FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$

represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$

represents a circle centre (a, b) and radius r .

Scalar Product: $a.b = |a||b|\cos\theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\sin 2A = 2\sin A\cos A$$
$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Algebra Unit Practice

1.(a)(i) Simplify $\log_5 7a + \log_5 2b$

(ii) Simplify $\log_6 4b + \log_6 3c$

(iii) Simplify $\log_4 9d + \log_4 5a$

(iv) Simplify $\log_8 7y + \log_8 3s$ [1]

1.(b)(i) Express $\log_a x^3 - \log_a x^2$ in the form $k \log_a x$

(ii) Express $\log_a x^5 - \log_a x^2$ in the form $k \log_a x$

(iii) Express $\log_a x^3 - \log_a x$ in the form $k \log_a x$

(iv) Express $\log_a x^6 - \log_a x^5$ in the form $k \log_a x$ [2]

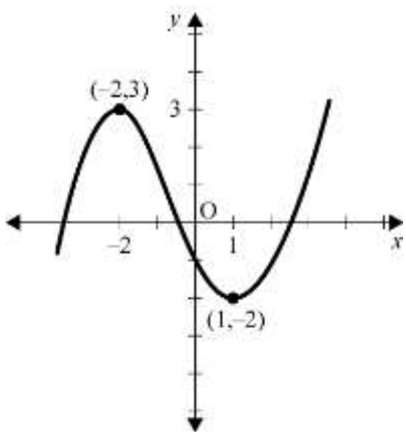
2. (a) Solve $\log_2(x - 5) = 5$

(b) Solve $\log_5(y + 2) = 2$

(c) Solve $\log_3(z - 1) = 3$

(d) Solve $\log_3(d + 2) = 2$

3. The diagram shows the graph of $y = f(x)$ with a maximum turning point at $(-2, 3)$ and a minimum turning point at $(1, -2)$.



(a) Sketch the graph of $y = f(x + 3) - 2$

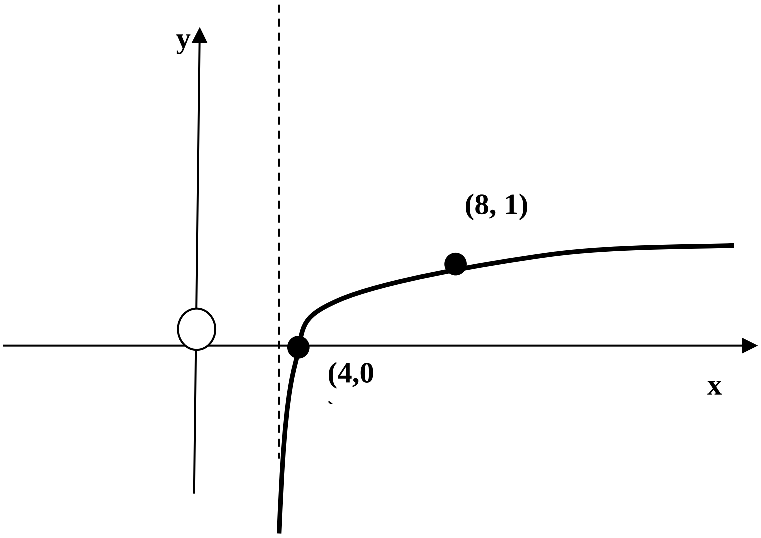
(b) Sketch the graph of $y = f(x + 4) - 3$

(c) Sketch the graph of $y = f(x - 2) + 3$

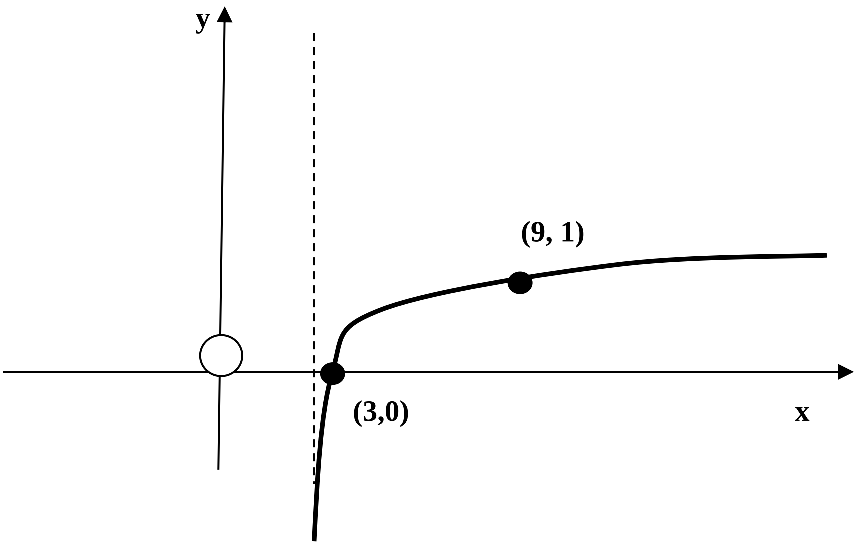
(d) Sketch the graph of $y = f(x - 3) - 6$

[[3]

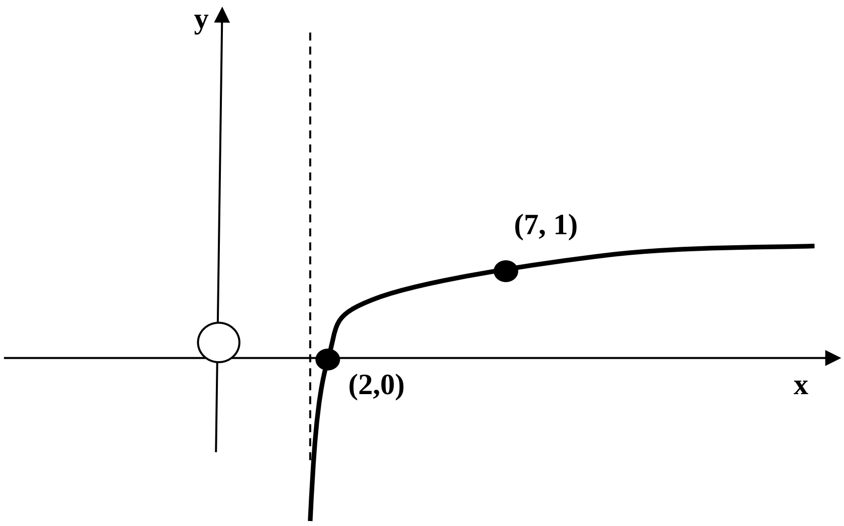
4. **(a)** The diagram shows the graph of $y = \log_b(x - a)$
Determine the values of *a* and *b*



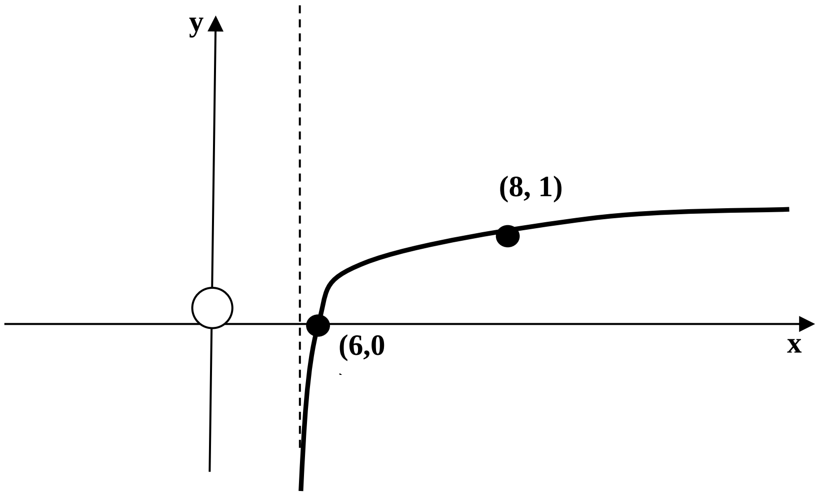
4.(b) The diagram shows the graph of $y = \log_b(x - a)$
Determine the values of *a* and *b*



4.(c) The diagram shows the graph of $y = \log_b(x - a)$
Determine the values of *a* and *b*



4.(d) The diagram shows the graph of $y = \log_b(x - a)$
Determine the values of *a* and *b*



[2]

5.(a) The functions f and g defined on suitable domains, are given by

$f(x) = 2x + 5$ and $g(x) = \sqrt{x}$.

A third function $h(x)$ is defined as $h(x) = g(f(x))$.

(i) Find an expression for $h(x)$.

(ii) Explain why the largest domain for $h(x)$ is given by $x \geq -2{\cdot}5$.

(b) The functions f and g defined on suitable domains, are given by

$f(x) = 3x + 6$ and $g(x) = \sqrt{x}$.

A third function $h(x)$ is defined as $h(x) = g(f(x))$.

(i) Find an expression for $h(x)$.

(ii) Explain why the largest domain for $h(x)$ is given by $x \geq -2$

(c) The functions f and g defined on suitable domains, are given by

$f(x) = 4x + 10$ and $g(x) = \sqrt{x}$.

A third function $h(x)$ is defined as $h(x) = g(f(x))$.

(i) Find an expression for $h(x)$.

(ii) Explain why the largest domain for $h(x)$ is given by $x \geq -2{\cdot}5$

(d) The functions f and g defined on suitable domains, are given by

$f(x) = 2x + 7$ and $g(x) = \sqrt{x}$.

A third function $h(x)$ is defined as $h(x) = g(f(x))$.

(i) Find an expression for $h(x)$.

(ii) Explain why the largest domain for $h(x)$ is given by $x \geq -3{\cdot}5$

[2,#2.2]

6. **(a)** A function is given by f (x) = 6x + 7. Find the inverse function $f^{-1}(x)$.

(b) A function is given by f (x) = 5x + 8. Find the inverse function $f^{-1}(x)$.

(c) A function is given by f (x) = 8x + 9. Find the inverse function $f^{-1}(x)$.

(d) A function is given by f (x) = 2x + 1. Find the inverse function $f^{-1}(x)$.

[3]

7a) A function f is defined by the formula $f(x) = x^3 - 3x^2 - 6x + 8$ where x is a real number.

- **i)** Show that $x - 1$ is a factor of $f(x)$.
- **ii)** Hence factorise $f(x)$ fully.
- **iii)** Solve $f(x) = 0$.

b) A function f is defined by the formula $f(x) = x^3 - 4x^2 + x + 6$ where x is a real number.

- **i)** Show that $x - 3$ is a factor of $f(x)$.
- **ii)** Hence factorise $f(x)$ fully.
- **iii)** Solve $f(x) = 0$.

c) A function f is defined by the formula $f(x) = x^3 - 2x^2 - 11x + 12$ where x is a real number.

- **i)** Show that $x - 1$ is a factor of $f(x)$.
- **ii)** Hence factorise $f(x)$ fully.
- **iii)** Solve $f(x) = 0$.

d) A function f is defined by the formula $f(x) = x^3 + 9x^2 + 24x + 16$ where x is a real number.

- **i)** Show that $x + 4$ is a factor of $f(x)$.
- **ii)** Hence factorise $f(x)$ fully.
- **iii)** Solve $f(x) = 0$.

8.a) Solve the cubic equation $f(x)=0$ given the following:

- when $f(x)$ is divided by $x+2$, the remainder is zero
- when the graph of $y=f(x)$ is drawn, it passes through the point (-6, 0)
- $(x+3)$ is a factor of $f(x)$.

b) Solve the cubic equation $f(x)=0$ given the following:

- when $f(x)$ is divided by $x+4$, the remainder is zero
- when the graph of $y=f(x)$ is drawn, it passes through the point (2, 0)
- $(x-5)$ is a factor of $f(x)$.

c) Solve the cubic equation $f(x)=0$ given the following:

- when $f(x)$ is divided by $x+7$, the remainder is zero
- when the graph of $y=f(x)$ is drawn, it passes through the point (-1, 0)
- $(x+11)$ is a factor of $f(x)$.

d) Solve the cubic equation $f(x)=0$ given the following:

- when $f(x)$ is divided by $x-6$, the remainder is zero
- when the graph of $y=f(x)$ is drawn, it passes through the point (10, 0)
- $(x-12)$ is a factor of $f(x)$.

