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Q1. a) $f(x) = (x+1)(x-2)^3$

$u = x+1 \quad v = (x-2)^3$
$u' = 1 \quad v' = 3(x-2)^2$
$u'v + uv'$

$$f'(x) = 1.(x-2)^3 + 3(x-2)^2(x+1)$$

$$= (x-2)^2\left[(x-2) + 3(x+1)\right]$$

$$= (x-2)^2\left[x-2+3x+3\right]$$

$$f'(x) = (x-2)^2(4x+1) = 0$$

$(x-2)^2 = 0 \qquad (4x+1) = 0$

$x - 2 = 0 \qquad 4x = -1$

$\underline{\underline{x = 2}} \qquad \underline{\underline{x = \frac{-1}{4}}}$

3

Q1. b) $\dfrac{x^2}{y} + x = y - 5$

$u = x^2 \quad v = y$
$u' = 2x \quad v' = \frac{dy}{dx}$
$\left(\frac{u'v - uv'}{v^2}\right)$

$$\frac{2xy - x^2\frac{dy}{dx}}{y^2} + 1 = \frac{dy}{dx}$$

$$2xy - x^2\frac{dy}{dx} + y^2 = y^2\frac{dy}{dx}$$

4

$$(2xy + y^2) = (x^2 + y^2)\,dy/dx$$

$$\therefore \quad \frac{dy}{dx} = \frac{(2xy + y^2)}{(x^2 + y^2)} = \frac{(2\times3\times-1) + (-1)^2}{((3)^2 + (-1)^2)}$$

At $(3,-1) \Rightarrow$ Gradient $m = \dfrac{-6+1}{9+1} = \dfrac{-5}{10} = \underline{\underline{\dfrac{-1}{2}}}$

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Q2. $A = \begin{pmatrix} t+4 & 3t \\ 3 & 5 \end{pmatrix}$

$$\begin{aligned} \text{Det } A &= 5(t+4) - 3(3t) \\ &= 5t + 20 - 9t \\ &= 20 - 4t \end{aligned}$$

$\therefore \text{ Det } A = 4(5-t)$

$$\boxed{A^{-1} = \frac{1}{\text{Det} A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$$

$$A^{-1} = \frac{1}{4(5-t)}\begin{pmatrix} 5 & -3t \\ -3 & t+4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5}{4(5-t)} & \frac{-3t}{4(5-t)} \\ \frac{-3}{4(5-t)} & \frac{(t+4)}{4(5-t)} \end{pmatrix}$$

3

b) Singular when Det $A = 0$

ie $20 - 4t = 0$ 1

$-4t = -20$

$t = 5$

c) $A^T = \begin{pmatrix} t+4 & 3 \\ 3t & 5 \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 6 & 5 \end{pmatrix}$

$\therefore \ t + 4 = 6$ or $3t = 6$

$t = 2$ $\quad$ $t = 2$ 1

Q3. $x^2 e^y \frac{dy}{dx} = 1$

$$\int e^y\,dy = \int \frac{1}{x^2}\,dx$$

$$\int e^y\,dy = \int x^{-2}\,dx \quad \bullet$$

$$e^y = \frac{x^{-1}}{-1} + C$$

$$e^y = \frac{-1}{x} + C \quad \bullet$$

$\left.\begin{matrix} y=0 \\ x=1 \end{matrix}\right\}$ $e^0 = \frac{-1}{1} + C$

$$1 = -1 + C$$

$$\therefore\ 2 = C \quad \bullet$$

Thus, $e^y = \frac{-1}{x} + 2$ (4)

