

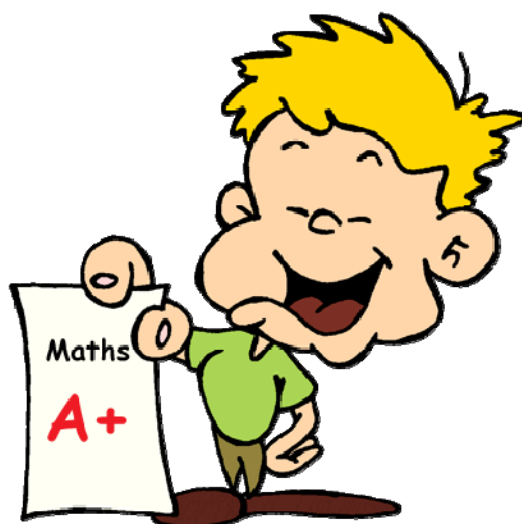
# **St Andrew's Academy**

## **CfE Senior Phase**



## **HIGHER MATHEMATICS**

### **Course Summary & Self Assessment Booklet**



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

This booklet lists the skills required for each topic covered in the Higher Maths course.

When you complete a topic in class you should tick the column with the faces to indicate how well you understand the work.

😊 means I have a good understanding of this work.

😐 means I can do this work but I am not confident with it.

😞 means that I have struggled with this work.




The school website lists a brief summary of each topic and a list of websites that can be used for revision and practice.




At the end of each topic you will complete a formal homework exercise. This exercise assesses your understanding of the topic. You must give maximum effort to your homework and hand it in to your teacher on time.




A scientific calculator is an essential tool for the course. It is recommended that you buy a scientific calculator so that you can use it for your class work and homework. A mobile phone is not acceptable for use as a calculator.

## Straight Line

I CAN	😊	😐	☹️
find the distance between 2 points using the distance formula. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$			
find a mid-point. $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$			
find a gradient between 2 given points. $m = \frac{y_2 - y_1}{x_2 - x_1}$			
find a gradient when given the angle that the line makes with the positive direction of the x-axis. $m = \tan\theta$			
find the gradient of a line when given the gradient, or equation, of a parallel line. $m_1 = m_2$			
find the gradient of a line when given the gradient, or equation, of a perpendicular line. $m_1 \times m_2 = -1$			
find the equation of a line when given the gradient and y-intercept. $y = mx + c$			
find the equation of a line when given the gradient and a point on the line. $y - b = m(x - a)$			
find the point of intersection between lines.			
find the equation of a median.			
find the equation of an altitude.			
find the equation of a perpendicular bisector.			
solve problems involving any of the above.			

Quadratics			
I CAN			
complete the square for a quadratic expression.			
state the max/min value of a function completed square form.			
sketch the graph of a quadratic function.			
solve a quadratic equation.			
use the discriminant to determine the nature of the roots.			
use the discriminant to prove tangency.			
solve quadratic inequalities.			
solve problems involving any of the above.			

Polynomials			
I CAN			
use synthetic division to factorise a polynomial.			
use the remainder and factor theorems.			
find the roots of polynomials.			
use iteration to find approximate roots of polynomials.			
solve problems involving any of the above.			

Functions			
I CAN			
understand function terminology and notation.			
find a composite function e.g. $f(g(x))$ .			
find the equation of an inverse function, $f^{-1}(x)$ .			
solve problems involving any of the above.			

## Differentiation 1

I CAN	😊	😐	☹️
differentiate a function of the form $f(x) = ax^n$ .			
differentiate function with negative and fractional powers.			
use differentiation to find the gradient of a tangent to a function.			
find the equation of a tangent to a function.			
find an expression for velocity ( $v = \frac{dx}{dt}$ ) when given the displacement, $x$ .			
find an expression for acceleration ( $a = \frac{dv}{dt}$ ) when given the velocity, $v$ .			
use the Chain Rule to differentiate a function of the form $f(x) = (ax + b)^n$ .			
solve problems involving any of the above.			