9 a) The graph of the function $f(x) = kx^2 + 3x + 3$ touches the *x*-axis at one point.

What is the range of values for k ?

b) The graph of the function $f(x) = kx^2 + 2x - 5$ touches the *x*-axis at two points.

What is the range of values for k ?

c) The graph of the function $f(x) = kx^2 - 8x + 2$ does not touch the *x*-axis.

What is the range of values for k ?

d)The graph of the function $f(x) = kx^2 - 2x + 7$ touches the *x*-axis at one point.

What is the range of values for k ?

10. (a) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 2, u_2 = 4$ and $u_3 = 14.$

Find the recurrence relation described by the sequence and use it to find the value of u_6.

(b) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 10, u_2 = 35$ and $u_3 = 47.5.$

Find the recurrence relation described by the sequence and use it to find the value of u_6.

(c) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 5, u_2 = 9.5$ and $u_3 = 20.75$

Find the recurrence relation described by the sequence and use it to find the value of u_6.

(d) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 12, u_2 = 10$ and $u_3 = 8.$

Find the recurrence relation described by the sequence and use it to find the value of u_6.

11. (a) On a particular day at 07:00, a doctor injects a first dose of 300mg of medicine into a patients bloodstream. The doctor then continues to administer the medicine in this way at 07:00 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 20% of what it was at the start.

(i) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their bloodstream exceeds 390mg.

(ii) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

(b) On a particular day at 06:00, a doctor injects a first dose of 150mg of medicine into a patients bloodstream. The doctor then continues to administer the medicine in this way at 06:00 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 10% of what it was at the start.

(i) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their bloodstream exceeds 170mg.

(ii) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

(c) On a particular day at 09:00, a doctor injects a first dose of 50mg of medicine into a patients bloodstream. The doctor then continues to administer the medicine in this way at 09:00 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 25% of what it was at the start.

(i)Set up a recurrence relation which shows the amount of
medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their bloodstream exceeds 70mg.

(ii) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

(d) On a particular day at 08:30, a doctor injects a first dose of 225mg of medicine into a patients bloodstream. The doctor then continues to administer the medicine in this way at 08:30 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be17% of what it was at the start.

(i) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their bloodstream exceeds 275mg.

(ii) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

Answers

1(a) (i) $\log_5 14ab$ (ii) $\log_6 12bc$ (iii) $\log_4 45ad$ (iv) $\log_8 21sy$

1(b) (i) $\log_a x$ (ii) $3\log_a x$ (iii) $2\log_a x$ (iv) logax

2. (a) x = 37 (b) y = 23

(c) z = 28 (d) d = 7

3. Correct x coordinates, correct y coordinates and correct shape and annotation

4. (a) a = 3, b = 5 (b) a = 2, b = 7 (c) a = 1, b = 6 (d) a = 5, b = 3

5. (a) $g(f(x)) = \sqrt{2x + 5}$, Square root of a negative cannot be found

(b) $g(f(x)) = \sqrt{3x + 6}$ (c) $g(f(x)) = \sqrt{4x + 10}$ (d) $g(f(x)) = \sqrt{2x + 7}$

6. (a) $f^{-1}(x) = \frac{x-7}{6}$ (b) $f^{-1}(x) = \frac{x-8}{5}$

(c) $f^{-1}(x) = \frac{x-9}{8}$ (d) $f^{-1}(x) = \frac{x-1}{2}$

7a) **i)** remainder =0 **ii)** $(x - 1)(x + 2)(x - 4)$

iii) $x = 1, x = -2, x = 4$

b) **i)** remainder =0 **ii)** $(x - 3)(x - 2)(x + 1)$

iii) $x = 3, x = 2, x = -1$

c) **i)** remainder =0 **ii)** $(x - 1)(x - 4)(x + 3)$

iii) $x = 1, x = 4, x = -3$

d) **i)**remainder =0 **ii)** $(x + 4)(x + 4)(x + 1)$

iii) $x = -4, x = -4, x = -1$

8. **a)** $x = -2,\ x = -3,\ x = -6$ **b)** $x = 2,\ x = 5,\ x = -4$

c) $x = -1,\ x = -7,\ x = -11$ **d)** $x = 6,\ x = 10,\ x = 12$

9. **a)** $k = \frac{3}{4}$ **b)** $k > -\frac{1}{5}$ **c)** $k > 8$ **(d)** $k = \frac{1}{7}$

10 (a) $U_{n+1} = 5U_n - 6$ (ii) $U_6 = 1564$

(b) $U_{n+1} = 0 \cdot 5U_n + 30$ (ii) $U_6 = 58 \cdot 44$

(c) $U_{n+1} = 2 \cdot 5U_n - 3$ (ii) $U_6 = 294 \cdot 97$

(d) $U_{n+1} = U_n - 2$ (ii) $U_6 = 2$

11 (a) $U_{n+1} = 0 \cdot 2U_n + 300$ (ii) $L = 375$ ∴ No danger

(b) $U_{n+1} = 0 \cdot 1U_n + 150$ (ii) $L = 166.67$ ∴ No danger

(c) $U_{n+1} = 0 \cdot 25U_n + 50$ (ii) $L = 66.67$ ∴ No danger

(d) $U_{n+1} = 0 \cdot 17U_n + 225$ (ii) $L = 271\frac{7}{83}$ ∴ No danger

ALGEBRA MIN. COMPETENCE SOLUTIONS

① ⓐ $\log_5 7a + \log_5 2b = \log_5 14ab$

② ⓐ $\log_2 (x-5) = 5$

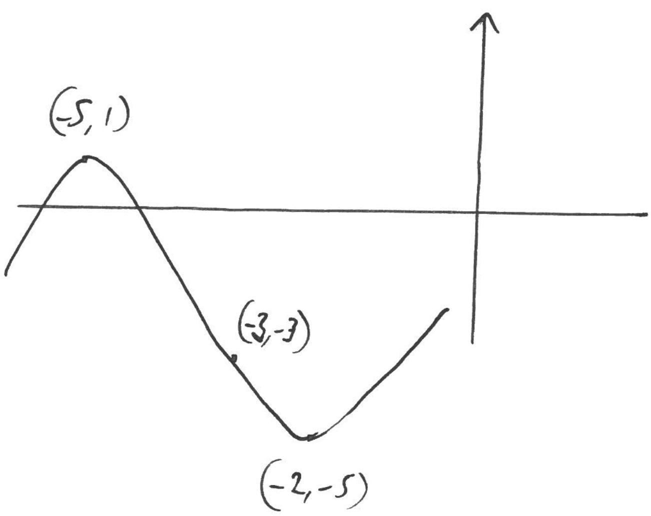
$2^5 = x - 5$

$x - 5 = 32$

$x = 37$

③ ⓐ

Orig Points	$f(x+3)$ [3 ←]	-2 [−2 ↓]
$(-2, 3)$	$(-5, 3)$	$(-5, 1)$
$(0, -1)$	$(-3, -1)$	$(-3, -3)$
$(1, -2)$	$(-2, -2)$	$(-2, -5)$



(4)(a) $y = \log_b (x-a)$

@ (4, 0) $0 = \log_b (4-a)$

$b^0 = 4 - a$

$1 = 4 - a$

$-a = -3$

$\underline{\underline{a = 3}}$

$y = \log_b (x-3)$

@ (8, 1) $1 = \log_b (8-3)$

$1 = \log_b 5$

$b^1 = 5$

$\underline{\underline{b = 5}}$

(5) $g(f(x)) = g(2x+5)$

$h(x) = \underline{\sqrt{2x+5}}$

(ii) Cannot take square root of a negative

$\Rightarrow \quad 2x + 5 \geq 0$

$2x \geq -5$

$\underline{\underline{x \geq -2.5}}$

(6) (a)

$$f(x) = 6x + 7$$

$$y = 6x + 7$$

At inverse .

$$x = 6y + 7$$

$$6y = x - 7$$

$$y = \frac{x-7}{6}$$

So $$f^{-1}(x) = \frac{x-7}{6}$$

(7) (a)

$$f(x) = x^3 - 3x^2 - 6x + 8$$

1	1	-3	-6	8
		1	-2	-8
	1	-2	-8	0

Remainder $= 0$

$\Rightarrow x - 1$ is a factor.

(b) $$x^3 - 3x^2 - 6x + 8$$

$$= (x-1)(x^2 - 2x - 8)$$

$$= (x-1)(x-4)(x+2)$$

(c) $f(x) = 0$

$(x-1)(x-4)(x+2) = 0$

$x-1=0 \qquad x-4=0 \qquad x+2=0$

$x=1 \qquad x=4 \qquad x=-2$

(8)

- $x+2$ is a factor
- $x+6$ is a factor
- $x+3$ is a factor

$\Rightarrow$ Cubic is $(x+2)(x+6)(x+3)$

$f(x) = (x+2)(x+6)(x+3)$

$f(x) = 0$

$(x+2)(x+6)(x+3) = 0$

$x+2=0 \qquad x+6=0 \qquad x+3=0$

$x=-2 \qquad x=-6 \qquad x=-3$

(9) Equal roots $\Rightarrow b^2-4ac=0$

$a=k, \quad b=3, \quad c=3$

$3^2-4(k)(3)=0$

$9-12k=0$

$12k=9 \qquad k=\frac{9}{12} = \frac{3}{4}$

(10) (a) $U_{n+1} = mU_n + c$

$4 = 2m + c$ $\quad\quad$ $14 = 4m + c$

$14 = 4m + c$ ①

$4 = 2m + c$ ②

① − ②

$10 = 2m$

$m = 5$

Sub $m = 5$ into ①

$14 = 4(5) + c$

$14 = 20 + c$

$c = -6$

$U_{n+1} = 5U_n - 6$

(11) (a) (i) $U_{n+1} = 0.2\,U_n + 300$

(ii) Limit as $-1 < 0.2 < 1$

At limit $L = 0.2L + 300$

$0.8L = 300$ $\quad\quad$ $375 < 390$

$L = \frac{300}{0.8} = 375g$ $\quad\quad$ so no danger

FORMULAE LIST

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represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$

represents a circle centre (a, b) and radius r.

Scalar Product: $a.b = |a||b|\cos\theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Calculus Unit Practice

1. **a)** Find $f'(x)$, given that $f(x) = 5\sqrt{x} - \frac{2}{x^3}, x > 0.$

b) Find $f'(x)$, given that $f(x) = 2\sqrt{x} + 3x^{-4}, x > 0.$

c) Find $f'(x)$, given that $f(x) = 2x^{\frac{1}{2}} - \frac{3}{x^5}, x > 0.$

d) Find $f'(x)$, given that $f(x) = 6\sqrt{x} - \frac{5}{x^6}, x > 0.$

2 **a)** Differentiate the function $f(x) = 4cosx$ with respect to *x*.

b) Differentiate the function $f(x) = 7sinx$ with respect to *x*.

c) Differentiate the function $f(x) = -2cosx$ with respect to *x*.

d) Differentiate the function $f(x) = 3\cos x$ with respect to *x*.

3.a) A curve has equation $y = 3x^2 + 2x + 2$, find the equation of the tangent to the curve at $x = -1$.

b) A curve has equation $y = 5x^2 - 3x + 2$, find the equation of the tangent to the curve at $x = 2$.

c) A curve has equation $y = 4x^2 + 2x - 1$, find the equation of the tangent to the curve at $x = -2$.

d) A curve has equation $y = 3x^2 - 2x + 5$, find the equation of the tangent to the curve at $x = 1$.