$$\ln|e^y| = \ln|2 - 1/x|$$

$$y = \ln|2 - 1/x| \quad \bullet$$

$\left(\text{or} \quad y = \ln\left|\frac{2x-1}{x}\right| \right)$

Q4. Let $n=1$ $\frac{1}{1(1+1)} = 1 - \frac{1}{1+1}$

$$\frac{1}{1\times 2} = 1 - \frac{1}{2}$$

$$\frac{1}{2} = \tfrac{1}{2}$$ ✓ true for $n=1$ •

Assume true for $n=k$

$$\sum_{r=1}^{k} \frac{1}{r(r+1)} = 1 - \frac{1}{k+1}$$ •

Consider $n=k+1$

$$\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \sum_{r=1}^{k} \frac{1}{r(r+1)} + \frac{1}{(k+1)((k+1)+1)} = 1 - \frac{1}{k+1} + \frac{1}{(k+1)(k+2)}$$ •

$$= 1 + \frac{1}{(k+1)(k+2)} - \frac{1}{(k+1)} \cdot \frac{(k+2)}{(k+2)}$$

$$= 1 + \frac{1-k-2}{(k+1)(k+2)}$$

$$= 1 + \frac{-k-1}{(k+1)(k+2)}$$

(5)

$$= 1 + \frac{-(k+1)}{(k+1)(k+2)}$$

$$= 1 - \frac{1}{(k+2)}$$ •

$$= 1 - \frac{1}{(k+1)+1}$$ Let $(k+1)+1 = N$ to compare to original

$$= 1 - \frac{1}{N+1}$$ as required •

As true for $n=1$ & assumed true for $n=k$. By Proof of Mathematical Induction true for $n=k+1$, thus true $\forall n \in N$

Q5. $\displaystyle\int_{\ln 3/2}^{\ln 2} \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$

$= \displaystyle\int_{5/6}^{3/2} \frac{e^x+e^{-x}}{u} \cdot \frac{du}{e^x+e^{-x}}$

$= \displaystyle\int_{5/6}^{3/2} \frac{du}{u}$ •

$= \left[\ln|u|\right]_{5/6}^{3/2}$

$= \ln\left|\frac{3}{2}\right| - \ln\left|5/6\right|$

$= \ln\left|\frac{3}{2} \div \frac{5}{6}\right|$

$= \ln\left|\frac{3}{2} \times \frac{6}{5}\right|$

$= \ln\left|\frac{18}{10}\right|$ •

$= \underbrace{\ln\left|\frac{9}{5}\right|}$ as required

Let $u = e^x - e^{-x}$

$\frac{du}{dx} = e^x + e^{-x}$ •

$\frac{du}{e^x+e^{-x}} = dx$

If $x = \ln 2$

$u = e^x - e^{-x}$

$= e^{\ln 2} - e^{-\ln 2}$

$= e^{\ln 2} - e^{\ln 2^{-1}}$

$= 2 - 2^{-1}$

$= 2 - 1/2$

$= \underline{\underline{3/2}}$

•

If $\ln 3/2$

$u = e^x - e^{-x}$

$= e^{\ln 3/2} - e^{-\ln 3/2}$

$= e^{\ln \frac{3}{2}} - e^{\ln (\frac{3}{2})^{-1}}$

$= \frac{3}{2} - \left(\frac{3}{2}\right)^{-1}$

$= \frac{3}{2} - \frac{2}{3}$

$= \frac{9-4}{6}$

$= \underline{\underline{5/6}}$

(4)

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Q6.
$$\frac{(1+2i)^2}{7-i} = \frac{(1+2i)(1+2i)}{(7-i)} \times \frac{(7+i)}{(7+i)}$$