## Trigonometry – Radians & Equations

I CAN	😊	😐	☹️
state the exact values of sin/cos/tan for angles: $0^\circ$ , $30^\circ$ , $45^\circ$ , $60^\circ$ & $90^\circ$ .			
convert between degrees and radians.			
solve simple linear trigonometric equations.			
solve trigonometric equations with multiple angles.			
solve trigonometric equations involving squared functions.			
solve quadratic trigonometric equations.			
solve trigonometric equations with phase angles			
use trigonometric identities to simplify and solve trigonometric equations.			
solve problems involving any of the above.			

## Vectors

I CAN	😊	😐	😞
use vectors to prove collinearity of points.			
write column vectors in terms of the unit vectors $i, j$ and $k$ .			
use vectors to divide a line in a given ratio.			
use the scalar product to show that vectors are perpendicular.			
use the scalar product to find the angle between vectors.			
solve problems involving any of the above.			

## Differentiation 2

I CAN	😊	😐	😞
sketch the graph of a derivative.			
find stationary points and their nature.			
identify when a function is increasing/decreasing.			
use the stationary points and points to intersection of the axes to sketch the graph of a curve.			
find the maximum/minimum values in a closed interval.			
solve optimization problems.			

## Trigonometry – Addition & Double Angle Formulae

I CAN	😊	😐	😞
use the addition formulae to solve problems.			
use the additional formulae to find the related angle formulae.			
use the double angle formulae to solve problems.			
use the addition & double angle formulae to simplify and solve trigonometric equations.			

## Exponentials & Logarithms

I CAN	😊	😐	☹️
convert between logarithmic and exponential forms. $(y = a^x \leftrightarrow x = \log_a y)$			
use the rules of logs. ( $\log_a 1 = 0$ , $\log_a a = 1$ , $\log_a x + \log_a y = \log_a xy$ , $\log_a x - \log_a y = \log_a \frac{x}{y}$ , $\log_a x^n = n \log_a x$ )			
solve logarithmic equations.			
solve exponential problems.			
use the graph of $\log_a y$ against $x$ to find the equation connecting $x$ and $y$ in the form $y = ab^x$ .			
use the graph of $\log_a y$ against $\log_a x$ to find the equation connecting $x$ and $y$ in the form $y = ax^n$ .			

## Integration 1

I CAN	😊	😐	☹️
integrate a function of the form $f(x) = ax^n$ .			
integrate a function of the form $f(x) = (ax + b)^n$ .			
solve differential equations to find a general or particular solution.			






## Circles

I CAN	😊	😐	😞
find the equation of a circle with centre O and radius $r$ . $x^2 + y^2 = r^2$			
find the equation of a circle with centre $(a,b)$ and radius $r$ . $(x-a)^2 + (y-b)^2 = r^2$			
find the equation of a circle in general form. $(x^2 + y^2 + 2gx + 2fy + c = 0)$			
find the centre and radius of a circle in any form.			
find the equation of a tangent to a circle.			
use the discriminant to determine the number of points of contact between a line and circle.			
find the point/points of intersection between a line and a circle.			




## Trigonometry – Wave Function

I CAN	😊	😐	😞
convert expression of the form $a\cos x + b\sin x$ into the form $k \cos(x \pm \alpha)$ or $k \sin(x \pm \alpha)$			
identify the maximum/minimum value of a function in the form $k \cos(x \pm \alpha)$ or $k \sin(x \pm \alpha)$			
use the wave function to solve trigonometric equations in degrees or radians.			

## Integration 2

I CAN			
integrate functions of the form $a \sin(bx + c)$ or $a \cos(bx + c)$			
evaluate a definite integral.			
find the area between a curve and the x-axis.			
find the area between a line and a curve or 2 curves.			

## Recurrence Relations

I CAN			
explain the difference between arithmetic and geometric sequences.			
construct a recurrence relation when given relevant information.			
find the limit, where it exists, for a recurrence relation.			
solve problems involving any of the above.			