4a) A yachtsman in distress fires a flare vertically upwards to signal for help. The height (in metres) of the flare t seconds after it is fired can be represented by the formula

$h = 40t - 2t^2$.

The velocity of the flare at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the flare 10 seconds after it is set off.

b) A yachtsman in distress fires a flare vertically upwards to signal for help. The height (in metres) of the flare t seconds after it is fired can be represented by the formula

$h = 30t - t^2$.

The velocity of the flare at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the flare 15 seconds after it is set off.

c) A yachtsman in distress fires a flare vertically upwards to signal for help. The height (in metres) of the flare t seconds after it is fired can be represented by the formula

$h = 60t - 6t^2$.

The velocity of the flare at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the flare 5 seconds after it is set off.

d) A yachtsman in distress fires a flare vertically upwards to signal for help. The height (in metres) of the flare t seconds after it is fired can be represented by the formula

$h = 16t - 4t^2$.

The velocity of the flare at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the flare 2 seconds after it is set off.

5.a) Find $\int 5x^{\frac{3}{2}} + \frac{1}{x^3}\, dx, x \neq 0.$

b) Find $\int 2x^{\frac{1}{2}} - \frac{1}{x^5}\, dx, x \neq 0.$

c) Find $\int 4x^{\frac{2}{3}} + \frac{1}{x^2}\, dx, x \neq 0.$

d) Find $\int 5x^{\frac{1}{4}} - \frac{1}{x^7}\, dx, x \neq 0.$

6. **a)** $f'(x) = (x+3)^{-7}$, find f(x), x ≠-3.

b) $f'(x) = (x-1)^{-6}$, find f(x), x ≠1.

c) $f'(x) = (x+4)^{-3}$, find f(x), x ≠-4.

d) $f'(x) = (x-9)^{-2}$, find f(x), x ≠9.

7. **a)** Find $\int 3\cos\theta\ d\theta$

b) Find $\int 2\sin\theta\ d\theta$

c) Find $\int -6\cos\theta\ d\theta$

d) Find $\int 4\cos\theta\ d\theta$

8.a) $\int_1^3 (x+1)^3$

b) $\int_1^2 (x-5)^4$

c) $\int_2^3 (x+2)^6$

d) $\int_1^4 (x-7)^2$

9. (a) A box with a square base and open top has a surface area of 192cm^{2}. The volume of the box can be represented by the formula:

$$V(x) = 48x - \frac{1}{4}x^3 \; where \; x > 0$$

Find the value of x which maximises the volume of the box.

(b) A box with a square base and open top has a surface area of 972cm^{2}. The volume of the box can be represented by the formula:

$$V(x) = 243x - \frac{1}{4}x^3 \; where \; x > 0$$

Find the value of x which maximises the volume of the box.

(c) A box with a square base and open top has a surface area of 432cm^{2}. The volume of the box can be represented by the formula:

$$V(x) = 108x - \frac{1}{4}x^3 \; where \; x > 0$$

Find the value of x which maximises the volume of the box.

(d) A box with a square base and open top has a surface area of 484cm^{2}. The volume of the box can be represented by the formula:

$$V(x) = 121x - \frac{1}{4}x^3 \; where \; x > 0$$

Find the value of x which maximises the volume of the box.

10. (a) The curve with equation $y = x^2(3 - x)$ is shown below.

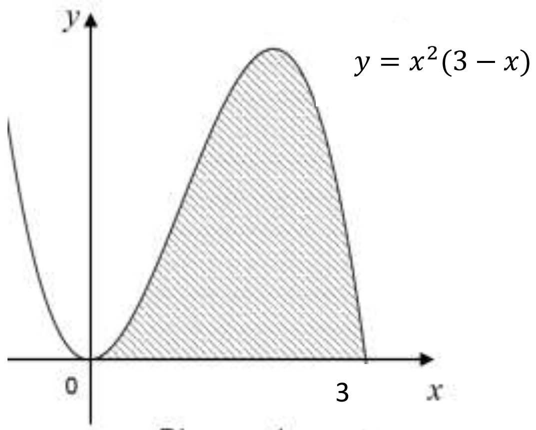


Diagram 1

5

Calculate the shaded area.

(b) The curve with equation $y = x^2(5 - x)$ is shown below.

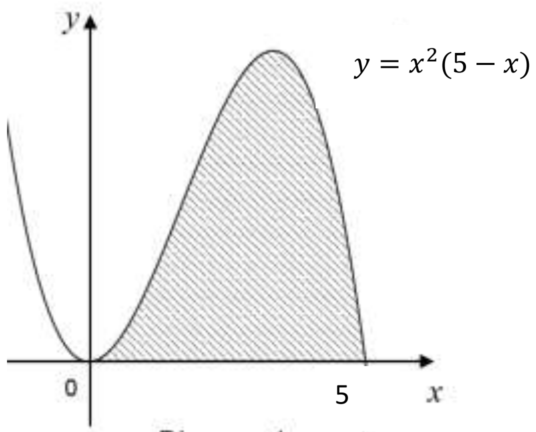


Diagram 1

5

Calculate the shaded area.

(c) The curve with equation $y = x^2(6 - x)$ is shown below.

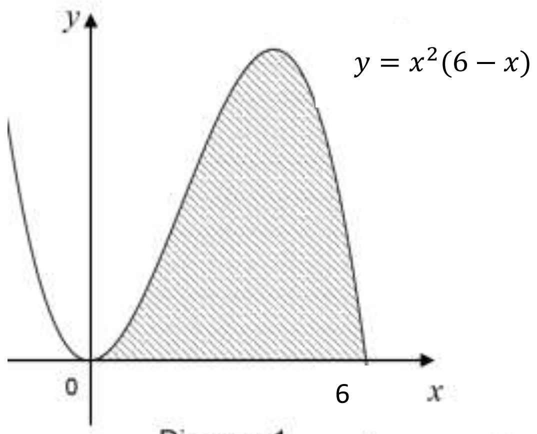


Diagram 1

5

Calculate the shaded area.

(d) The curve with equation $y = x^2(10 - x)$ is shown below

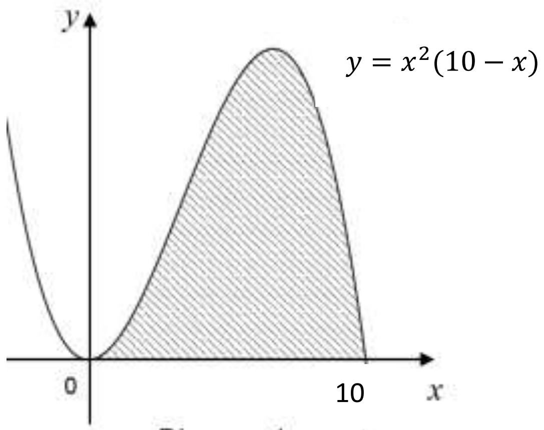
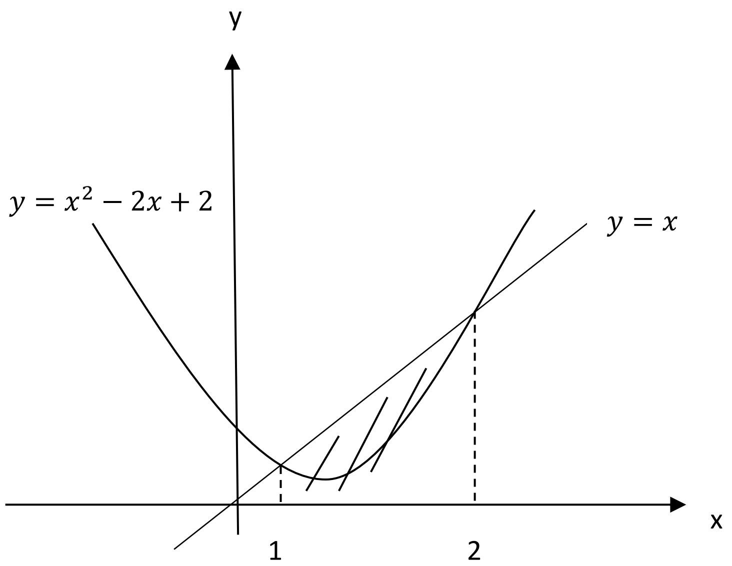


Diagram 1

5

Calculate the shaded area.

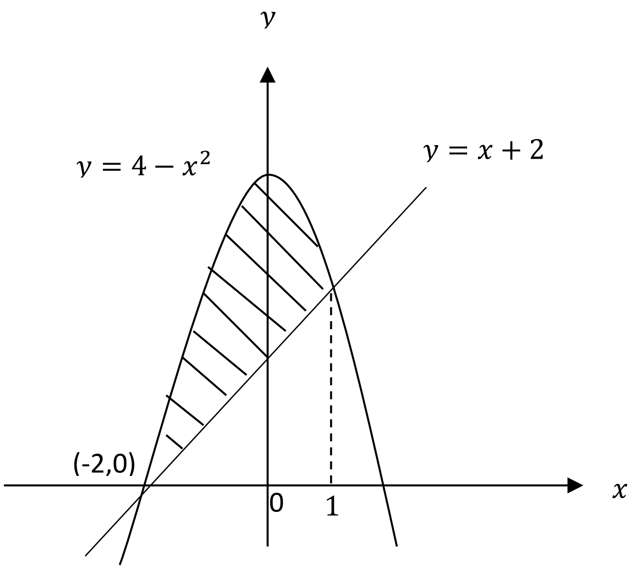
11. (a) The line with equation $y = x$ and the curve with equation $y = x^2 - 2x + 2$ are shown below



The line and the curve meet at the points where $x = 1$ *and* $x = 2$.

Calculate the shaded area.

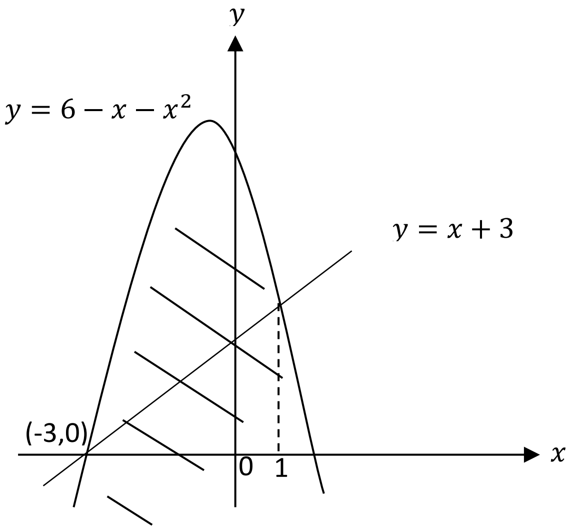
(b) The line with equation $y = x + 2$ and the curve with the equation $y = 4 - x^2$ are shown below.



The line and the curve meet at the points where $x = -2$ and $x = 1$.

Calculate the shaded area.

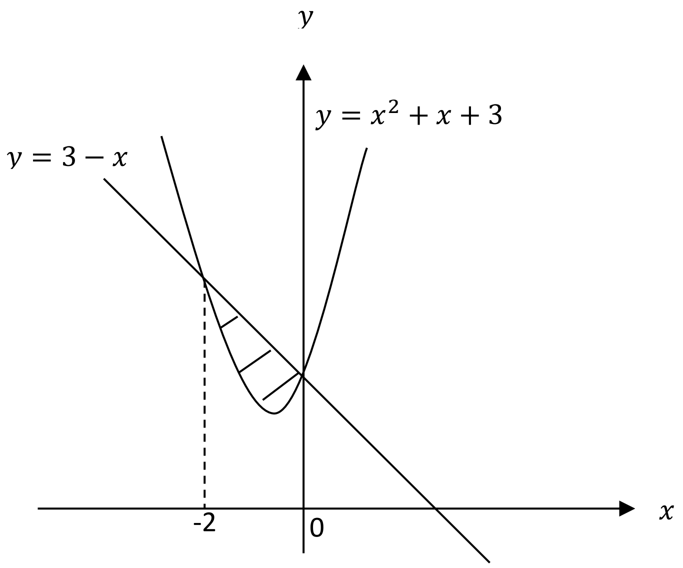
(c) The line with equation $y = x + 3$ and the curve with equation $y = 6 - x - x^2$ are shown below.