$$= \frac{(1+4i+4i^2)(7+i)}{(7-i)(7+i)}$$

$$= \frac{(1+4i-4)(7+i)}{(49+7i-7i-i^2)}$$

$$= \frac{(-3+4i)(7+i)}{49-(-1)}$$

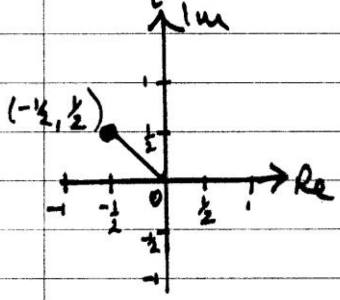
$$= \frac{-21-3i+28i+4i^2}{50}$$

$$= \frac{-21+25i-4}{50}$$

$$= \frac{-25+25i}{50}$$

$$= \underline{\underline{-\frac{1}{2}+\frac{1}{2}i}}$$

6



$$|z| = \sqrt{a^2+b^2}$$
$$= \sqrt{(-\tfrac{1}{2})^2+(\tfrac{1}{2})^2}$$
$$= \sqrt{\tfrac{1}{4}+\tfrac{1}{4}}$$
$$= \sqrt{\tfrac{1}{2}}$$
$$= \underline{\underline{\frac{1}{\sqrt{2}}}} \left(\text{or } \frac{\sqrt{2}}{2}\right)$$

$$\arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$$
$$= \tan^{-1}\left(\frac{\frac{1}{2}}{-\frac{1}{2}}\right)$$
$$= \tan^{-1}(-1)$$
$$= 135^\circ \text{ or } \frac{3\pi}{4}$$

Q7. $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\,dx$

$= \int_0^{\pi/4} \frac{4\sin^2\theta}{\sqrt{4-4\sin^2\theta}} \times 2\cos\theta\, d\theta$

$= \int_0^{\pi/4} \frac{8\sin^2\theta\cos\theta\, d\theta}{\sqrt{4(1-\sin^2\theta)}}$

$= \int_0^{\pi/4} \frac{8\sin^2\theta\cos\theta\, d\theta}{\sqrt{4}\sqrt{\cos^2\theta}}$

$= \int_0^{\pi/4} \frac{8\sin^2\theta\cos\theta\, d\theta}{2\cos\theta}$

$= 4\int_0^{\pi/4} \sin^2\theta\, d\theta$

$= 4\int_0^{\pi/4} \frac{1}{2}\left[1-\cos 2\theta\right] d\theta$

$= 2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/4}$

$= 2\left[\left(\frac{\pi}{4} - \frac{\sin \pi/2}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$

$= 2\left[\frac{\pi}{4} - \frac{1}{2} - 0\right]$

$= \frac{\pi}{2} - 1$

Let $x = 2\sin\theta$
$x^2 = 4\sin^2\theta$

$x = 2\sin\theta$
$\frac{dx}{d\theta} = 2\cos\theta$
$\therefore dx = 2\cos\theta\, d\theta$

If $x = 0$: $x = 2\sin\theta$
$2\sin\theta = 0$
$\therefore \theta = 0$

If $x = \sqrt{2}$: $x = 2\sin\theta$
$2\sin\theta = \sqrt{2}$
$\sin\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$
$\therefore \theta = \frac{\pi}{4}$

$\cos 2\theta = 1 - 2\sin^2\theta$
$2\sin^2\theta = 1 - \cos 2\theta$
$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

6

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$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \end{array}$$

Q8.

$$(1+x)^5 = \sum_{r=0}^{5} \binom{5}{r} (1)^{5-r} (x)^r$$

$$= \binom{5}{0}(1)^5(x)^0 + \binom{5}{1}(1)^4(x)^1 + \binom{5}{2}(1)^3(x)^2 + \binom{5}{3}(1)^2(x)^3 + \binom{5}{4}(1)^1(x)^4 + \binom{5}{5}(1)^0(x)^5$$

$$= (1\times1\times1) + (5\times1\times x) + (10\times1\times x^2) + (10\times1\times x^3) + (5\times1\times x^4) + (1\times1\times x^5)$$

