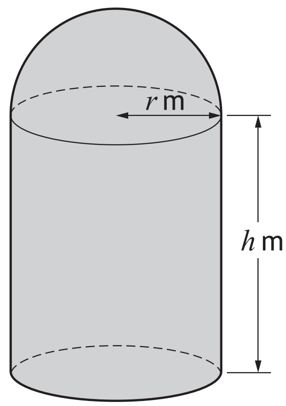
Question	Generic scheme	Illustrative scheme	Max mark
7. (a)	•[1] evidence of reflecting in x-axis •[2] vertical translation of 2 units identifiable from graph	•[1] reflection of graph in x-axis •[2] graph moves parallel to y-axis by 2 units upwards 	**2**
7. (b)	•[3] identify roots •[4] interpret point of inflexion •[5] complete cubic curve	•[3] 0 **and** 2 only •[4] turning point at $(2,0)$ •[5] cubic passing through origin with negative gradient 	**3**

Question	Generic scheme	Illustrative scheme	Max mark
8. (a)	•[1] use compound angle formula	•[1] $k\cos x\cos a - k\sin x\sin a$ **stated explicitly**	**4**
	•[2] compare coefficients	•[2] $k\cos a = 5, k\sin a = 2$ **stated explicitly**	
	•[3] process for k	•[3] $k = \sqrt{29}$	
	•[4] process for a and express in required form	•[4] $\sqrt{29}\cos(x + 0\cdot38)$	
8. (b)	•[5] equate to 12 and simplify constant terms	•[5] $5\cos x - 2\sin x = 2$ or $5\cos x - 2\sin x - 2 = 0$	**4**
	•[6] use result of part (a) and rearrange	•[6] $\cos(x + 0\cdot3805\ldots) = \dfrac{2}{\sqrt{29}}$	
	•[7] solve for $x + a$	•[7] $1\cdot1902\ldots,$ | $5\cdot0928\ldots$	
	•[8] solve for x	•[8] $0\cdot8097\ldots,$ | $4\cdot712\ldots$	

MARKS

9. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.



(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$ 3

(b) Determine the value of r which minimises the amount of metal needed to build the container. 6

10. Given that

$$\int_{\frac{\pi}{8}}^{a} \sin\left(4x - \frac{\pi}{2}\right) dx = \frac{1}{2}, \quad 0 \leq a < \frac{\pi}{2},$$

calculate the value of a. 6

Question	Generic scheme	Illustrative scheme	Max mark
9. (a)	•[1] equate volume to 100 •[2] obtain an expression for h •[3] demonstrate result	•[1] $V=\pi r^2 h=100$ •[2] $h=\dfrac{100}{\pi r^2}$ •[3] $A=\pi r^2+2\pi r^2+2\pi r\times\dfrac{100}{\pi r^2}$ leading to $A=\dfrac{200}{r}+3\pi r^2$	**3**
9. (b)	•[4] start to differentiate •[5] complete differentiation •[6] set derivative to zero •[7] obtain r •[8] verify nature of stationary point •[9] interpret and communicate result	•[4] $A'(r)=6\pi r\ldots$ •[5] $A'(r)=6\pi r-\dfrac{200}{r^2}$ •[6] $6\pi r-\dfrac{200}{r^2}=0$ •[7] $r=\sqrt[3]{\dfrac{100}{3\pi}}\ (\approx 2\cdot 20)$ metres •[8] table of signs for a derivative when $r=2\cdot 1974\ldots$ •[9] minimum when $r\approx 2\cdot 20$ (m) or •[8] $A''(r)=6\pi+\dfrac{400}{r^3}$ •[9] $A''(2\cdot 1974\ldots)>0$ $\therefore$ minimum when $r\approx 2\cdot 20$ (m)	**6**

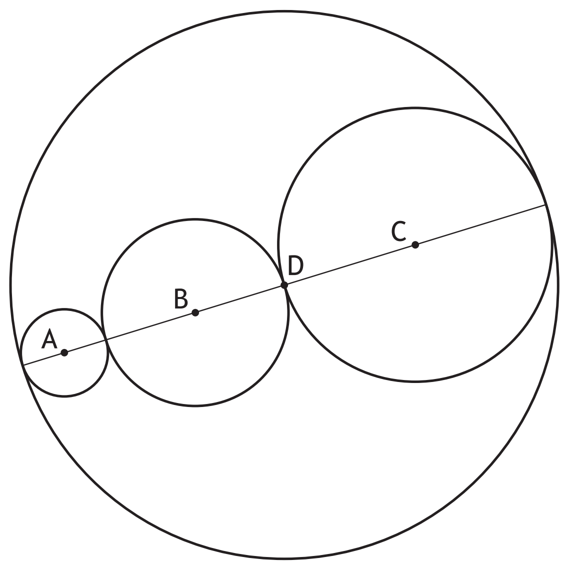
10.	$\bullet^1$ start to integrate	$\bullet^1$ $-\frac{1}{4}\cos\ldots$	**6**
	$\bullet^2$ complete integration	$\bullet^2$ $-\frac{1}{4}\cos\left(4x-\frac{\pi}{2}\right)$	
	$\bullet^3$ process limits	$\bullet^3$ $-\frac{1}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{1}{4}\cos\left(\frac{4\pi}{8}-\frac{\pi}{2}\right)$	
	$\bullet^4$ simplify numeric term and equate to $\frac{1}{2}$	$\bullet^4$ $-\frac{1}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{1}{4}=\frac{1}{2}$	
	$\bullet^5$ start to solve equation	$\bullet^5$ $\cos\left(4a-\frac{\pi}{2}\right)=-1$	
	$\bullet^6$ solve for a	$\bullet^6$ $a=\frac{3\pi}{8}$	