The line and the curve meet at the points where $x = -3$ and $x = 1$.

Calculate the shaded area.

(d) The line with equation $y = 3 - x$ and the curve with equation $y = x^2 + x + 3$ are shown below.



The line and the curve meet at the points where $x = -2$ and $x = 0$.

Calculate the shaded area.

Answers

1. **a)** $f'(x) = \frac{5}{2}x^{-\frac{1}{2}} + 6x^{-4}$ **b)** $f'(x) = x^{-\frac{1}{2}} - 12x^{-5}$

c) $f'(x) = x^{-\frac{1}{2}} + 15x^{-6}$ **d)** $f'(x) = 3x^{-\frac{1}{2}} + 30x^{-7}$

2. **a)** $f'(x) = -4sinx$ **b)** $f'(x) = 7cosx$

c) $f'(x) = 2sinx$ **d)** $f'(x) = -3sinx$

3.a) $y - 3 = -4(x + 1)$ **b)** $y - 16 = 17(x - 2)$

c) $y - 11 = -14(x + 2)$ **d)** $y - 6 = 4(x - 1)$

4. **a)** **i)** $v = 0$ **ii)** The flare has stopped rising

b) **i)** $v = 0$ **ii)** The flare has stopped rising

c) **i)** $v = 0$ **ii)** The flare has stopped rising

d) **i)** $v = 0$ **ii)** The flare has stopped rising

5. a) $2x^{\frac{5}{2}} - \frac{1}{2}x^{-2} + c$ **b)** $\frac{4}{3}x^{\frac{3}{2}} + \frac{1}{4}x^{-4} + c$

c) $\frac{12}{5}x^{\frac{5}{3}} - x^{-1} + c$ **d)** $4x^{\frac{5}{4}} + \frac{1}{6}x^{-6} + c$

6. **a)** $f'(x) = -\frac{1}{6}(x + 3)^{-6} + c$ **b)** $f'(x) = -\frac{1}{5}(x - 1)^{-5} + c$

c) $f'(x) = -\frac{1}{2}(x + 4)^{-2} + c$ **d)** $f'(x) = -(x - 9)^{-1} + c$

7. **a)** $3sin\boldsymbol{\theta} + \boldsymbol{c}$ **b)** $-2cos\boldsymbol{\theta} + \boldsymbol{c}$ **c)** $-6sin\boldsymbol{\theta} + \boldsymbol{c}$ **d)** $4sin\boldsymbol{\theta} + \boldsymbol{c}$

8. **a)** 60 **b)** $\frac{781}{5}$ **c)** $\frac{61741}{7}$ d**)** 63

9 (a) Max at $x = 8$

(b) Max at $x = 18$

(c) Max at $x = 12$

(d) Max at $x = 12 \cdot 7$

10 (a) $6\frac{3}{4}$ (b) $52\frac{1}{12}$ (c) 108 (d) $833\frac{1}{3}$

11 (a) $\frac{1}{6}$ (b) $4\frac{1}{2}$ (c) $10\frac{2}{3}$ (d) $\frac{4}{3}$

CALCULUS MIN. COMPETENCE SOLUTIONS

(1)(a) $f(x) = 5\sqrt{x} - \dfrac{2}{x^3}$

$= 5x^{\frac{1}{2}} - 2x^{-3}$

$f'(x) = \dfrac{5}{2}x^{-\frac{1}{2}} + 6x^{-4}$

$= \dfrac{5}{2x^{\frac{1}{2}}} + \dfrac{6}{x^4}$

(2)(a) $f(x) = 4\cos x$

$f'(x) = -4\sin x$

(3)(a) $y = 3x^2 + 2x + 2$

$\dfrac{dy}{dx} = 6x + 2$

$m_{x=-1} = 6(-1) + 2 = -6 + 2 = -4$

When $x = -1$ $y = 3(-1)^2 + 2(-1) + 2$

$= 3 - 2 + 2 = 3$

So point of contact is $(-1, 3)$

$y - b = m(x - a)$

$y - 3 = -4(x - (-1))$

$$y - 3 = -4(x+1)$$

$$y - 3 = -4x - 4$$

$$\underline{\underline{y = -4x - 1}}$$

(4) (a)

$$h = 40t - 2t^2$$

$$\frac{dh}{dt} = 40 - 4t$$

$$v = 40 - 4t$$

When $t = 10$, $v = 40 - 4(10)$

$$= 0.$$

$\underline{\underline{\text{Velocity} = 0}}$

(5) (a)

$$\int 5x^{3/2} + \frac{1}{x^3} \, dx$$

$$= \int 5x^{3/2} + x^{-3}$$

$$= \frac{5x^{5/2}}{5/2} + \frac{x^{-2}}{-2} + c$$

$$= \frac{2(5x^{5/2})}{5} - \frac{x^{-2}}{2} + c$$

$$= \underline{\underline{2x^{5/2} - \frac{1}{2x^2} + c}}$$

⑥ ⓐ $f'(x) = (x+3)^{-7}$

$f(x) = \int (x+3)^{-7}$

$= \dfrac{(x+3)^{-6}}{-6} + c$

$= -\frac{1}{6}(x+3)^{-6} + c$

$= \dfrac{-1}{6(x+3)^{6}} + c$

⑦ ⓐ $\int 3\cos\theta = 3\sin\theta + c$

⑧ ⓐ $\int_1^3 (x+1)^3 = \left[\dfrac{(x+1)^4}{4}\right]_1^3$

$= \left(\dfrac{(3+1)^4}{4} - \dfrac{(1+1)^4}{4}\right)$

$= \dfrac{4^4}{4} - \dfrac{2^4}{4} = 4^3 - \dfrac{16}{4}$

$= 64 - 4 = 60$

sq. units

9) a) $V(x) = 48x - \frac{1}{4}x^3$

Max when $V'(x) = 0$

$V'(x) = 48 - \frac{3}{4}x^2$

At max $48 - \frac{3}{4}x^2 = 0$

$\frac{3}{4}x^2 = 48$

$3x^2 = 192$

$x^2 = 64$

$x = \pm 8$

x	$\to$	-8	$\to$	8	$\to$
$V'(x)$	$-$	0	$+$	0	$-$
Shape	\	—	/	—	\

Max when $x = 8$

$V'(-10) = 48 - \frac{3}{4}(-10)^2$
$= 48 - 75$
negative.

$V'(0) = 48 - \frac{3}{4}(0)$
$= 48$
positive

$V'(10) = 48 - \frac{3}{4}(10)^2$
$= 48 - 75$
$=$ negative.

(10) (a) $$\int_1^3 x^2(3-x)$$

$$= \int_0^3 3x^2 - x^3$$

$$= \left[\frac{3x^3}{3} - \frac{x^4}{4}\right]_0^3$$

$$= \left[x^3 - \frac{1}{4}x^4\right]_0^3$$

$$= \left(3^3 - \frac{1}{4}(3)^4\right) - \left(0^3 - \frac{1}{4}(0)^4\right)$$

$$= \left(27 - \frac{81}{4}\right) - (0)$$

$$= \left(27 - 20\tfrac{1}{4}\right) -$$

$$= 6\tfrac{3}{4}$$

$$= \underline{\underline{6}}$$

(11) (a) $$\text{Area} = \int_1^2 \text{Top} - \text{Bottom}$$

$$= \int_1^2 x - (x^2 - 2x + 2)$$

$$= \int_1^2 x - x^2 + 2x - 2$$

$$= \int_1^2 \left(-x^2 + 3x - 2\right) \, dx$$

$$= \left[\frac{-x^3}{3} + \frac{3x^2}{2} - 2x \right]_1^2$$

$$= \left(\frac{-2^3}{3} + \frac{3(2)^2}{2} - 2(2) \right) - \left(\frac{-1^3}{3} + \frac{3(1)^2}{2} - 2(1) \right)$$

$$= \left(-\frac{8}{3} + 6 - 4 \right) - \left(\frac{-1}{3} + \frac{3}{2} - 2 \right)$$

$$= \left(-2\tfrac{2}{3} + 2 \right) - \left(\frac{7}{6} - 2 \right)$$

$$= -2\tfrac{2}{3} + 2 - \frac{7}{6} + 2$$

$$= 4 - 2\tfrac{2}{3} - \frac{7}{6}$$

$$= 1\tfrac{1}{3} - 1\tfrac{1}{6}$$

$$= \frac{1}{3} - \frac{1}{6} = \underline{\underline{\frac{1}{6}}}$$

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$

represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$

represents a circle centre (a, b) and radius r .

Scalar Product: $a.b = |a||b|\cos\theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \, \mathrm{m} \sin A\sin B$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Geometry Unit Practice

1. (a) A straight line has the equation $5x + y - 3 = 0$.

   Write down the equation of the line parallel to the given line, which passes through the point (4,-8)

   (b) A straight line has the equation $y = -4x + 7$.

   Write down the equation of the line parallel to the given line, which passes through the point (3,-12)

   (c) A straight line has the equation $3x + y - 1 = 0$.

   Write down the equation of the line parallel to the given line, which passes through the point (6,-4)

   (d) A straight line has the equation $y = -5x + 2$.

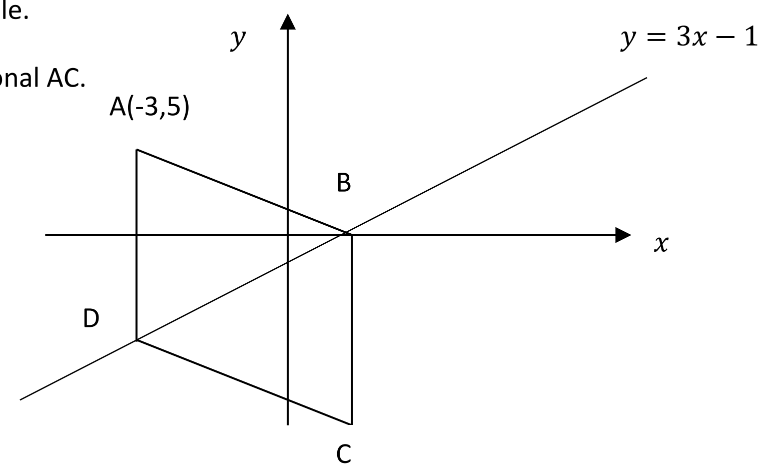
   Write down the equation of the line parallel to the given line, which passes through the point (3,-7)

2.(a) ABCD is a rhombus

Diagonal BD has equation $y = 3x - 1$ and point A has coordinates (-3,5).

Note the diagram is not to scale.

Find the equation of the diagonal AC.

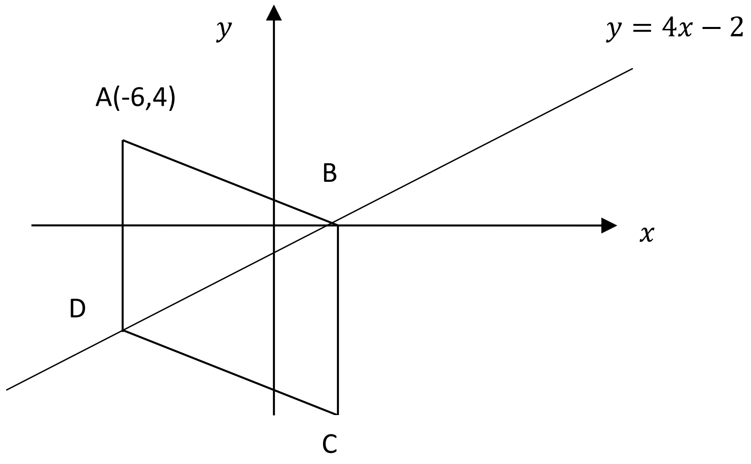


(b) ABCD is a rhombus

Diagonal BD has equation $y = 4x - 2$ and point A has coordinates

(-6,4).
Note the diagram is not to scale.