$$= \underline{1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5}$$

1

$$(0.9)^5 = (1+(-0.1))^5 \qquad \underline{\text{ie } x = -0.1}$$

$$(1+(-0.1))^5 = 1 + 5(-0.1) + 10(-0.1)^2 + 10(-0.1)^3 + 5(-0.1)^4 + (-0.1)^5$$

$$= 1 - 0.5 + (10\times0.01) + (10\times-0.001) + (5\times0.0001) - 0.00001$$

$$= 1 - 0.5 + 0.1 - 0.01 + 0.0005 - 0.00001$$

$$= 1.1005 - 0.51001$$

$$= \underline{0.59049}$$

as req'd

2

$$\begin{array}{r} 1.10050 \\ -\,0.51001 \\ \hline 0.59049 \end{array}$$

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9

Q9. $\int_0^1 x\tan^{-1}(x^2)\,dx$

$\int u'v = uv - \int uv'$

Let $u = x^2/2$ $\quad v = \tan^{-1}(x^2)$

$u' = x$ $\quad v' = \frac{1}{1+(x^2)^2}.2x$

$= \frac{2x}{1+x^4}$

$= \left[\frac{x^2}{2}\tan^{-1}(x^2)\right]_0^1 - \int_0^1 \frac{x^2}{2}.\frac{2x\,dx}{(1+x^4)}$

$= \left[\frac{x^2}{2}\tan^{-1}(x^2)\right]_0^1 - \int_0^1 \frac{x^3\,dx}{(1+x^4)}$

$= \left[\frac{1}{2}\tan^{-1}(1) - 0\right] - \int_1^2 \frac{x^3}{u}.\frac{du}{4x^3}$

Let $u = 1+x^4$

$\frac{du}{dx} = 4x^3$

$dx = \frac{du}{4x^3}$

If $x = 0$

$u = 1+0^4 = 1$

If $x = 1$

$u = 1+1^4 = 2$

$= \frac{1}{2}\tan^{-1}(1) - \frac{1}{4}\int_1^2 \frac{du}{u}$

$= \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{4}\left[\ln|u|\right]_1^2$

$= \frac{\pi}{8} - \frac{1}{4}\left[\ln|2| - \ln|1|\right]$

$= \frac{\pi}{8} - \frac{1}{4}\ln|2|$

or $\frac{1}{8}(\pi - 2\ln|2|)$

5

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Q10. $14654 = 11 \cdot 1326 + 68$ •
$1326 = 19 \cdot 68 + 34$
$68 = 2 \cdot 34 + 0$

$\therefore \ \gcd(14654, 1326) = \underline{\underline{34}}$ •

$$\begin{aligned}
34 &= 1326 - 19 \cdot 68 \\
&= 1326 - 19 \cdot [14654 - 11 \cdot 1326] \\
&= 1326 - 19 \cdot 14654 + 209 \cdot 1326 \\
&= 210 \cdot 1326 - 19 \cdot 14654 \\
&= -19 \cdot 14654 + 210 \cdot 1326 \\
&= 14654a + 1326b
\end{aligned}$$ •

$\therefore \ a = -19 \ \& \ b = 210$ •

4

Q11. $y = x^{2x^2+1}$

$\ln|y| = \ln|x|^{2x^2+1}$

$\ln|y| = (2x^2+1)\ln|x|$

$u = 2x^2+1 \quad v = \ln x$
$u' = 4x \quad v' = \frac{1}{x}$

$\frac{1}{y}\frac{dy}{dx} = 4x\ln|x| + \frac{2x^2+1}{x}$

$\frac{dy}{dx} = \left(4x\ln|x| + 2x + \frac{1}{x}\right) \times y$

$\frac{dy}{dx} = \left(4x\ln|x| + 2x + \frac{1}{x}\right) \times x^{2x^2+1}$

At $x=1$ $\quad y = x^{2x^2+1}$

$= (1)^{2+1}$

$= 1$

$\frac{dy}{dx} = \left(4x\ln|x| + 2x + \frac{1}{x}\right) \times x^{2x^2+1}$

$= \left(4\ln|1| + 2 + \frac{1}{1}\right) \times 1^{(3)}$

$= (0 + 2 + 1) \times 1$

$= 3$

5

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12

Q 12. $a_1 = p \quad a_2 = p^2$

Common ratio $r = \frac{p^2}{p} = p$

& Initial value $a = p$

$$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$$

$$\therefore \; S_n = \frac{p(1-p^n)}{1-p} \quad \& \quad S_{2n} = \frac{p(1-p^{2n})}{1-p}$$