MARKS

11. Show that $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \dfrac{\pi}{2}$. **3**

12. (a) Show that the points A(–7, –2), B(2, 1) and C(17, 6) are collinear. **3**

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.



The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D. **4**

11.	**Method 1**	**Method 1**	**3**
	•1 substitute for $\sin 2x$	•1 $\dfrac{2\sin x\cos x}{2\cos x} - \sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above	
	•2 simplify and factorise	•2 $\sin x\left(1-\cos^2 x\right)$	
	•3 substitute for $1-\cos^2 x$ and simplify	•3 $\sin x \times \sin^2 x$ leading to $\sin^3 x$	
	Method 2	**Method 2**	**3**
	•1 substitute for $\sin 2x$	•1 $\dfrac{2\sin x\cos x}{2\cos x} - \sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above	
	•2 simplify and substitute for $\cos^2 x$	•2 $\sin x - \sin x\left(1-\sin^2 x\right)$	
	•3 expand and simplify	•3 $\sin x - \sin x + \sin^3 x$ leading to $\sin^3 x$	

Question	Generic scheme	Illustrative scheme	Max mark
12. (a)	**Method 1**	**Method 1**	**3**
	•1 calculate m_{AB}	•1 eg $m_{AB}=\frac{3}{9}=\frac{1}{3}$	
	•2 calculate m_{BC}	•2 eg $m_{BC}=\frac{5}{15}=\frac{1}{3}$	
	•3 interpret result and state conclusion	•3 $\ldots\Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	
	Method 2	**Method 2**	**3**
	•1 calculate an appropriate vector, eg $\overrightarrow{AB}$	•1 eg $\overrightarrow{AB}=\begin{pmatrix}9\\3\end{pmatrix}$	
	•2 calculate a second vector, eg $\overrightarrow{BC}$ and compare	•2 eg $\overrightarrow{BC}=\begin{pmatrix}15\\5\end{pmatrix}\therefore\ \overrightarrow{AB}=\frac{3}{5}\overrightarrow{BC}$	
	•3 interpret result and state conclusion	•3 $\ldots\Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	
	Method 3	**Method 3**	**3**
	•1 calculate m_{AB}	•1 $m_{AB}=\frac{3}{9}=\frac{1}{3}$	
	•2 find equation of line and substitute point	•2 eg, $y-1=\frac{1}{3}(x-2)$ leading to $6-1=\frac{1}{3}(17-2)$	
	•3 communication	•3 since C lies on line A, B and C are collinear	
12. (b)	•4 find radius	•4 $6\sqrt{10}$	**4**
	•5 determine an appropriate ratio	•5 eg $2:3$ or $\frac{2}{5}$ (using B and C) or $3:5$ or $\frac{8}{5}$ (using A and C)	
	•6 find centre	•6 $(8,3)$	
	•7 state equation of circle	•7 $(x-8)^2+(y-3)^2=360$	

MARKS

13. The concentration of a pesticide in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

(a) It takes 25 days for the concentration of the pesticide to be reduced to one half of its initial concentration.

Calculate the value of k. **4**

(b) Eighty days after the initial application, what is the percentage decrease in concentration of the pesticide? **3**

[END OF SPECIMEN QUESTION PAPER]

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Question	Generic scheme	Illustrative scheme	Max mark
13. (a)	•[1] interpret half-life •[2] process equation •[3] write in logarithmic form •[4] process for k	•[1] $\frac{1}{2}P_0 = P_0 e^{-25k}$ **stated or implied by •[2]** •[2] $e^{-25k} = \frac{1}{2}$ •[3] $\log_e \frac{1}{2} = -25k$ •[4] $k \approx 0\cdot028$	**4**
13. (b)	•[5] interpret equation •[6] process •[7] state percentage decrease	•[5] $P_t = P_0 e^{-80\times0\cdot028}$ •[6] $P_t \approx 0\cdot1065P_0$ •[7] 89%	**3**

[END OF SPECIMEN MARKING INSTRUCTIONS]