Find the equation of the diagonal AC.



(c) ABCD is a rhombus

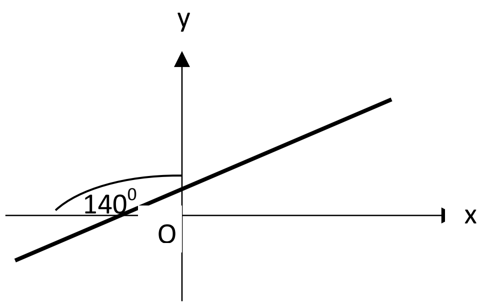
Diagonal BD has equation $y = 5x - 3$ and point A has coordinates

A ski slope is categorised by its gradient as shown in the table.

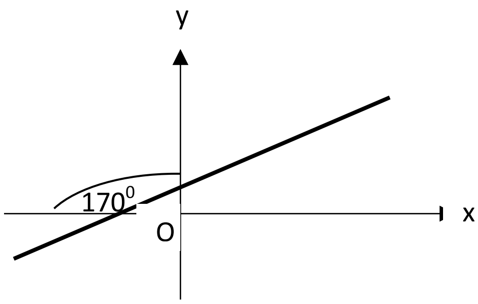
Dry slope category	**Gradient (*m*) of slope**
Teaching and general skiing	$0 < m \leq 0 \cdot 4$
Extreme skiing	$m > 0 \cdot 4$

2 What is the gradient of each ski slope shown below and which category does each ski slope belong to?
Explain your answer fully.

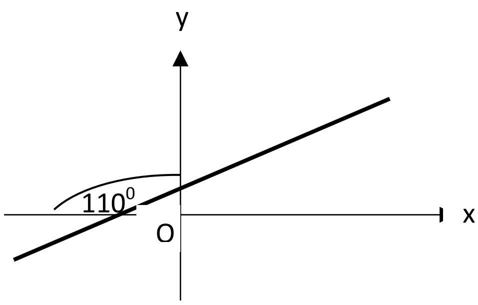
(a)



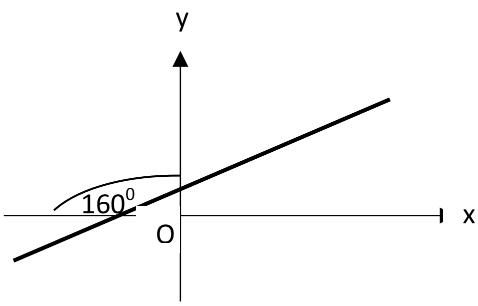
(b)



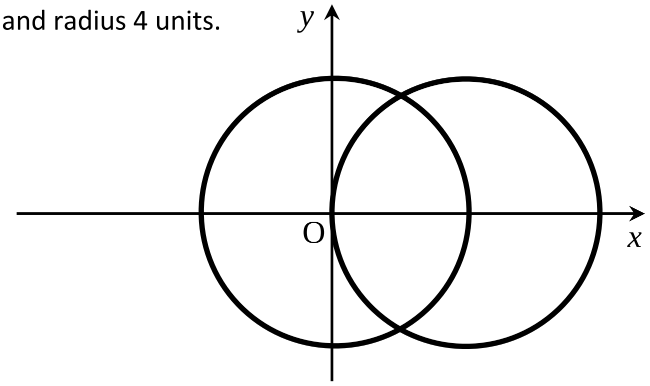
(c)



(d)



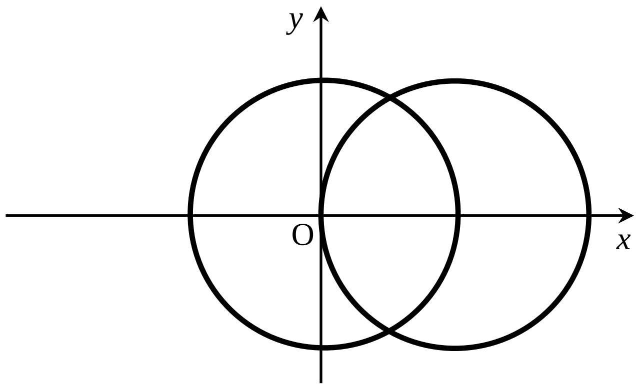
4. The diagram shows two congruent circles. One circle has centre the origin and radius 4 units.



Find the equation of the other circle which passes through the origin

whose centre lies on the x-axis.

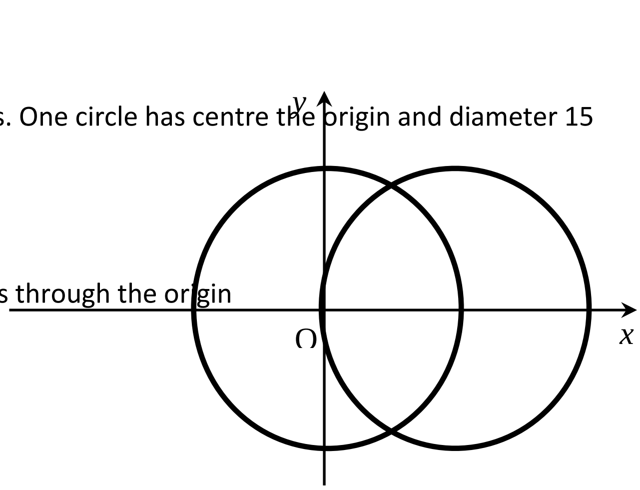
(b) The diagram shows two congruent circles. One circle has centre the origin and diameter 12 units.



Find the equation of the other circle which passes through the origin

whose centre lies on the x-axis.

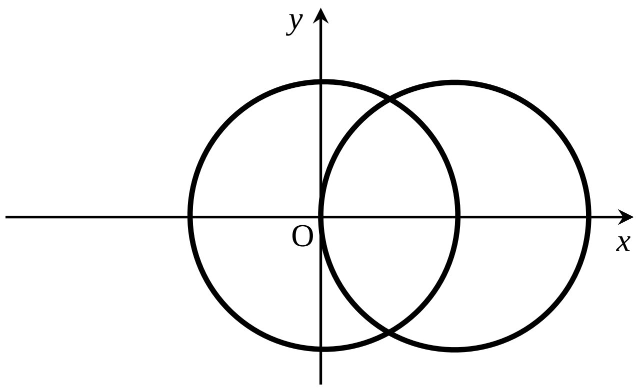
(c) The diagram shows two congruent circles. One circle has centre the origin and diameter 15 units.



Find the equation of the other circle which passes through the origin

whose centre lies on the x-axis.

(d) The diagram shows two congruent circles. One circle has centre the origin and radius 2.5 units.



Find the equation of the other circle which passes through the origin

whose centre lies on the x-axis.

5.(a) Determine algebraically if the line $y = x - 1$ is a tangent to the circle

$(x + 4)^2 + (y - 2)^2 = 49$

(b) Determine algebraically if the line $y = x + 1$ is a tangent to the circle

$(x + 1)^2 + (y - 2)^2 = 20$

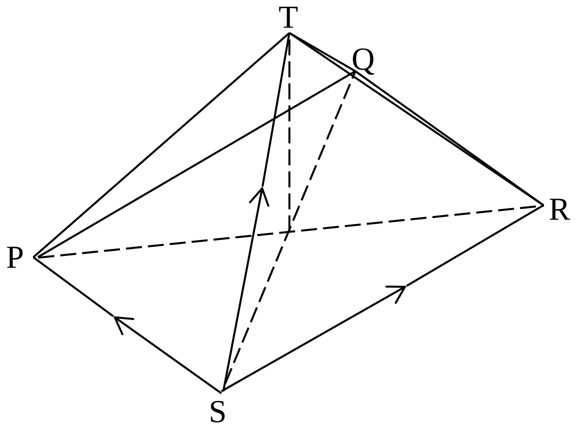
(c) Determine algebraically if the line $y = 3x + 10$ is a tangent to the circle

$(x - 4)^2 + (y - 2)^2 = 40$

(d) Determine algebraically if the line $y = x + 3$ is a tangent to the circle

$(x + 2)^2 + (y - 4)^2 = 9$

12. TPQRS is a pyramid with rectangular base PQRS.



(a) TPQRS is a pyramid with rectangular base PQRS (as above).
If the vectors $\overrightarrow{SP}$, $\overrightarrow{SR}$, $\overrightarrow{ST}$ are given by:

$\overrightarrow{SP} = 3\boldsymbol{i} - 8\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{SR} = \boldsymbol{i} + 12\boldsymbol{j} + 9\boldsymbol{k}$

$\overrightarrow{ST} = -7\boldsymbol{i} + 11\boldsymbol{k}$

Express $\overrightarrow{PT}$ in component form

(b) TPQRS is a pyramid with rectangular base PQRS (see diagram on left)
If the vectors $\overrightarrow{SP}$, $\overrightarrow{SR}$, $\overrightarrow{ST}$ are given by:

$\overrightarrow{SP} = 4\boldsymbol{i} - 6\boldsymbol{j} - 5\boldsymbol{k}$

$\overrightarrow{SR} = \boldsymbol{i} + 12\boldsymbol{j} + 9\boldsymbol{k}$

$\overrightarrow{ST} = -6\boldsymbol{i} + 2\mathbf{j} + 12\boldsymbol{k}$

Express $\overrightarrow{PT}$ in component form

(c) TPQRS is a pyramid with rectangular base PQRS (see diagram on left)

If **t**he vectors $\overrightarrow{\mathrm{SP}}$, $\overrightarrow{\mathrm{SR}}$, $\overrightarrow{\mathrm{ST}}$ are given by:

$\overrightarrow{SP}$ = 2***i*** + ***j***

$\overrightarrow{SR}$ = ***i*** + 12***j*** + 9***k***

$\overrightarrow{ST}$ = –4***i*** + 6**j** + 5***k***

Express $\overrightarrow{PT}$ in component form

(d) TPQRS is a pyramid with rectangular base PQRS. (see diagram on left)

If the vectors $\overrightarrow{\mathrm{SP}}$, $\overrightarrow{\mathrm{SR}}$, $\overrightarrow{\mathrm{ST}}$ are given by:

$\overrightarrow{SP}$ = 8***i*** + ***j*** + 6***k***

$\overrightarrow{SR}$ = ***i*** + 12***j*** + 9***k***

$\overrightarrow{ST}$ = –2***i*** + 5**j + 6*k***

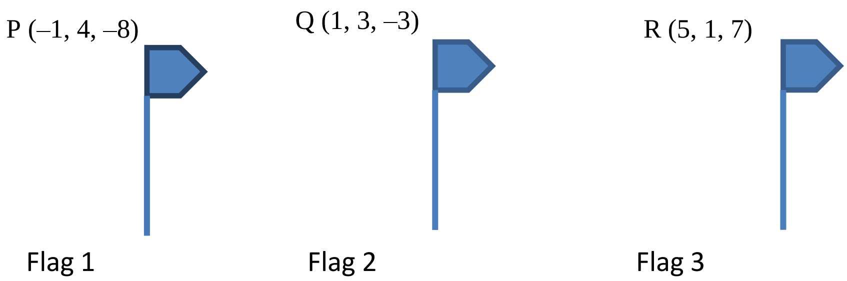
Express $\overrightarrow{PT}$ in component form

[2]

13. (a) An architect laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is twice the distance between Flag 1 and Flag 2.

Relative to suitable axes, the top-left corner of each flag can be represented by the points P (–1, 4, –8), Q (1, 3, –3), and R (5, 1, 7) respectively. All three flags point vertically upwards.