(2)

$$\boxed{S_{2n} = 65 S_n}$$

$$\frac{p(1-p^{2n})}{1-p} = \frac{65p(1-p^n)}{1-p}$$

$$(1-p^{2n}) = 65(1-p^n)$$

$$(1-(p^n)^2) = 65(1-p^n)$$

$$(1-p^n)(1+p^n) = 65(1-p^n)$$

$$1+p^n = 65$$

$$\therefore \; p^n = 64 \quad \text{as required}$$

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Q12 (Ctd...)

$\boxed{\text{Sequence: } p, p^2, p^3, p^4 \ldots}$

$\therefore \; a_3 = p^3$ but $a_3 = 2p$ also

$$\therefore \quad p^3 = 2p$$

$$p^2 = 2$$

$$p = \pm\sqrt{2}$$

As $p > 0$, $p = \sqrt{2}$ is only solution • 1

If $p = \sqrt{2}$

$$p^n = 64$$

$$(\sqrt{2})^n = 64$$

$$(2^{1/2})^n = 64$$

$$2^{n/2} = 64$$

$\boxed{\begin{aligned} &2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^6 \\ &= 64 \end{aligned}}$

$$\therefore \quad 2^{n/2} = 2^6$$

$$\frac{n}{2} = 6$$

$$\therefore \quad \underline{\underline{n = 12}}$$ • 1

Q13. (cont...)

$$f(x) = \frac{x^2+2x}{x^2-1}$$

$$x^2-1 \overline{\big) \, x^2 + 2x + 0} \quad \text{quotient } 1$$
$$\underline{x^2 \quad\quad -1}$$
$$\underline{\underline{2x + 1}}$$

$$f(x) = 1 + \frac{2x+1}{(x+1)(x-1)}$$

$\therefore$ 3 Asymptotes

2 Vertical Asymptotes at $x=1$ & $x=-1$ (2)

$x \to 1^+ \quad y \to \infty^+ \qquad x \to -1^+ \quad y \to \infty^+$

$x \to 1^- \quad y \to \infty^- \qquad x \to -1^- \quad y \to \infty^-$

$$f(x) = \frac{x^2+2x}{x^2-1} = \frac{1+2/x}{1-1/x^2}$$ $\div$ by x^2 and subst values to determine nature of asymptotes

Non-Vertical / Horizontal Asymptote at $y=1$ (1)

As $x \to +\infty$, $y \to 1^+$

$x \to -\infty$, $y \to 1^-$

$u = x^2+2x \quad v = x^2-1$

$u' = 2x+2 \quad v' = 2x$

$$f'(x) = \frac{(2x+2)(x^2-1) - 2x(x^2+2x)}{(x^2-1)^2}$$

$$= \frac{+2x^3 - 2x + 2x^2 - 2 - 2x^3 - 4x^2}{(x^2-1)^2}$$

$$= \frac{-2x^2 - 2x - 2}{(x^2-1)^2} < 0$$

$\therefore$ $f'(x) < 0$ for all x $\Rightarrow$ Strictly Decreasing

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Q13. (Ctd/..)

(i) Cuts x-axis at $y=0$

$$\frac{x^2+2x}{x^2-1} = 0$$

$$x^2+2x = 0$$

$$x(x+2) = 0$$

$$x=0 \qquad x=-2$$

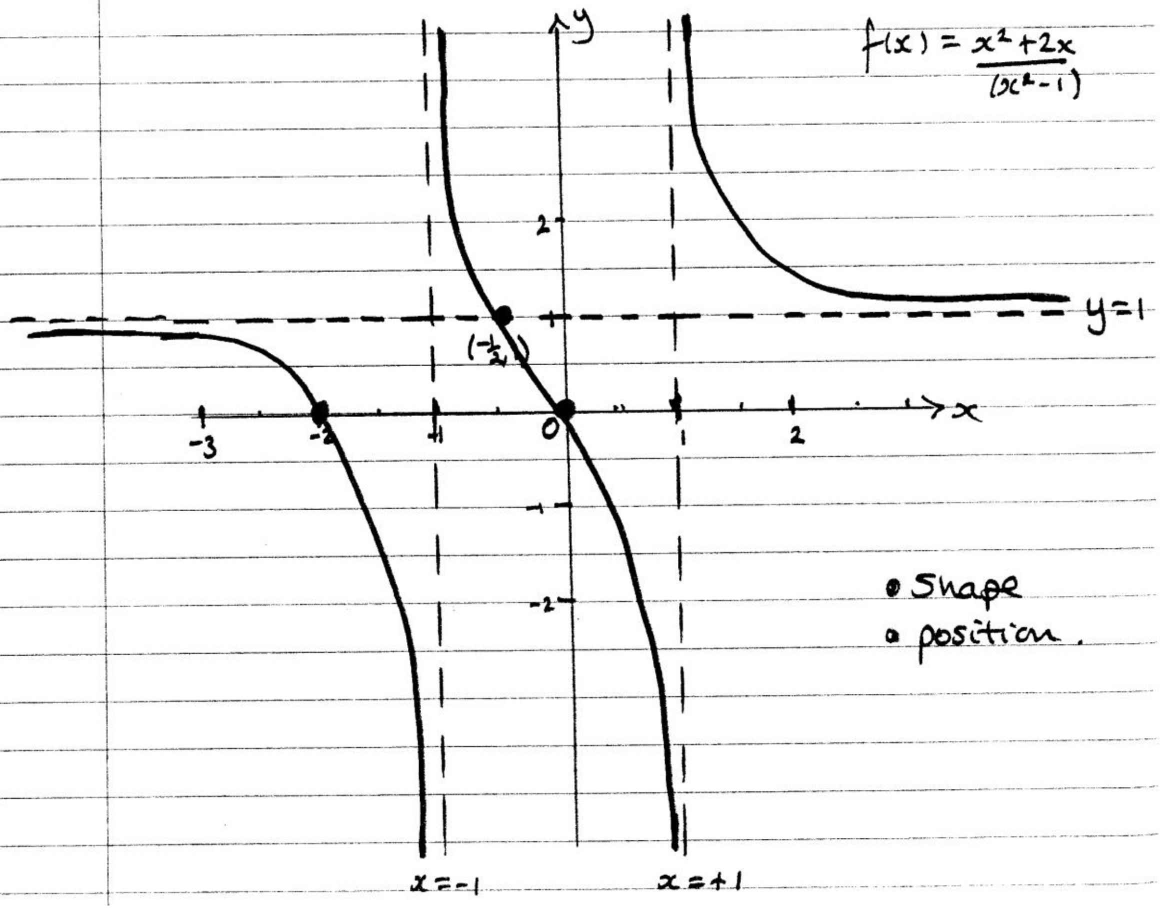
(ii) Cuts Horizontal Asymptote at $y=1$

$$\frac{x^2+2x}{x^2-1} = 1$$

$$x^2+2x = x^2-1$$

$$2x = -1$$

$$x = -\frac{1}{2}$$



- Shape
- position.

Q14. $$\frac{x^2+6x-4}{(x+2)^2(x-4)} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x-4)}$$

$$x^2+6x-4 = A(x+2)(x-4) + B(x-4) + C(x+2)^2$$

Let $x=-2$

$$(-2)^2+6(-2)-4 = 0 + B(-2-4) + 0$$
$$4 - 12 - 4 = -6B$$
$$-12 = -6B$$
$$\therefore B = 2$$

Let $x = 4$

$$4^2+6(4)-4 = 0+0+C(4+2)^2$$
$$16+24-4 = 36C$$
$$36 = 36C$$
$$\therefore C = 1$$