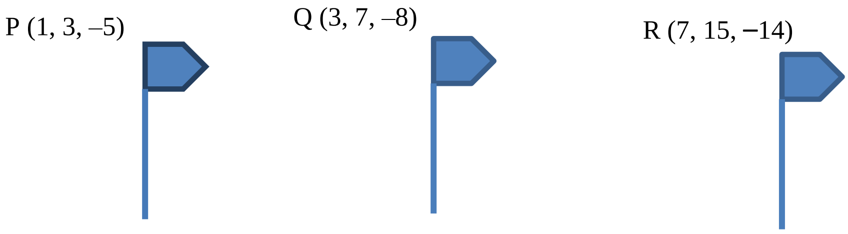


Has the architect laid the flags correctly? You must justify your answer.

13. **(b)** An architect laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is twice the distance between Flag 1 and Flag 2.

Relative to suitable axes, the top-left corner of each flag can be represented by the points P (1, 3, –5), Q (3, 7, –8), and R (7, 15, –14) respectively. All three flags point vertically upwards.



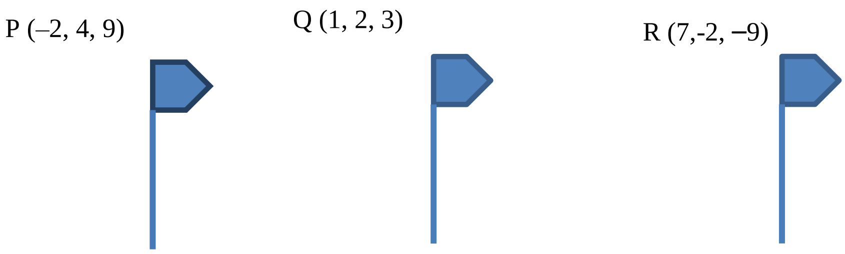
Flag 1 Flag 2 Flag 3

Has the architect laid the flags correctly? You must justify your answer.

13. **(c)** An architect laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is twice the distance between Flag 1 and Flag 2.

Relative to suitable axes, the top-left corner of each flag can be represented by the points P (–2, 4, 9), Q (1, 2, 3), and R (7, –2, –9) respectively. All three flags point vertically upwards.



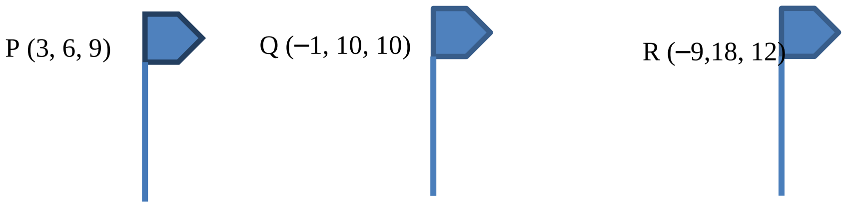
Flag 1 Flag 2 Flag 3

Has the architect laid the flags correctly? You must justify your answer.

13. (d) An architect laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is twice the distance between Flag 1 and Flag 2.

Relative to suitable axes, the top-left corner of each flag can be represented by the points P (3, 6, 9), Q (–1, 10, 10), and R (–9, 18, 12) respectively. All three flags point vertically upwards.



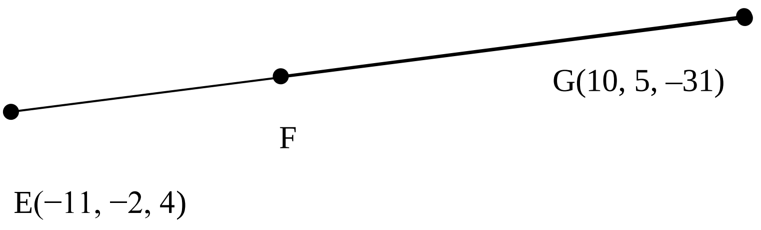
Flag 1 Flag 2 Flag 3

Has the architect laid the flags correctly? You must justify your answer.

[#2.1,2,#2.2]

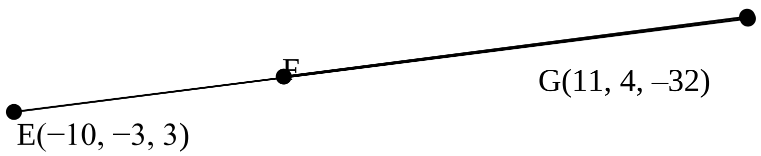
14. (a) The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 3:4.

Find the coordinates of F.



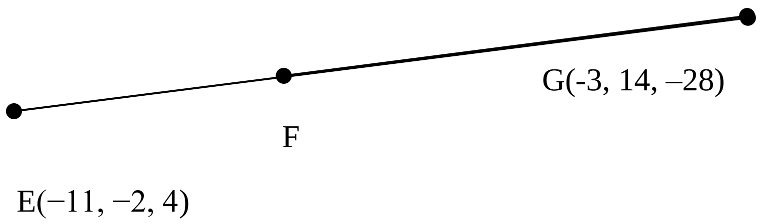
14. (b) The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 2:5.

Find the coordinates of F.



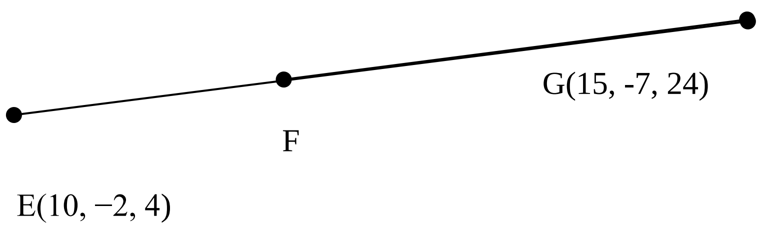
(c) The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 3:5.

Find the coordinates of F.

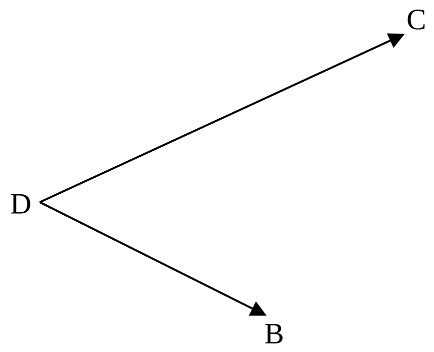


(d) The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 1:4.

Find the coordinates of F.

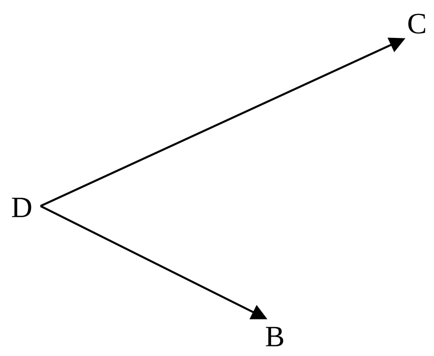


15.(a) Points B, C and D have coordinates B(21, –8, 0), C(20, –7, 7) and D(17, –6, 2).



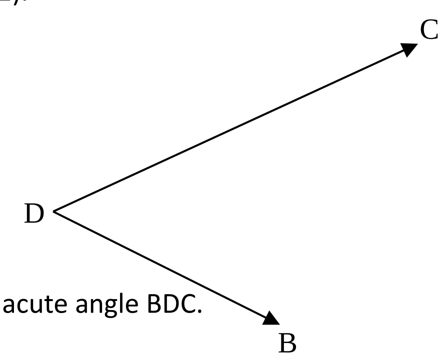
Find the size of the acute angle BDC.

(b) Points B, C and D have coordinates B(15, –7, 4), C(10, –2, 2) and D(18, –1, 3).



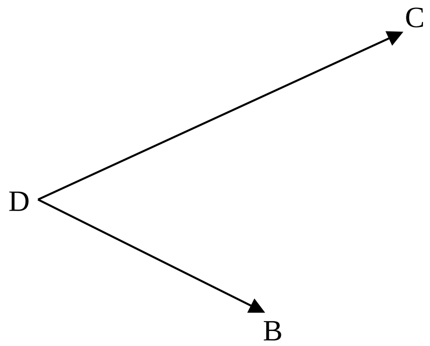
Find the size of the acute angle BDC.

(c) Points B, C and D have coordinates B(11, –4, 1), C(16, –5, 3) and D(12, –8, 1).



Find the size of the acute angle BDC.

(d) Points B, C and D have coordinates B(15, –4, 2), C(30, –8, 8) and D(14, –3, 1).



Find the size of the acute angle BDC.

[5]

Answers

1 (a) $y = -5x + 12$ (b) $y = -4x$ (c) $y = -3x$ (d) $y = -5x + 8$

3 (a) $m = 0 \cdot 83$ (b) $m = 0 \cdot 17$ (c) $m = 2 \cdot 75$ (d) $m = 0 \cdot 36$

4 (a) $(x - 4)^2 + y^2 = 16$ (b) $(x - 6)^2 + y^2 = 36$

(c) $(x - 7.5)^2 + y^2 = 56.25$ (d) $(x - 2.5)^2 + y^2 = 6.25$

5 (a) Line meets at $x = -4,3$ => Not a tangent.

(b) Line meets at $x = -3,3$ => Not a tangent.

(c) Line meets at $x = -2(twice)$ => A tangent.

(d) Line meets at $x = -2,1$ => Not a tangent.

12.(a) -10i + 8j + 17k (b) -10i + 8j + 17k

(c) -6i + 5j + 5k (d) -10i + 4j

13. For each question show they are collinear, interpret ratio and write a suitable solution

14. (a) F = (-2, 1,-11) (b) F = (-4, -1,-7) (c) F = (-8, 4,-8) (d) F = (11, -3, 8)

15. (a) 82.1 degrees (b) 58.2 degrees

(c) 68.9 degrees (d) 27.1 degrees

GEOMETRY MIN. COMPETENCE SOLUTIONS.

① ⓐ

$$5x + y - 3 = 0$$

$$y = -5x + 3$$

$$m = -5$$

$(4, -8)$

(a, b)

$$y - b = m(x - a)$$

$$y - (-8) = -5(x - 4)$$

$$y + 8 = -5x + 20$$

$$y = -5x + 12$$

② a Rhombus diagonals are perpendicular

$$m_{BD} = 3$$

$$\Rightarrow m_{AC} = -\frac{1}{3}$$

$(-3, 5)$

$a \quad b$

$$y - b = m(x - a)$$

$$y - 5 = -\frac{1}{3}(x - (-3))$$

$$y - 5 = -\frac{1}{3}(x + 3)$$

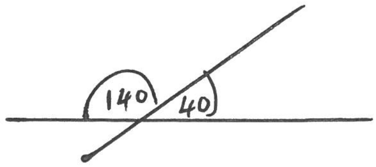
$(\times 3) \qquad (\times 3)$

$$3y - 15 = -1(x + 3)$$

$$3y - 15 = -x - 3$$

$$3y = -x + 12$$

(3) (a)



$$m = \tan\theta$$
$$= \tan 40$$
$$= 0.84$$

$0.84 > 0.4$ so extreme skiing

(4) (a) Each circle has radius 4 units

$\Rightarrow$ Centre of other circle is $(4, 0)$

$$(x-4)^2 + (y-0)^2 = 4^2$$
$$(x-4)^2 + y^2 = 16$$

(5) (a) Simultaneous Equations

$$(x+4)^2 + (y-2)^2 = 49$$
$$(x+4)^2 + ((x-1)-2)^2 = 49$$
$$(x+4)^2 + (x-3)^2 = 49$$
$$x^2 + 8x + 16 + x^2 - 6x + 9 = 49$$
$$2x^2 + 2x + 25 = 49$$
$$2x^2 + 2x - 24 = 0$$
$$(2x-6)(x+4) = 0$$

$2x - 6 = 0$
$2x = 6$
$x = 3$

$x + 4 = 0$
$x = -4$

Line meets circle at 2 points where $x = 3$ + $x = -4$

$\Rightarrow$ Not a tangent.

(6)(a) $\overrightarrow{PT} = -\overrightarrow{PS} + \overrightarrow{ST}$

$$= -\begin{pmatrix}3\\-8\\-6\end{pmatrix} + \begin{pmatrix}-7\\0\\11\end{pmatrix}$$

$$= \begin{pmatrix}-3\\8\\6\end{pmatrix} + \begin{pmatrix}-7\\0\\11\end{pmatrix} = \begin{pmatrix}-10\\8\\17\end{pmatrix}$$

$$= \underline{\underline{-10i + 8j + 17k}}$$

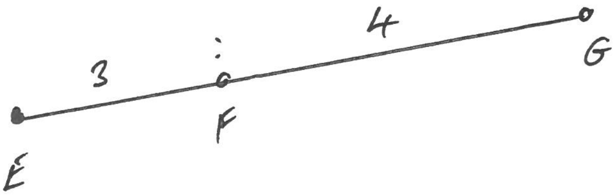
(7)(a) $\overrightarrow{PQ} = \underline{q} - \underline{p} = \begin{pmatrix}1\\3\\-3\end{pmatrix} - \begin{pmatrix}-1\\4\\-8\end{pmatrix} = \begin{pmatrix}2\\-1\\5\end{pmatrix}$

$\overrightarrow{QR} = \underline{r} - \underline{q} = \begin{pmatrix}5\\1\\7\end{pmatrix} - \begin{pmatrix}1\\3\\-3\end{pmatrix} = \begin{pmatrix}4\\-2\\10\end{pmatrix}$

$\overrightarrow{QR} = 2\overrightarrow{PQ}$ $\Rightarrow$ 3 points are collinear and distance between flag 2 + 3 is twice the distance between flags 1 + 2.

$\Rightarrow$ Flags have been laid correctly.

8 (a)



$$3\overrightarrow{FG} = 4\overrightarrow{EF}$$

$$3(\underline{g} - \underline{f}) = 4(\underline{f} - \underline{e})$$

$$3g - 3f = 4f - 4e$$

$$3g = 7f - 4e$$

$$7f = 3g + 4e$$

$$= 3\begin{pmatrix}10\\5\\-31\end{pmatrix} + 4\begin{pmatrix}-11\\-2\\4\end{pmatrix}$$

$$= \begin{pmatrix}30\\15\\-93\end{pmatrix} + \begin{pmatrix}-44\\-8\\16\end{pmatrix}$$

$$7\underline{f} = \begin{pmatrix}-14\\7\\-77\end{pmatrix}$$

$$f = \begin{pmatrix}-2\\1\\-11\end{pmatrix}$$

(9) (a)

$\overrightarrow{DC} = \underline{c} - \underline{d}$

$= \begin{pmatrix} 20 \\ -7 \\ 7 \end{pmatrix} - \begin{pmatrix} 17 \\ -6 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$

$\overrightarrow{DB} = \underline{b} - \underline{d}$

$= \begin{pmatrix} 21 \\ -8 \\ 0 \end{pmatrix} - \begin{pmatrix} 17 \\ -6 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}$

$\overrightarrow{DC} \cdot \overrightarrow{DB} = (3 \times 4) + ((-1) \times (-2)) + (5 \times (-2))$

$= 12 + 2 + (-10)$

$= \underline{\underline{4}}$

$|\overrightarrow{DC}| = \sqrt{3^2 + (-1)^2 + 5^2}$

$= \sqrt{9 + 1 + 25}$

$= \underline{\underline{\sqrt{35}}}$

$|\overrightarrow{DB}| = \sqrt{4^2 + (-2)^2 + (-2)^2}$

$= \sqrt{16 + 4 + 4}$

$= \sqrt{24}$.

$\cos\theta = \dfrac{\overrightarrow{DC} \cdot \overrightarrow{DB}}{|\overrightarrow{DC}| . |\overrightarrow{DB}|} = \dfrac{4}{\sqrt{35} . \sqrt{24}}$

$= 0.138$

$\theta = \cos^{-1}(0.138)$

$= \underline{\underline{82.1^\circ}}$

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$

represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$

represents a circle centre (a, b) and radius r.

Scalar Product: $a.b = |a||b|\cos\theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\sin 2A = 2\sin A\cos A$$
$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Trigonometry Unit Assessment

1.(a) Express 4 cos x + 8 sin x in the form *k* sin (x – a)

where k > 0 and 0 ≤ a ≤ 360. Calculate the values of k and a.

(b) Express 3 cos x + 8 sin x in the form *k* sin (x – a)

where k > 0 and 0 ≤ a ≤ 360. Calculate the values of k and a.

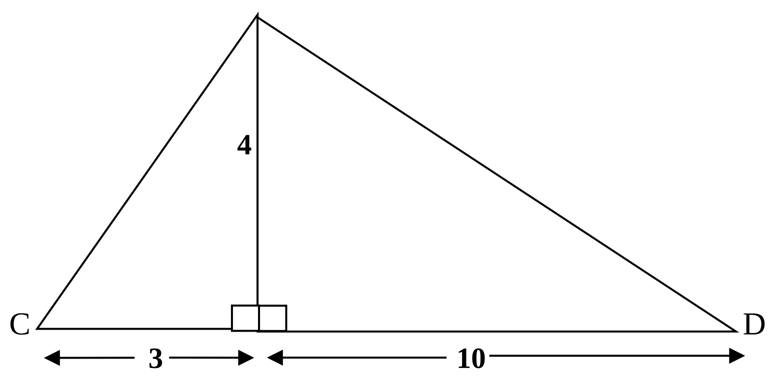
(c) Express 2 cos x + 6 sin x in the form *k* sin (x – a)

where k > 0 and 0 ≤ a ≤ 360. Calculate the values of k and a.

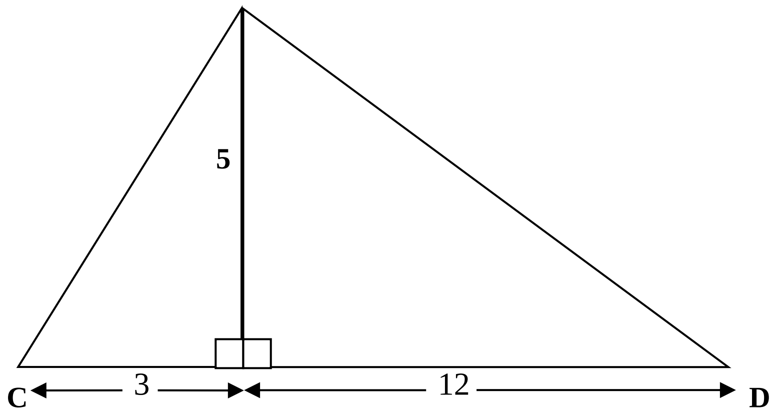
(d) Express 4 cos x + 7 sin x in the form *k* sin (x – a)

where k > 0 and 0 ≤ a ≤ 360. Calculate the values of k and a. [4]

2. (a) The diagram below shows two right-angled triangles.
Find the exact value of cos (C – D)



(b) The diagram below shows two right-angled triangles.
Find the exact value of cos (C + D)



3. **(a)** Show that $(7 - 4 \sin x)(7 + 4 \sin x) = 16 \cos^2 x + 33$

(b) Show that $(8 - 3 \sin x)(8 + 3 \sin x) = 9 \cos^2 x + 55$

(c) Show that $(3 - 2 \cos x)(3 + 2 \cos x) = 4 \sin^2 x + 5$

(d) Show that $(6 - 5 \sin x)(6 + 5 \sin x) = 25 \cos^2 x + 11$ [2,#2.1]

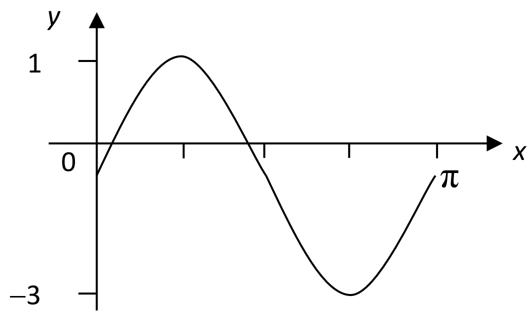
4. **(a)** Sketch the graph of $y = a \cos (x + \pi/3)$ for $a > 0$ and $0 \le x \le 2\pi$,
Show clearly the intercepts on the x-axis and the coordinates of the turning points.

(b) Sketch the graph of $y = b \sin (x + \pi/4)$ for $a > 0$ and $0 \le x \le 2\pi$,
Show clearly the intercepts on the x-axis and the coordinates of the turning points.

(c) Sketch the graph of $y = a \cos (x - \pi/3)$ for $a > 0$ and $0 \le x \le 2\pi$,
Show clearly the intercepts on the x-axis and the coordinates of the turning points.

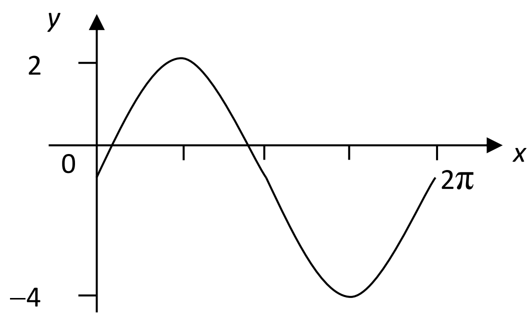
(d) Sketch the graph of $y = b \sin (x - \pi/4)$ for $a > 0$ and $0 \le x \le 2\pi$,
Show clearly the intercepts on the x-axis and the coordinates of the turning points.

5.(a) The diagram below shows the graph of $y = a \sin (bx) + c$



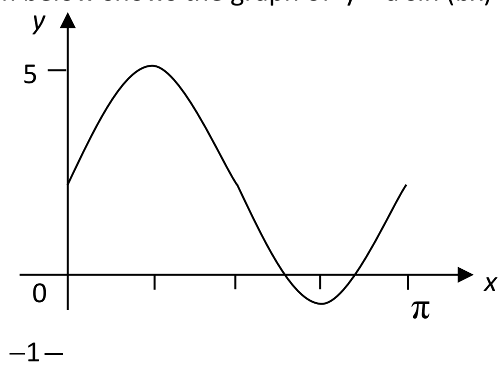
Write down the values of a, b and c.

(b) The diagram below shows the graph of $y = a \sin (bx) + c$



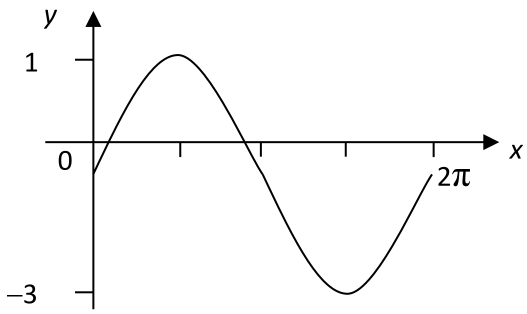
Write down the values of a, b and c.

5. (c) The diagram below shows the graph of y = a sin (bx) + c



Write down the values of a, b and c.

(d) The diagram below shows the graph of y = a sin (bx) + c



Write down the values of a, b and c.