Let $x=0$

$$0+0-4 = A(2)(-4) + B(-4) + C(2)^2$$
$$-4 = -8A + (2)(-4) + (1)(4)$$
$$-4 = -8A - 8 + 4$$
$$-8A = 0$$
$$\therefore A = 0$$

(4)

$$\therefore \frac{x^2+6x-4}{(x+2)^2(x-4)} = \frac{2}{(x+2)^2} + \frac{1}{(x-4)}$$

Q14
(Cont..) $f(x) = \frac{2}{(x+2)^2} + \frac{1}{(x-4)} = 2(x+2)^{-2} + (x-4)^{-1}$

$f'(x) = -4(x+2)^{-3} - (x-4)^{-2} = \frac{-4}{(x+2)^3} - \frac{1}{(x-4)^2}$

$f''(x) = 12(x+2)^{-4} + 2(x-4)^{-3} = \frac{12}{(x+2)^4} + \frac{2}{(x-4)^3}$

$f'''(x) = -48(x+2)^{-5} - 6(x-4)^{-4}$, etc.

$f(0) = \frac{2}{(0+2)^2} + \frac{1}{(0-4)} = \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$ •

$f'(0) = \frac{-4}{(0+2)^3} - \frac{1}{(0-4)^2} = \frac{-4}{8} - \frac{1}{16} = \frac{-8-1}{16} = \frac{-9}{16}$ •

$f''(0) = \frac{12}{(0+2)^4} + \frac{2}{(0-4)^3} = \frac{12}{16} - \frac{2}{64} = \frac{24-1}{32} = \frac{23}{32}$ •

$$\boxed{f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + \ldots + f^n(0)\frac{x^n}{n!}}$$ •

$\therefore f(x) = \frac{x^2+6x-4}{(x+2)^2(x-4)} = \left(\frac{1}{4}\right) + \left(\frac{-9}{16}\right) \times \frac{x}{1!} + \left(\frac{23}{32}\right) \times \frac{x^2}{2!}$

$= \frac{1}{4} - \frac{9x}{16} + \frac{23x^2}{64} \ldots$ • (5)

are the first 3 non-zero terms.

Alt 2009

Q15 (b).

$y = (x+1)^4$ or $f(x) = (1+x)^4$

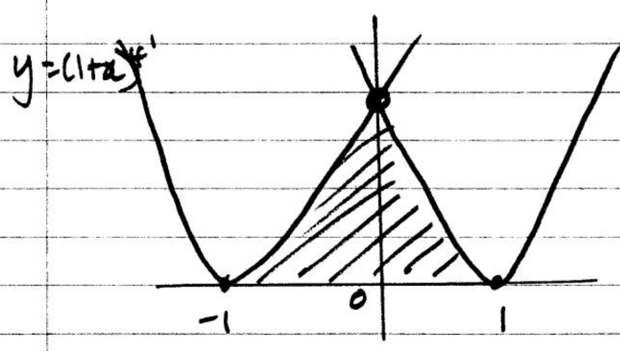
& $y = (1-x)^4$

$\underline{y=y}$ $(1+x)^4 = (1-x)^4$

$1+x = 1-x$

$2x = 0$

$\underline{\underline{x = 0}}$ $\therefore$ meet at $x=0$.



4

Symmetrical $\Rightarrow$ Can find $2\int_0^1 (1-x)^4 dx$

$A_1 = \int_0^1 (1-x)^4 dx$

$= \left[\frac{(1-x)^5}{5}\right]_0^1$

$= \left(0 - \frac{1}{5}\right)$

$= 1/5$ as Area >0

$\boxed{\begin{array}{l}\text{Thus area altogether} = 2A_1 \\ A = 2 \times \frac{1}{5} = \underline{\underline{\frac{2}{5}\text{ units}^2}}\end{array}}$

(20)