[3]

6.a) Solve 2cos $2x = \sqrt{3}$, for 0≤ x ≤180

b) Solve 4sin $2x = 2$, for 0≤ x ≤180

c) Solve $\sqrt{2}$cos $2x = 1$, for 0≤ x ≤180

d) Solve 3sin $2x = 3$, for 0≤ x ≤180

7. a) Solve , $2\sin 2t - \sin t = 0$, for 0≤ t ≤180

b) Solve , $3sin2x + sinx = 0$, for 0≤ x ≤180

c) Solve , $4sin2\alpha - sin\alpha = 0$, *for* 0≤ α ≤180

d) Solve , $5sin2x - sinx = 0$, for 0≤ x ≤180

8.a) Given $\sqrt{3}cosx + sinx = 2\cos(x - 30)^{\circ}$,

solve $\sqrt{3}cosx + sinx = \sqrt{2}$, for 0≤ x ≤360

b) Given $4cos + 3sinx = 5\cos(x - 36.9)^{\circ}$,

solve $4cosx + 3sinx = 1.5$, for 0≤ x ≤360

c) Given $2sinx - 5cosx = \sqrt{29}\sin(\mathrm{x} - 68.2)^{\circ}$,

solve $2sinx - 5cosx = 2.5$, for 0≤ x ≤360

d) Given $2sinx + 2cosx = \sqrt{8}\cos(x - 45)^{\circ}$,

solve $2sinx + 2cosx = 2.7$, for 0≤ x ≤360

Answers

1. (a) $k = 4\sqrt{5}$, a = 333·4° (b) $k = \sqrt{58}$, a = 339·4°

(c) $k = 2\sqrt{10}$, a = 341·6° (d) $k = \sqrt{65}$, a = 330·3°

2. (a) $\frac{23}{5\sqrt{29}}$ (c) $\frac{11}{13\sqrt{34}}$

3. Solution is shown

4. Correct Max and Min, correct x intercepts and correct shape.

5. (a) a = 2, b = 2, c = -1 (b) a = 3, b = 1, c = -1

(c) a = 3, b = 2, c = 2 (d) a = 2, b = 1, c = -1

6. **a)** $x = 15^\circ \; and \; 165^\circ$ **b)** $x = 15^\circ \; and \; 75^\circ$

c) $x = 22.5^\circ and \; 157.5^\circ$ **d)** $x = 45^\circ$

7. **a)** $t = 0^\circ, 75.5^\circ, 180^\circ$ **b)** $t = 0^\circ, 99.6^\circ, 180^\circ$

c) $t = 0^\circ, 82.8^\circ, 180^\circ$ **d)** $t = 0^\circ, 84.3^\circ, 180^\circ$

8. **a)** $x = 75^\circ, x = 345^\circ$ **b)** $x = 109.4^\circ, x = 324.4^\circ$

c) $x = 95.5^\circ, 220.5^\circ$ **d)** $x = 62.3^\circ$

TRIG MIN. COMPETENCE SOLUTIONS

(1)(a) $4\cos x + 8\sin x = k\sin(x - a)$

$4\cos x + 8\sin x = k\sin x\cos a - k\cos x\sin a$

$-k\sin a = 4$ $\qquad$ $k\cos a = 8$

$k\sin a = -4$

$k^2 = (-4)^2 + 8^2$

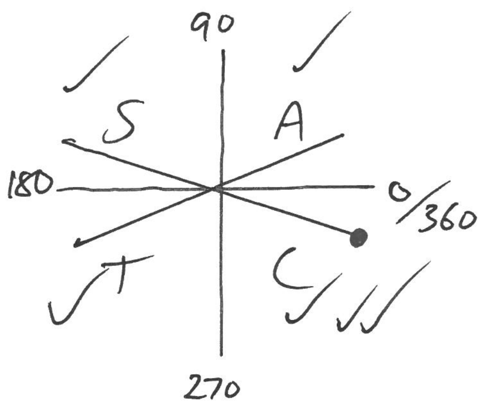
$= 16 + 64$

$= 80$

$k = \sqrt{80}$

$$\frac{k\sin a}{k\cos a} = \frac{-4}{8} = -\frac{1}{2}$$

$\tan a = -\frac{1}{2}$

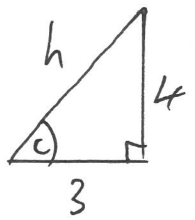


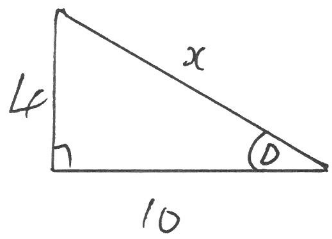
Related Acute Angle $\tan^{-1}\left(\frac{1}{2}\right)$
$= 26.6°$

$a = 360 - 26.6$

$= 333.4°$

② ⓐ $\cos(C-D) = \cos C \cos D + \sin C \sin D$





$h^2 = 3^2 + 4^2$
$= 25$
$h = \sqrt{25}$
$= 5$

$x^2 = 4^2 + 10^2$
$= 116$
$x = \sqrt{116}$

$\cos(C-D) = \left(\frac{3}{5} \times \frac{10}{\sqrt{116}}\right) + \left(\frac{4}{5} \times \frac{4}{\sqrt{116}}\right)$

$= \frac{30}{5\sqrt{116}} + \frac{16}{5\sqrt{116}}$

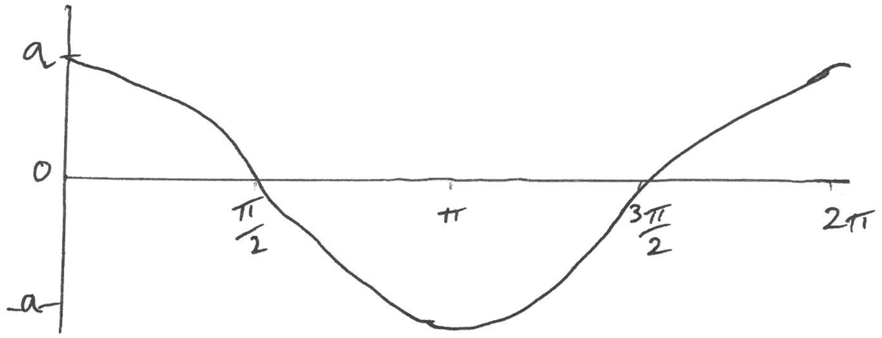
$= \frac{46}{5\sqrt{116}}$

$\sqrt{116} = \sqrt{4}\sqrt{29}$
$= 2\sqrt{29}$

$= \frac{46}{10\sqrt{29}} = \frac{23}{5\sqrt{29}}$

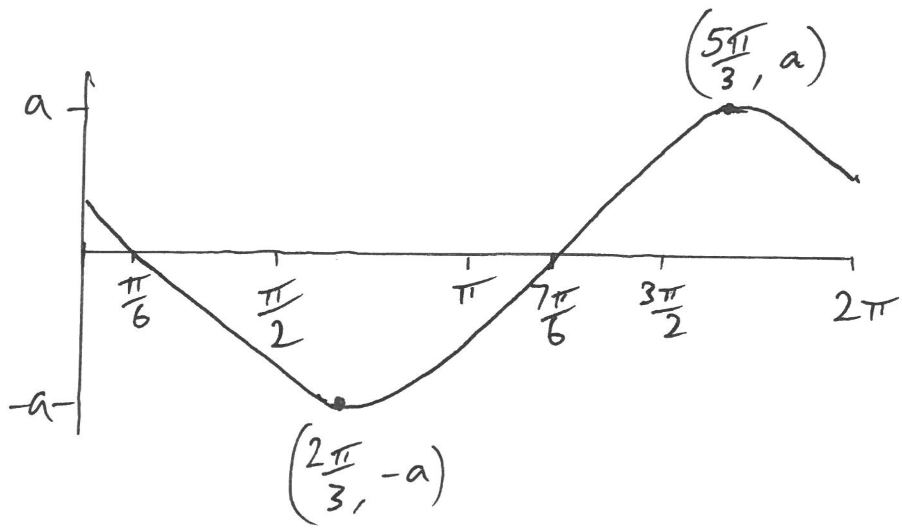
③ a) $LHS = (7 - 4\sin x)(7 + 4\sin x)$
$= 49 + 28\sin x - 28\sin x - 16\sin^2 x$
$= 49 - 16\sin^2 x$
$= 49 - 16(1 - \cos^2 x)$
$= 49 - 16 + 16\cos^2 x$
$= 16\cos^2 x + 33 = RHS$

4a)



$y = a\cos x$

$y = a\cos x$	$y = a\cos\left(x + \frac{\pi}{3}\right)$	
$(0, a)$	$\left(-\frac{\pi}{3}, a\right)$	$= \left(-\frac{\pi}{3}, a\right)$
$\left(\frac{\pi}{2}, 0\right)$	$\left(\frac{\pi}{2} - \frac{\pi}{3}, 0\right)$	$= \left(\frac{\pi}{6}, 0\right)$
$(\pi, -a)$	$\left(\pi - \frac{\pi}{3}, -a\right)$	$= \left(\frac{2\pi}{3}, -a\right)$
$\left(\frac{3\pi}{2}, 0\right)$	$\left(\frac{3\pi}{2} - \frac{\pi}{3}, 0\right)$	$= \left(\frac{7\pi}{6}, 0\right)$
$(2\pi, a)$	$\left(2\pi - \frac{\pi}{3}, a\right)$	$= \left(\frac{5\pi}{3}, a\right)$

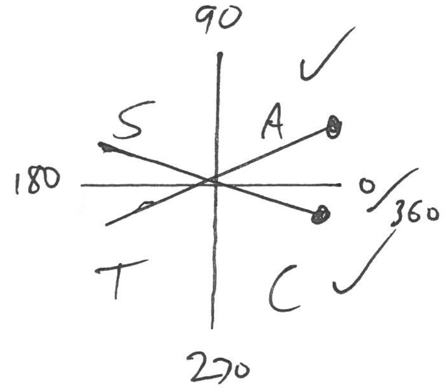


$y = a\cos\left(x + \frac{\pi}{3}\right)$

5 (a) $a = 2$, $b = 2$, $c = -1$

6 (a) $2\cos 2x = \sqrt{3}$

$\cos 2x = \frac{\sqrt{3}}{2}$



$2x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$2x = 30, 330$

$\underline{\underline{x = 15°, 165°}}$

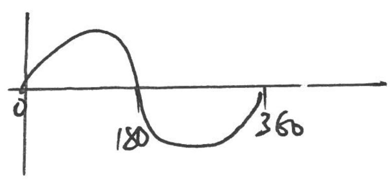
7 (a) $2\sin 2t - \sin t = 0$

$2(2\sin t \cos t) - \sin t = 0$

$4\sin t \cos t - \sin t = 0$

$\sin t(4\cos t - 1) = 0$

$\sin t = 0$

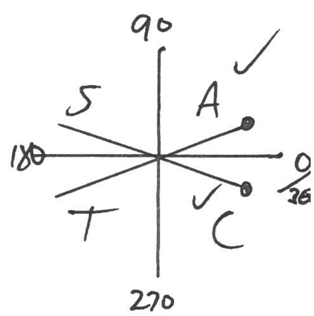


$\underline{\underline{t = 0, 180,}}$ ~~360~~

$4\cos t - 1 = 0$

$4\cos t = 1$

$\cos t = \frac{1}{4}$



$t = \cos^{-1}\left(\frac{1}{4}\right)$

$= 75.5°, 360 - 75.5$

$= \underline{\underline{75.5°, \text{~~284.5°~~}}}$

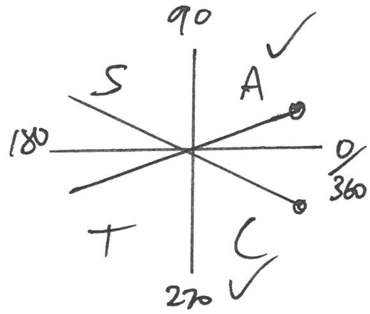
(8) (a)

$$\sqrt{3}\cos x + \sin x = \sqrt{2}$$

$$2\cos(x-30) = \sqrt{2}$$

$$\cos(x-30) = \frac{\sqrt{2}}{2}$$

$$(x-30) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$



$$= 45, \quad 360-45$$

$$= 45°, \ 315°$$

$$x = 75°, \ 345°$$