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Q16.
a)

$$x + y - z = 6$$
$$2x - 3y + 2z = 2$$
$$-5x + 2y - 4z = 1$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 2 & -3 & 2 & 2 \\ -5 & 2 & -4 & 1 \end{array}\right) \begin{array}{l} r_1 \\ r_2 \\ r_3 \end{array}$$

$$\begin{array}{l} \\ r_2 - 2r_1 \\ r_3 + 5r_1 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -5 & 4 & -10 \\ 0 & 7 & -9 & 31 \end{array}\right) \begin{array}{l} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{l} \\ 7r_2 \\ 5r_3 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -35 & 28 & -70 \\ 0 & 35 & -45 & 155 \end{array}\right) \begin{array}{l} r_1 \\ r_2 \\ r_3 \end{array}$$

$$\begin{array}{l} \\ \\ r_2 + r_3 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -35 & 28 & -70 \\ 0 & 0 & -17 & 85 \end{array}\right) \begin{array}{l} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{l} \\ \frac{1}{7}r_2 \\ \\ \end{array} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -5 & 4 & -10 \\ 0 & 0 & -17 & 85 \end{array}\right) \begin{array}{l} r_1 \\ r_2 \\ r_3 \end{array}$$

$$\therefore \quad -17z = 85$$
$$\underline{\underline{z = -5}}$$

If $z = -5$:
$$-5y + 4z = -10$$
$$-5y - 20 = -10$$
$$-5y = 10$$
$$\underline{\underline{y = -2}}$$

(5)

If $z = -5$ & $y = -2$:
$$x + y - z = 6$$
$$x - 2 + 5 = 6$$
$$\underline{\underline{x = 3}}$$

So $(3, -2, -5)$ is Solution

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Q16(b) $\pi_1: x + y - z = 6$
$\pi_2: 2x - 3y + 2z = 2$

Let $x = \lambda$

Then $\pi_1: \lambda + y - z = 6$
$\pi_2: 2\lambda - 3y + 2z = 2$

① ×2 $\quad 2\lambda + 2y - 2z = 12$
$2\lambda - 3y + 2z = 2$

Adding : $4\lambda - y = 14$

$\therefore y = 4\lambda - 14$

$3\pi_1: 3\lambda + 3y - 3z = 18$
$\pi_2: 2\lambda - 3y + 2z = 2$

Adding: $5\lambda - z = 20$
$\therefore z = 5\lambda - 20$

$\therefore x = \lambda$
$y = 4\lambda - 14$
$z = 5\lambda - 20$ as required

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Q16(c) Line L: $x = \lambda$

$y = 4\lambda - 14$

$z = 5\lambda - 20$

In Symmetric form:

$$\frac{x-0}{1} = \frac{y+14}{4} = \frac{z+20}{5} \; (=\lambda)$$

$\therefore \; \underset{\sim}{d} = \underline{i} + 4\underline{j} + 5\underline{k}$ *

$\underset{\sim}{\pi}: \; -5x + 2y - 4z = 1$

$\therefore \; \underset{\sim}{n} = -5\underline{i} + 2\underline{j} - 4\underline{k}$ *

$$\cos\theta = \frac{\underline{n} \cdot \underline{d}}{|\underline{n}||\underline{d}|} = \frac{\begin{pmatrix}-5\\2\\-4\end{pmatrix} \cdot \begin{pmatrix}1\\4\\5\end{pmatrix}}{\sqrt{25+4+16}\,\sqrt{1+16+25}}$$

$$= \frac{-5+8-20}{\sqrt{45}\,\sqrt{42}}$$

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$$= \frac{-17}{\sqrt{1890}}$$

$$\theta = \cos^{-1}\left(\frac{-17}{\sqrt{1890}}\right) = 113.02^\circ$$

Need acute angle $\therefore \; 180^\circ - 113.02^\circ = 66.98^\circ$

Angle between line & plane is Complement

$\therefore$ Angle $= 90^\circ - 66.98^\circ = \underline{\underline{23.02^\circ}}$